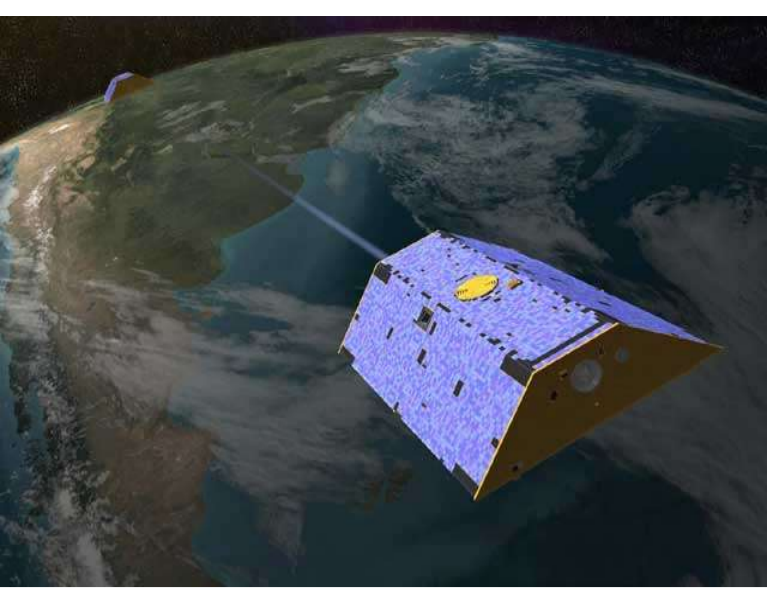


Degradation of Grace Monthly Geopotentials in 2004 Explained

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Summary:

In September 2004 the Grace orbit contracted to yield a repeat track of 61 revolutions every 4 days. As a result Geopotential information previously available to fully resolve a 120x120 field every month was compressed into about 1/4 of the observation space previously available. Ideal resolution in Sept. 2004 is estimated to be 30x30 and future orbit contraction must avoid repeat cycles with orbit revolutions less than 2*L to achieve full LxL resolution.

Geopotential

$$U_e = \mu_e / r \sum_{l=2}^L \sum_{m=0}^l (r_e / r)^l P_{lm}(\phi, \lambda) [C_{lm} \cos(m\lambda) + S_{lm} \sin(m\lambda)]$$

(for r, ϕ, λ radius, latitude and longitude; P,C,S fully normalized Associated Legendre functions and Spherical Harmonic Coefficients)

Geopotential Frequencies on a Near Circular orbit (Band Limited)

$$\dot{\alpha} = k(\dot{\omega} + \dot{M}) + m(\dot{\Omega} - \dot{\theta}_e)$$

(where $-L \leq k \leq L$, $0 < m \leq L$ and ω, M , are the orbit's arguments of perigee, mean and right ascension of the ascending node, and θ_e is the Greenwich hour angle)

Tandem, Low-Low Intersatellite Range Rate Signal for a near circular Polar Orbit (Linear Perturbations)

$$\Delta \dot{R} = \sum_{\alpha} [C_{\alpha} \cos(\dot{\alpha}t) + S_{\alpha} \sin(\dot{\alpha}t)]$$

$$C_{\alpha}, S_{\alpha} = \sum_l H_{lm}(S_{lm}, -C_{lm}) : l-m, \text{ even}$$

$$C_{\alpha}, S_{\alpha} = \sum_l H_{lm}(C_{lm}, S_{lm}) : l-m, \text{ odd}$$

$$H_{lmk} = \dot{M} a (r_e / a)^l F_{l, m, (l-k)/2}(\pi/2)$$

$$\{ \delta u \cos(-k \delta u / 2) [\beta(l+1) - 2k] (\beta^2 - 1)^{-1} + 2 \sin(-k \delta u / 2) [2(l+1 + 2k\beta) (\beta^2 - 1)^{-1} + (2k\beta^{-1})] \},$$

for $l \geq \max(m, |k|)$, with the same parity as k , where δu is the separation of the two subsatellites, $F(\pi/2)$ are fully normalized inclination functions, 'a' is the mean semimajor axis of their orbits, and for simplicity (with no loss of generality) the tracks start (at $t=0$) with their mean position at Greenwich on the equator. β , the normalized frequency of each wave, is $\dot{\alpha} / \dot{M}$

As long as the orbit has a non-repeating ground track the non-zonal frequencies will all be distinct and the signal as a whole will be a-periodic and continue to yield new information. When the ground track repeats there can be a significant loss of information as the number of distinct frequencies decreases (depending on the resolved band width attempted).

