Geopotential Resolution in a degraded Repeat Orbit (R < 2L)

What happens to the resolution space when R < 2L? Evaluating the conditions for embedded extra frequencies in each wave during the Grace orbit for most of September, 2004 (R/D = 61/4) we find that most of them (see Table 1) contain 4 frequencies (m,k) resulting in a severe loss of precision compared to the ideal case though only for two sets of geopotential harmonics (of zonals and 61st order terms) is there an actual deficiency of observations to determine them formally (Table 2).

We tried three different inversion routines on the poorly conditioned normal matrices associated with the blocks of observations shown in Table 1. Even with the addition of a constraint of $10^{-5}/l^2$ for each geopotential coefficient (roughly the expected size of the term) the conditioning was still very poor for the great majority of terms (see the right panel of Figure 2). As the top curve of Figure 3 shows, the overall degradation in the degree variances of these solutions with respect to the 'ideal' ranges from more than two orders of magnitude for low degree to about one order of magnitude for high (where there is almost no improvement in the field errors over what is already assumed).

Table 1

Content of Block Matrices for 120x120 Recovery from a 61-rev/4-day Orbit

Summary: All Blocks :

waves	Observations (wave phases)	Freq-uencies	Geopotential Coefficients	Average String Length	Obs./Coeff.
7711	15421	29161	14637	77.548	1.054

Block #	Waves (Number in Block)	m's (Number in Block)	Obs. (Number in Block)	Geop. Coeffs. (Number in Block)	Avg. String Length	Obs./Coeffs.	orders (m) in block
1	125	2	249	239	102.944	1.042	0 61
2	249	3	498	478	73.723	1.042	1 62 60
3	253	4	506	480	73.498	1.054	2 63 59 120
4	253	4	506	480	73.466	1.054	3 64 58 119
5	253	4	506	480	73.893	1.054	4 65 57 118
....							
29	253	4	506	480	79.395	1.054	28 89 33 94
30	253	4	506	480	79.427	1.054	29 90 32 93
31	253	4	506	480	79.443	1.054	30 91 31 92

Table 2

Geopotential coefficients resolved in Block # 1

(sub block matrix:size- # observation equations)

k,l	Wave Phase: Cosine	Wave Phase: Sine
Odd	$C_{l,0} + S_{l,61}$ (89-62)	$C_{l,61}$ (30-62)
Even	$C_{l,61}$ (30-63)	$C_{l,0} + S_{l,6}$ (90-63)

Past and Future encounters of the Grace Satellites with critical repeat orbits.

Has Grace experienced reduced Geopotential precision in passages thru other critical repeat orbits and will it do so again in the future? Figure 6 shows the history of the altitude of the two subsatellites from launch in 2002 till late 2004 together with the dates of critical R,D resonances with respect to L=120 resolution. (Note none was as critical to monthly field determination as 61/4 in Sept. 2004 both because R was not as low and because the drag was stronger in the past, nearer the past high in the solar cycle in 2000 resulting in a more rapid pass thru the geostationary state).

To estimate the time-averaged semimajor axis in R,D repeat orbits we used the formula in Wagner, 1991 (accurate to about 100m):

$$a = a_0\{1 - C_{20}'(r_e/a_0)^2[4\cos^2(I) - (R/D)\cos(I) - 1]\}$$

Where

$$a_0 = \mu_e^{1/3}(\dot{\theta}_e R/D)^{-2/3}$$

and ' I ' is the mean inclination of the orbit and C_{20}' is here the unnormalized oblateness coefficient -1.08263×10^{-3}.

Finally in Figure 7 we show the full panoply of the Grace altitude in these critical (L=120) repeat orbits from launch to a minimum of 400km (based after December 2004 on a decline of 17m/day reached in the fall of 2004 which should be fairly accurate thru the minimum of the solar cycle in 2006-7). Note that only the R/D = 46/3 repeat will be as potentially severe as the Sept. 2004 event to monthly field resolution and since the time-frame for that will be near the solar high it's more rapid passage will ameliorate its severity.

