of the dependence $x(w)$ an additional condition fixing the parameterization. This can have the form of an algebraic equation

$$\omega(x, x') = 0 \ , \tag{B.14}$$

where ω is an arbitrary function. Spacetime variations δx^ι are then no more independent, but they must satisfy the linear equation

$$\delta\omega = \frac{\partial\omega}{\partial x}\delta x + \frac{\partial\omega}{\partial x'}\delta x' = 0 \ , \tag{B.15}$$

so that in (B.1) must be taken the variation of (B.10) with the condition (B.14). The Lagrangian variational principle for a parametric formulation of the motion thus reads

$$0 = \delta S = \delta \int f(x, x') + n(w)\omega(x, x')dw \ , \tag{B.16}$$

where n is the Lagrangian coefficient of the variational problem with a condition.

Similarly, the Hamiltonian formalism for a parametric description of the motion can be obtained from the action (B.10), the system of equations (B.12) and (B.13) for t' and x' does not have a unique solution (the condition from the footnote [1] on p. 50 is not satisfied for f given by (B.11)). The expression $p_\iota x'^\iota - f$ formally produced from f for x^ι analogously to the Hamiltonian (B.5) from L for x^i is identically equal to zero, which corresponds to the fact that the Hamiltonian made in this way, which can not be explicitely dependent on w, should be – in analogy with (B.7) – a trivial integral of motion. In view of (B.13) the integral in (B.6) can be expressed as $\int p_\iota x'^\iota dw$ (the Hamilton function for $x^\iota(w)$ would thus be identically equal to 0). However, p_ι are not independent because they must satisfy the condition (B.13). The Hamilton – Jacobi variational principle for parametric description of the motion must be thus in the form

$$0 = \delta \int (p_\iota x'^\iota - N(w)\mathcal{H}(x, p))dw \ , \tag{B.17}$$

where N again is the Lagrangian coefficient, and the so-called superhamiltonian

$$\mathcal{H}(x, p) = p_0 + H(t, x, p) \tag{B.18}$$

is the expression, which put equal to zero gives the condition (B.13). Taking the variation of (B.17) with respect to p_0 we get

$$0 = x'^0 - N(w) \ , \tag{B.19}$$

from where the physical meaning of $N(w)$ is obvious.

B.2 Fields

For a general spacetime field $\varphi_I = \varphi_I(x)$ (where the index "$_I$" denotes individual components of the field φ), the action S is the integral of the Lagrangian density $\mathcal{L} = \mathcal{L}(\varphi, \partial\varphi)$, i.e.,

$$S = \int \mathcal{L} d^4x \ , \tag{B.20}$$

hence its variation is

$$\begin{aligned} \delta S &= \int \delta\mathcal{L} d^4x = \int \sum_I \left(\frac{\partial\mathcal{L}}{\partial\varphi_I}\delta\varphi_I + \frac{\partial\mathcal{L}}{\partial\varphi_{I,\kappa}}\delta\varphi_{I,\kappa} \right) d^4x \\ &= \int \sum_I \left(\frac{\partial\mathcal{L}}{\partial\varphi_I} - \frac{\partial}{\partial x^\kappa}\frac{\partial\mathcal{L}}{\partial\varphi_{I,\kappa}} \right) \delta\varphi_I d^4x \ , \end{aligned} \tag{B.21}$$

and the field equations read

$$0 = \frac{\partial \mathcal{L}}{\partial \varphi_I} - \frac{\partial}{\partial x^\kappa}\left(\frac{\partial \mathcal{L}}{\partial \varphi_{I,\kappa}}\right) . \tag{B.22}$$

If the Lagrangian (density) does not depend explicitely on the coordinates, then its spacetime variation is given by the change of the field only

$$\frac{\partial \mathcal{L}}{\partial x^\iota} = \frac{\partial \mathcal{L}}{\partial \varphi}\varphi_{,\iota} + \frac{\partial \mathcal{L}}{\partial \varphi_{,\kappa}}\varphi_{,\kappa\iota} = \frac{\partial}{\partial x^\kappa}\left(\frac{\partial \mathcal{L}}{\partial \varphi_{,\kappa}}\varphi_{,\iota}\right) , \tag{B.23}$$

(The field index for which this expression should be summed is omitted for brevity). We can thus define the canonical stress-energy tensor

$$T_\iota{}^\kappa \equiv \frac{\partial \mathcal{L}}{\partial \varphi_{,\kappa}}\varphi_{,\iota} - \delta^\kappa_\iota \mathcal{L} + t_\iota{}^{[\kappa\lambda]}{}_{,\lambda} , \tag{B.24}$$

which satisfies the conservation law

$$T^{\iota\kappa}{}_{,\kappa} = 0 . \tag{B.25}$$

t is an arbitrary tensor with the marked antisymmetry due to which it cancels in the conservation law.

Index (English – Norwegian – Czech)

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