Repeat (Geostationary, Resonant) Orbits

The condition for a repeating trajectory (in geographic space), given a circular orbit, is that:

$$D(\dot{M} + \dot{\omega}) = R(\dot{\theta}_e - \dot{\Omega})$$

for R and D coprime integers (R/D irreducible). This condition results in a stationary ground track of R nodal revolutions in D synodic days (revolutions of Greenwich with respect to the satellite's node.) By comparison with the dominant frequencies $\alpha(m, k, q=0)$ it is also seen that in the same repeat cycle (of D synodic days) all these frequencies also repeat with each having a wave number with respect to that cycle time:

$$N = kR - mD$$

Now imagine the above signal generated over such a repeat cycle (D) by an LxL geopotential field and sampled at a certain data rate with Cosine and Sine phases of each wave recovered as observations. For simplicity we used a 'Kaula field' ($10^{-5}/l^2$) for each coefficient (beyond C_{20}) with the amplitude results shown in Figure 1. Because the geopotential is so attenuated at altitude the signal is severely restricted mainly to low order (m) effects of low frequency (k) but these are still numerous and easily seen by the 1 micron/sec Grace tracker. However they may not all be represented by distinct exclusive waves |N| in the repeat time D which we call 'ideal resolution'. What are its conditions?

Ideal and Degraded Geopotential Resolution in a Repeat Cycle

To assure 'ideal resolution' in a repeat cycle the |N| of all dominant frequencies must be distinct. Phrased in reverse, two different frequencies (m,k) and (m₁,k₁) have indistinguishable waves if either:

$$kR - mD = -(k_1 R - m_1 D) \quad \text{for } N = N_1 \text{ or}$$

$$kR - mD = k_1 R - m_1 D \quad \text{for } N = -N_1$$

The first condition is satisfied for:

$$k_1 = k + iD$$

And

$$m_1 = m + iR$$

with 'i' any non-zero integer as long as $0 \leq (m, m_1) \leq L$ and $(|k|, |k_1|) \leq L$, but which can never be satisfied as long as $R > L$. The second condition is satisfied for:

$$m_1 = iR - m$$

And

$$k_1 = iD - k$$

with 'i' any positive integer with the same band limitations as above, but now requiring $R > 2L$ to avoid duplicate waves. Thus to assure unique waves for all dominant geopotential frequencies in a repeat orbit (R,D) from a band-limited LxL field, ideal resolution requires that R must be greater than 2L.

Fig. 1: Grace Geopotential Signal from Kaula Field: Orbit: 243revs/16days (May 2002)