Figure 6: Altitude of Grace A in 2002-4

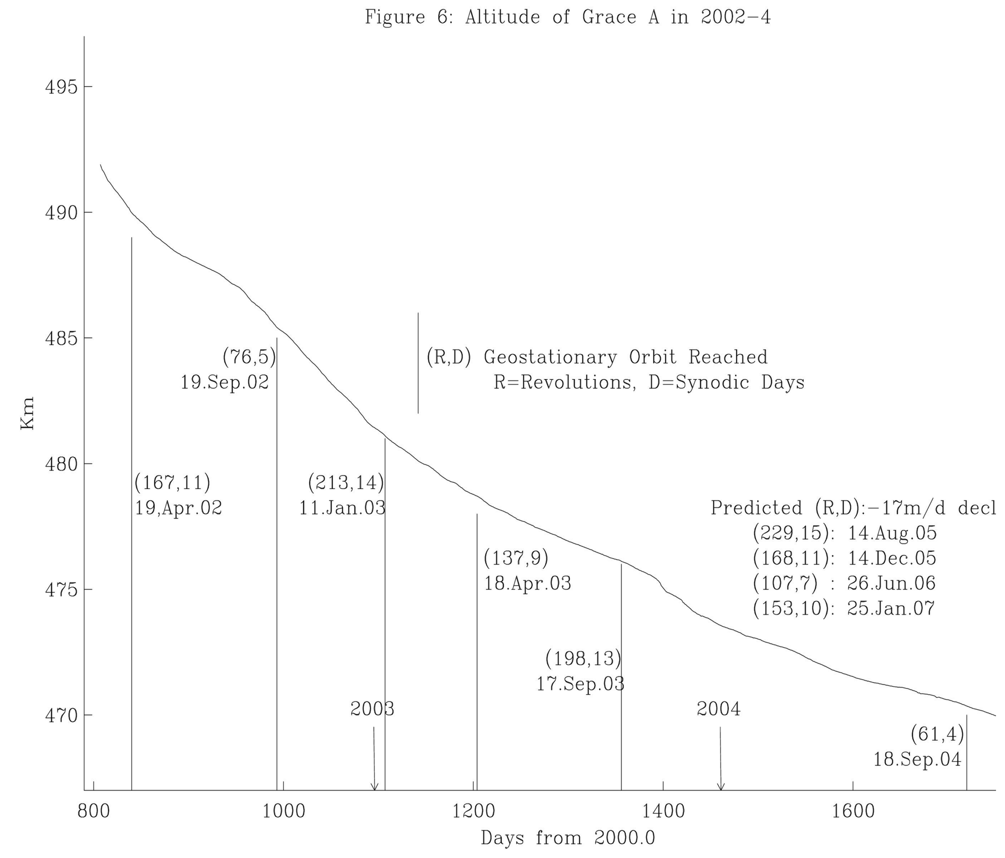


Fig. 5: Formal Errors for a Grace Monthly 120x120 Solution (3-04: UTCSR)

