SCATTERED LIGHT

A lecture based on the paper by
V. Martínez Pillet, Solar Phys. 140 (1992), 207

In addition to the wavefront distortion in the Earth's atmosphere, *the light is scattered on a dust and water droplets in the air, and on dust particles on optical surfaces.*
This results in a bright circumsolar annulus – the **aureole**.

Scattered light affects the far wings of the PSF. This is hard to detect in frames taken on the disk → aureole observations are necessary.

Scattered light causes spurious enhancement of intensity in sunspots. Since the light is scattered on relatively long distances (comparable with $R_\odot$), in the first approximation it appears as an additive constant to the observed intensity.

Scattered light as a radiative transfer problem

Formal solution:

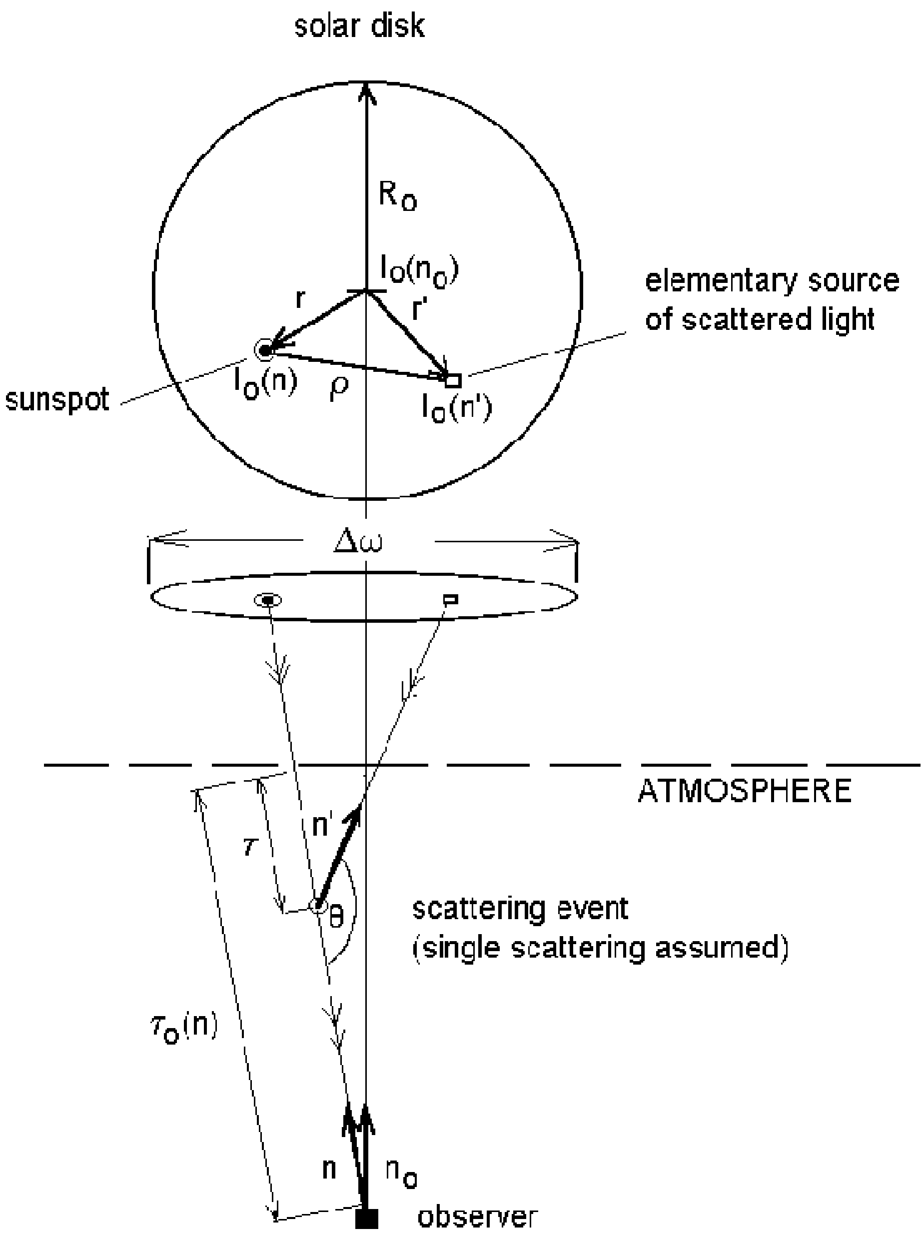
$$I(\tau_0, \mathbf{n}) = J_0(\mathbf{n}) \exp(-\tau_0(\mathbf{n})) + \int_0^{\tau_0(\mathbf{n})} S(\tau, \mathbf{n}) \, \exp(-(\tau_0 - \tau)) \, \mathrm{d}\tau \quad (1)$$

$J_0(\mathbf{n}) = I_0(\mathbf{n})$ = center-to-limb variation (CLV) inside $\Delta\omega$,
$J_0(\mathbf{n}) = 0$ outside $\Delta\omega$,
$\Delta\omega$ is the solid angle occupied by the sun.