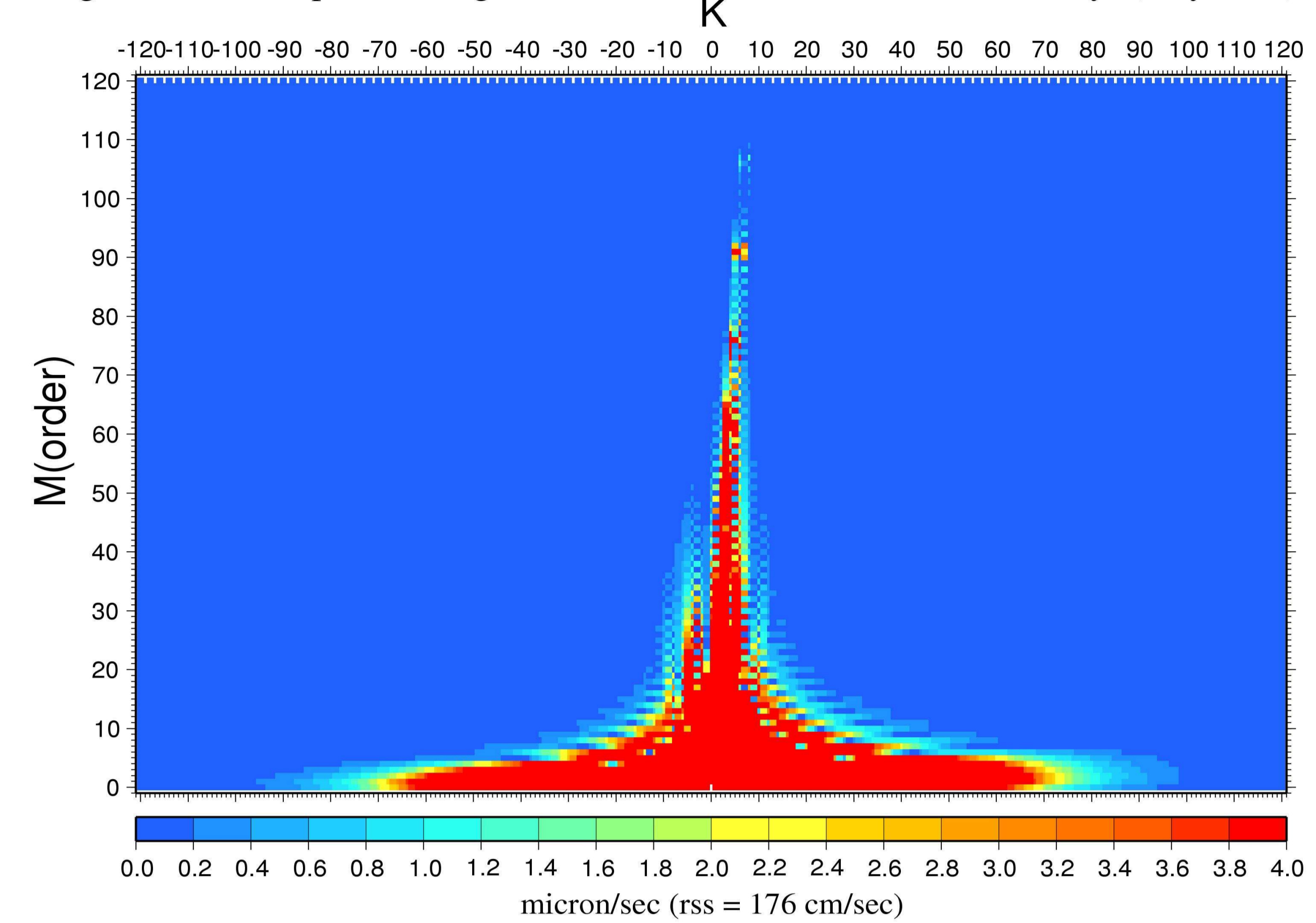


Fig. 2: Theoretical Geoid Precisions in Grace 120x120 