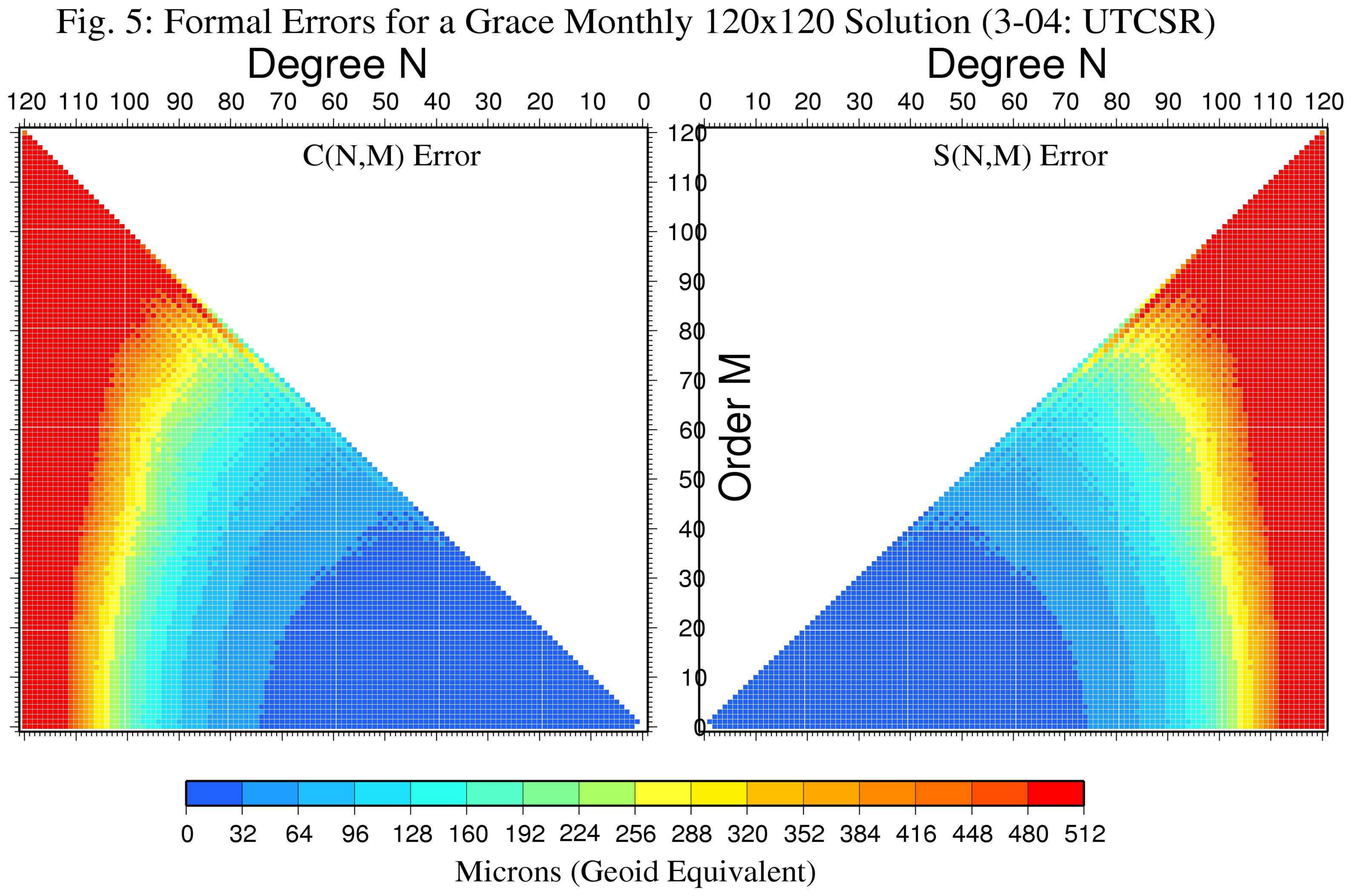
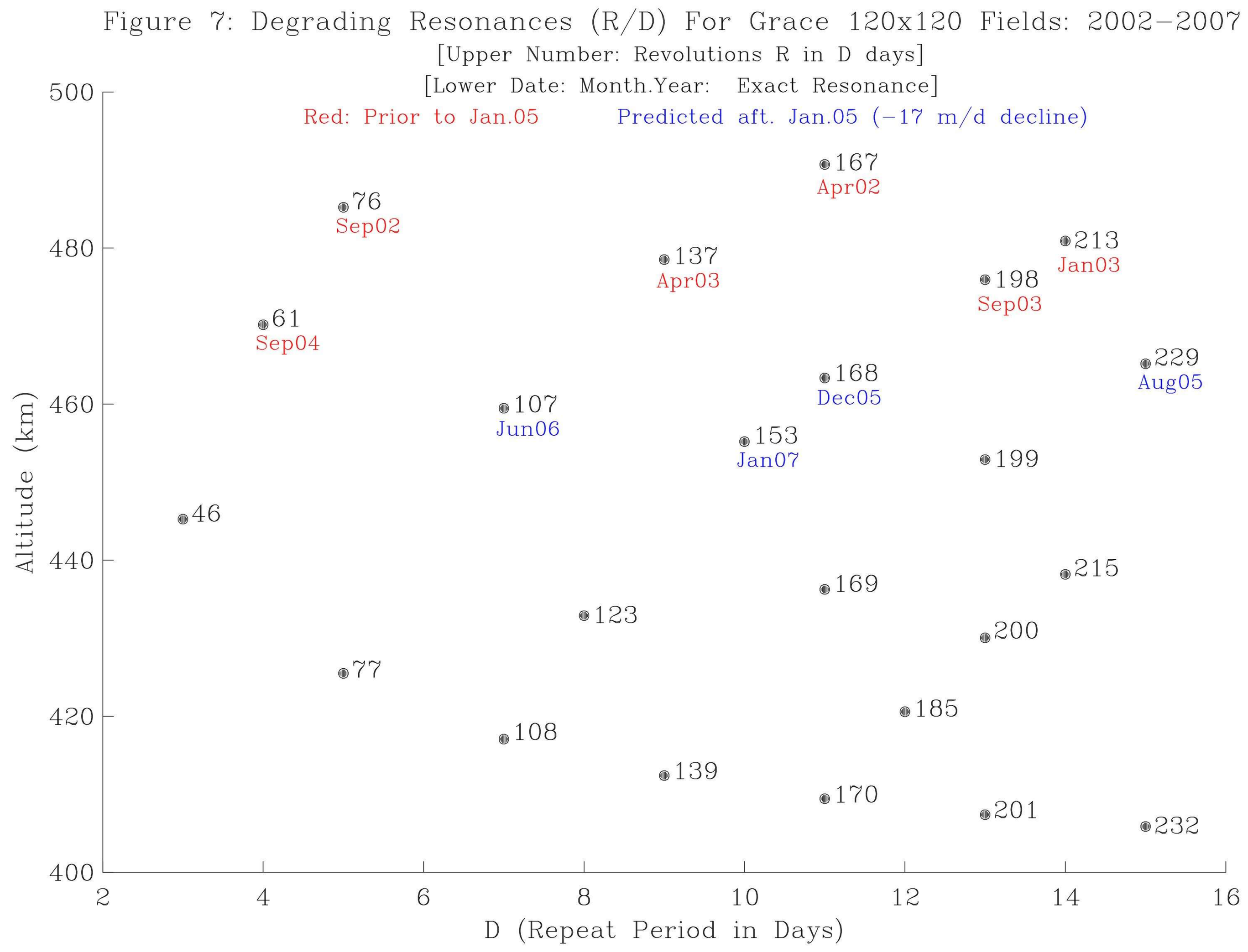


Figure 7: Degrading Resonances (R/D) For Grace 120x120 Fields: 2002-2007



Conclusions and Recommendations

The attempt to extract a complete high-degree Geopotential during an extended repeat-orbit period of insufficient coverage results in a degraded precision at all wavelengths. Monthly Grace fields presented prior to 2005 show this degradation for at least solutions in early to mid 2004. There is also evidence (not shown here) of similar degradation of the monthly solutions near previous repeat orbits. To avoid the reduced precision of the monthly fields in the future, the altitude of the Grace pair can be maneuvered lower to skip them or a reduced band limited solution LxL (L < R/2) accepted.

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