Scattering source function:

$$S(\tau, \mathbf{n}) = \frac{1}{4\pi} \int_{\Delta\omega} \gamma \, \psi(\theta) \, I(\tau, \mathbf{n}') \, \mathrm{d}\omega \quad (2)$$

γ – albedo = fraction of light lost from the incident ray,
$\psi(\theta)$ – spread function (phase function),
$I(\tau, \mathbf{n}')$ – local intensity.



solar disk
R₀
I₀(n₀)
r
r'
elementary source
of scattered light
sunspot
I₀(n)
ρ
I₀(n')
Δω
ATMOSPHERE
n'
τ
θ
scattering event
(single scattering assumed)
τ₀(n)
n
n₀
observer

Expressing the local intensity by the "original" one $I_0(\mathbf{n}')$: $I(\tau,\mathbf{n}') = I_0(\mathbf{n}')\,\exp(-\tau(\mathbf{n}'))$, we have

$$S(\tau,\mathbf{n}) = \frac{1}{4\pi}\exp(-\tau(\mathbf{n}_0))\int_{\Delta\omega}\gamma\,\psi(\theta)\,I_0(\mathbf{n}')\,\underbrace{\exp(-(\tau(\mathbf{n}')-\tau(\mathbf{n}_0)))}_{\simeq 1}\,\mathrm{d}\omega \tag{3}$$

where $\exp(-\tau(\mathbf{n}_0))$ is the measure of attenuation of the incident ray.

Normalizing (3) to the "true" disk-center intensity $I_0(\mathbf{n}_0)$, we get for the source function

$$S(\tau,\mathbf{n}) = I_0(\mathbf{n}_0)\,\exp(-\tau(\mathbf{n}_0))\,\phi(\mathbf{n},\mathbf{n}_0) \tag{4}$$

$$\phi(\mathbf{n},\mathbf{n}_0) = \frac{1}{4\pi}\int_{\Delta\omega}\gamma\,\psi(\theta)\,\frac{I_0(\mathbf{n}')}{I_0(\mathbf{n}_0)}\,\mathrm{d}\omega \tag{5}$$

where $I_0(\mathbf{n}')/I_0(\mathbf{n}_0)$ is the CLV normalized to the disk center intensity.

Taking into account that $\tau_0(\mathbf{n}) \simeq \tau_0(\mathbf{n}_0)$, and that

$$\begin{aligned}\int_0^{\tau_0(\mathbf{n})} S(\tau,\mathbf{n})\,\exp(-(\tau_0-\tau))\,\mathrm{d}\tau &= \\ = I_0(\mathbf{n}_0)\,\phi(\mathbf{n},\mathbf{n}_0)\int_0^{\tau_0(\mathbf{n})}\exp(-\tau(\mathbf{n}_0)-(\tau_0-\tau(\mathbf{n}_0)))\,\mathrm{d}\tau &= \\ = I_0(\mathbf{n}_0)\,\phi(\mathbf{n},\mathbf{n}_0)\,\exp(-\tau_0)\,\tau_0 &\end{aligned}$$

we can express the formal solution (1) as

$$I(\tau_0,\mathbf{n}) = J_0(\mathbf{n})\,\exp(-\tau_0(\mathbf{n}_0)) + I_0(\mathbf{n}_0)\,\exp(-\tau_0(\mathbf{n}_0))\,\tau_0(\mathbf{n}_0)\,\phi(\mathbf{n},\mathbf{n}_0) \tag{6}$$

Re-writing (6) for the disk center, i.e. $J_0(\mathbf{n}) \to I_0(\mathbf{n}_0)$, and dividing both equations, we obtain the observed intensity normalized to the observed disk-center intensity:

$$\frac{I(\tau_0,\mathbf{n})}{I(\tau_0,\mathbf{n}_0)} = \frac{1}{1+\tau_0(\mathbf{n}_0)\,\phi(\mathbf{n},\mathbf{n}_0)}\left(\frac{J_0(\mathbf{n})}{I_0(\mathbf{n}_0)} + \tau_0(\mathbf{n}_0)\,\phi(\mathbf{n},\mathbf{n}_0)\right) \tag{7}$$

This is the basic equation for the scattered-light correction. $\tau_0(\mathbf{n}_0)\,\phi(\mathbf{n},\mathbf{n}_0)$ is the amount of scattered light.

To perform the correction, we must know the integral $\phi(\mathbf{n}, \mathbf{n}_0)$ over the normalized CLV (known) and over the spread function $\psi(\theta)$.

Approximation of the spread function

as a sum of 2–3 Gaussians and one Lorentzian:

$$\psi(\theta) = \sum_{i=1}^{n} m_i\, a_i\, \exp(-\rho^2/b_i^2) + M \frac{A}{B^2 + \rho^2} \tag{8}$$

m_i, M – weights, $\sum m_i + M = 1$
a_i, A – normalization constants (can be computed)
ρ – observed angular distance between the sunspot and the elementary source of scattered light.