Monthly Solutions

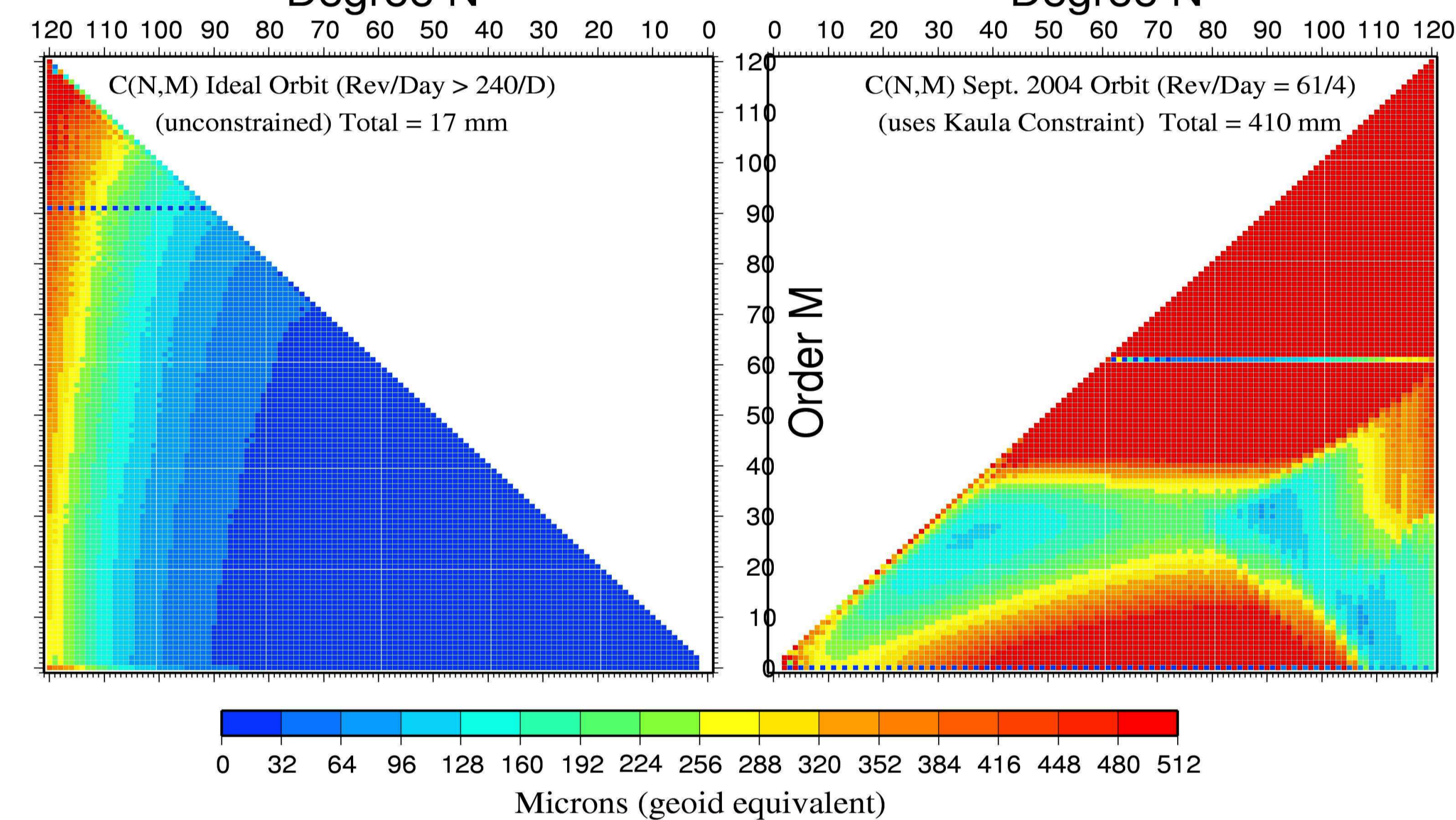
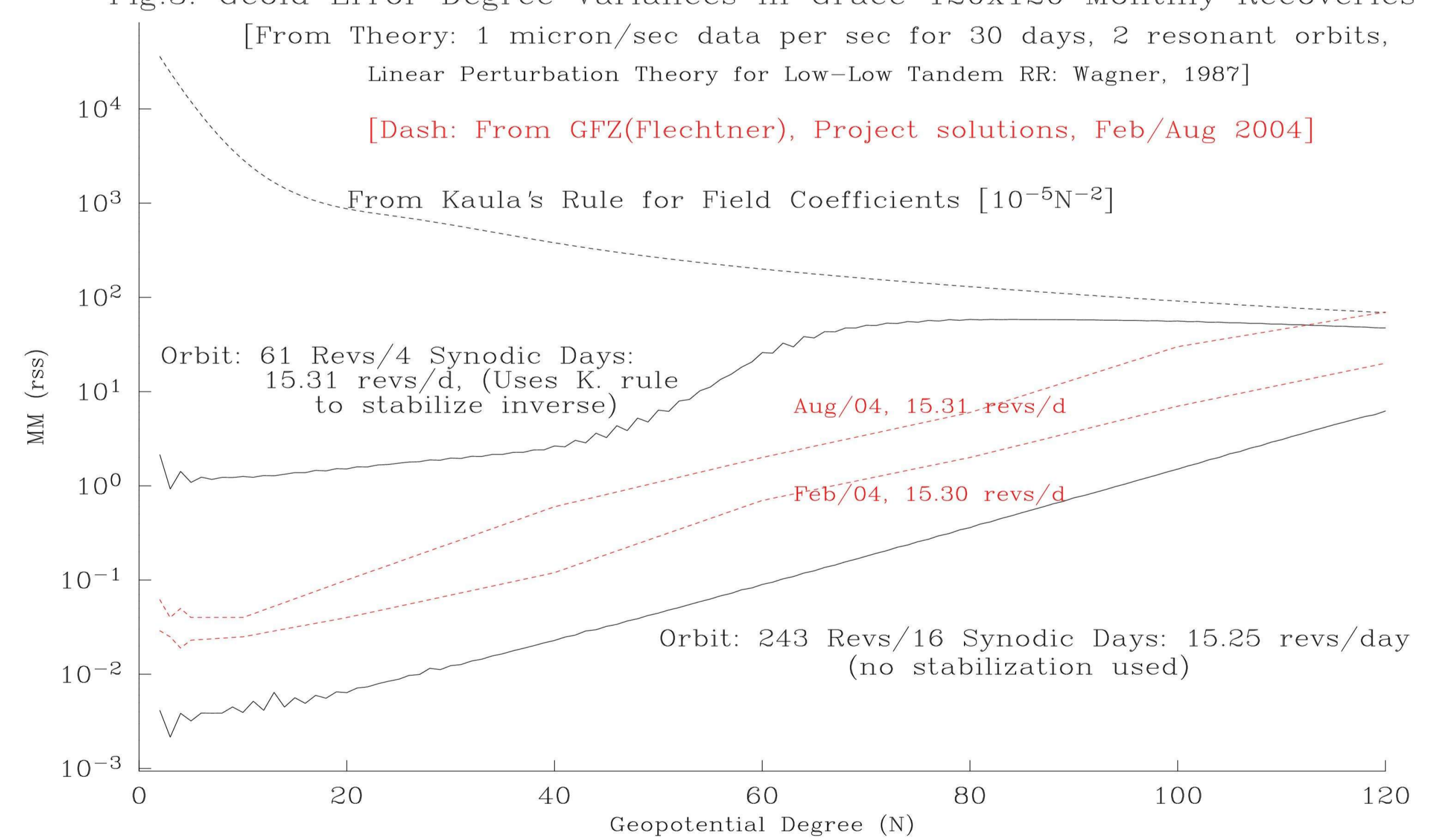


Fig.3: Geoid Error Degree Variances in Grace 120x120 Monthly Recoveries



Geopotential Resolution in an Ideal Repeat Orbit (R < 2L)

This case was first discussed by Colombo (1984). Summarizing: the observed wave phases C_{α}, S_{α} in D can be organized by order 'm' parity 'k' and kind C_{lm} or S_{lm} resolving all m,l (with the same parity as k) from these observations. Roughly, for a band limit of L the minimum size of the normal matrices from the observations (of the wave phases) will be only (L-m)/2 or at most 60 for L=120. Further, since the number of geopotential coefficients in an LxL field is about $(L+1)^2$ and there are $2(L+1)(2L+1)$ phase observations of them in the repeat cycle, for large L the ratio of observations to geopotential coefficients approaches 4/1. Figure 2 (left panel) and Figure 3 (bottom curve) illustrate the ideal geopotential variances obtainable in a near circular polar repeat orbit for a month near Grace's altitude in 2002 by inversion of these minimum block-diagonal matrices.

In Fig. 2 (left panel) note the general fall-off for each degree from the zonals (best determined) to the sectorials (worst) which follow from the polar orbit whose ground track is nearly north-south and so best suited to determining zonal harmonics. In Fig. 3 (bottom curve) we note its close similarity to Grace project solutions in early 2004 determined by far more complex data processing (see also Figure 5 where the more severe fall-off from zonals to sectorials in the project solution is due to further compromise of the sectorials by the sharing of low frequency information between the geopotential and the many empirical orbit and bias parameters of that solution.)