Parameters b_i, B, weights m_i, M, and $\gamma\,\tau_0$ have to be obtained by fitting computed photometric profiles, derived from (7), (5), and (8), to the observed aureola in the range from the disk center to about 2' outside the limb.

Typical values of parameters b_i, B:

$b_1 = 40" - 90"$
$b_2 = 5" - 20"$
$b_3 = 1" - 2"$, practically not used (replaced by image deconvolution)
$B = 1"$ — Lorentzian wings are practically insensitive to the value of B. All variations are introduced by changing M.

Correction of images for scattered light

Cartesian coordinates x, y with the origin (0,0) at the disk center. Define

$$c = \frac{1}{1 + \tau_0(0,0)\,\phi(0,0)} \tag{9}$$

Then, from (7) we obtain for observed intensity at (x, y):

$$\frac{I(x,y)}{I(0,0)} = c\,\frac{I_0(x,y)}{I_0(0,0)} + \tilde{\phi}(x,y) \tag{10}$$

and, using (5),

$$\tilde{\phi}(x,y) = c\,\frac{\gamma\,\tau_0}{4\pi}\int_y\int_x \psi(\theta)\,\frac{I_0(x',y')}{I_0(0,0)}\,\mathrm{d}x'\mathrm{d}y' \tag{11}$$

– here we integrate over the region of interest.

A case of a **small pore**:

The scattered light has a purely photospheric origin. Then, the "true" intensity is

$$\frac{I_0(x,y)}{I_0(0,0)} = \frac{1}{c}\left(\frac{I(x,y)}{I(0,0)} - \tilde{\phi}(x,y)\right) \tag{12}$$

This equation can be solved *by iterations* for each pixel (x, y) in the image:
An estimated value for the photospheric intensity $I_0(x,y)/I_0(0,0)$ (CLV) is set into (11) to calculate $\tilde{\phi}$, then a new "true" intensity distribution $I_0(x,y)/I_0(0,0)$ is computed from (12); this is used to get a new $\tilde{\phi}$, etc., until the difference between the "true" intensity distributions calculated in subsequent steps is less than about 1 %.

A case of a **sunspot with a penumbra**:

The scattered light comes from the photosphere, from the penumbra, and in case of an extended sunspot also from the umbra. Thus, a sunspot introduces a deficit of radiation into the integral $\tilde{\phi}^{spot}$. Inside the sunspot, we can decompose this integral:

$$\tilde{\phi}^{spot}(x,y) = \tilde{\phi}(x,y) - \Delta\tilde{\phi}(x,y)$$

where

$$\Delta\tilde{\phi}(x,y) = c\,\frac{\gamma\,\tau_0}{4\pi}\int\int \psi(\theta)\,\frac{I_0(x',y') - I_0^{spot}(x',y')}{I_0(0,0)}\,\mathrm{d}x'\mathrm{d}y' \qquad (13)$$

– here we integrate over the sunspot area ($\Delta\tilde{\phi}(x,y) = 0$ outside the sunspot).

The "true" intensity distribution can be obtained by iterations of the following equation:

$$\frac{I_0^{spot}(x,y)}{I_0(0,0)} = \frac{1}{c}\left(\frac{I(x,y)}{I(0,0)} - (\tilde{\phi}(x,y) - \Delta\tilde{\phi}(x,y))\right) \qquad (14)$$

We see that the scattered light in sunspots is lower than in small pores.

The initialization values of $I_0(x,y)/I_0(0,0)$ for the iteration can be given as

$I_0^{penumb}/I_0^{phot} = 0.75$

$I_0^{umbra}/I_0^{phot} = 0.05$ for $\lambda = 520$ nm.

Simplifications for the central zone of solar disk

Near the disk center: $(x, y) \to (0, 0)$

$I_0(x, y) \to I_0(0, 0)$

$\phi(\mathbf{n}, \mathbf{n}_0) \to \phi(\mathbf{n}_0, \mathbf{n}_0) = c\,\tau_0\,\tilde{\phi}(0, 0)$

Then, (10) converts to

$$\underbrace{\frac{I(0,0)}{I(0,0)}}_{=1} = c\,\underbrace{\frac{I_0(0,0)}{I_0(0,0)}}_{=1} + \tilde{\phi}(0,0)$$

and

$$\tilde{\phi}(0,0) = 1 - c \tag{15}$$

$$c = \frac{1}{1 + \tau_0(0,0)\,\phi(0,0)}$$

For a **small pore** we have for each pixel approximately

$$\frac{I^{pore}}{I^{phot}} = c\,\frac{I_0^{pore}}{I_0^{phot}} + (1 - c) \tag{16}$$

For a **sunspot with a penumbra** the penumbral contribution of scattered light is smaller than that of the photosphere. An empirical approximation gives with an accuracy of 4 %

$$\frac{I^{spot}}{I^{phot}} = c\,\frac{I_0^{spot}}{I_0^{phot}} + \frac{1 - c}{2} \tag{17}$$

So, roughly, the scattered light over a sunspot or a pore in the central zone of solar disk can be considered as a constant.

Michal Sobotka, 1999