1.7 Evolution of inhomogeneities

The perturbations of homogeneity of the Universe observed in the cosmic background radiation are of relative order 10^{-5}, while at the present epoch they are highly non-linear, i.e., of an order > 1. To understand the history of their evolution we can first compare it with the collapse of a protostellar cloud. In a gas with density ρ and pressure P, any local contraction δr of a volume with radius r leads to an increase of its density and pressure, and the gradient of the pressure intends to smear out this perturbation. However, for a sufficiently large r, the potential energy of the self-gravitation of the volume released by this contraction (which is of the order $\delta(GM^2/r) \sim G\rho^2 r^4 \delta r$) can prevail over the work ($Pr^2\delta r$) needed to compress the gas and the contraction thus continues by gravitational collapse of volumes with radii larger than the *Jeans length*

$$r_{\mathrm{J}} \sim \sqrt{\frac{P}{G\rho^2}} \,. \tag{34}$$

In the epoch dominated by the radiation, $P \sim \rho c^2$ and the Jeans length was of the order of the horizon, hence all inhomogeneities on smaller scales were efficiently smoothed out by the radiation pressure. The highly non-linear fluctuations of density like the galaxies and stars could thus start to develop after the decoupling of the matter from the radiation only, when $P \simeq \frac{kT}{m}\rho$ and the Jeans mass $M_{\mathrm{J}} \sim \rho r^3 \sim (\frac{kT}{Gm})^{3/2}\rho^{-1/2}$.

Because the pressure was non-relativistic in the subsequent matter dominated epoch of the Universe evolution, the evolution of the inhomogeneities can be calculated as a perturbation of the Newtonian cosmological model, i.e., by solution of hydrodynamic equations (continuity of mass, momentum, and the Poisson eq.)

$$\dot\rho + \nabla(\rho v) = 0 \,, \tag{35}$$

$$\dot v + (v\nabla)v = -\nabla\Phi - \frac{1}{\rho}\nabla P \,, \tag{36}$$

$$\nabla^2\Phi = 4\pi G\rho \,. \tag{37}$$

Assuming their solution to be in the form of the Newtonian solution plus a small perturbation

$$\rho(t,r) = \rho_0 R^{-3}(1+\delta) \,, \tag{38}$$

$$v(t,r) = \frac{\dot R}{R} r + u \,, \tag{39}$$

$$\Phi(t,r) = \frac{2}{3}\pi G\rho_0 R^{-3} r^2 + \varphi \,, \tag{40}$$

we get for the perturbations linear equations

$$\dot\delta + \frac{\dot R}{R}(r\nabla)\delta + \nabla u = 0 \,, \tag{41}$$

$$\dot u + \frac{\dot R}{R}(r\nabla)u + \frac{\dot R}{R}u = -\nabla\varphi - \left[\frac{dP}{d\rho}\right]\nabla\delta \,, \tag{42}$$

$$\nabla^2\varphi = 4\pi G\rho_0 R^{-3}\delta \,. \tag{43}$$

If we express this perturbation in comoving coordinates[25] and perform the Fourier transform, i.e., we suppose $\delta(t,r) = \tilde\delta(t,k)\exp(i\frac{kr}{R})$, etc., we get the equations in the form

$$\dot{\tilde\delta} + \frac{i}{R}k\tilde u = 0 \,, \tag{44}$$

$$\dot{\tilde u} + \frac{\dot R}{R}\tilde u = -\frac{i}{R}k\tilde\varphi - \left[\frac{dP}{d\rho}\right]\frac{i}{R}k\tilde\delta \,, \tag{45}$$

$$-\frac{1}{R^2}k^2\tilde\varphi = 4\pi G\rho_0 R^{-3}\tilde\delta \,, \tag{46}$$

[25] It means that we transform from t, r to $\tau = t, \chi = r/R(t)$ so that $\partial_r = \frac{1}{R}\partial_\chi$ and $\partial_t = \partial_\tau - \frac{\dot R}{R} r\partial_\chi$.

from where we finally get the differential equation

$$\ddot{\tilde{\delta}} + 2\frac{\dot{R}}{R}\dot{\tilde{\delta}} + \left(\left[\frac{dP}{d\rho}\right]\frac{k^2}{R^2} - 4\pi G\rho_0 R^{-3}\right)\tilde{\delta} = 0 \tag{47}$$

for the density perturbation. It is obvious from this that the modes with wavelengths smaller than the Jeans one are dumped (because the term in the parentheses is positive and the time derivatives must thus be negative) and the modes with the longest wavelengths ($k = 0$) behave in time as

$$\delta \simeq \delta_+ t^{2/3} + \delta_- t^{-1} \ . \tag{48}$$

There are several hypotheses about the nature of the initial perturbations. In the case of the so-called *adiabatic perturbations* for which $\delta n_\gamma / n_\gamma = \delta n_B / n_B$ (so that the entropy per baryon is constant), the characteristic mass of inhomogeneities is determined by the dimension of the particle horizon at that time and it is of the order of 10^{12} up to $10^{15} M_\odot$ which corresponds to the mass of large clusters of galaxies. According to this scenario the large-scale structure arose first, and then gradually fragmented so that the galaxies appeared at $z \simeq 10$ or 3 only. According to the *isothermic scenario*, $\delta n_\gamma = 0$, while δn_B could form already at the radiation-dominated epoch on the scale of Jeans' gravitational instability of the baryon gas, i.e. $\simeq 10^5 M_\odot$, which corresponds to the globular clusters. Inhomogeneities of this size could exist already at $z \simeq 1000$, and the more massive systems (rather of a spherical shape) were gradually built from them later. The formation of the structure of the Universe could be influenced by the blast waves arisen by the explosions of the primordial supermassive stars.

It follows from the observation of the cosmic background radiation that the amplitude of adiabatic perturbations was small at $z \simeq 1000$, despite the opposite regions were not causally connected in a case of power-law form of $R(t)$. This observed homogeneity of the Universe is involved in the initial conditions in Friedmann models. However, it is a natural desire to explain it by some relaxation processes in the very early epochs of the evolution of any initially inhomogeneous universe. One such possibility is offered by the inflation epoch of the evolution of the Universe, which is expressed by Eq. (17). If R increases for many orders (it is supposed to be up to 10^{30}) in the course of the inflation epoch, then the real horizon is larger than the observed sphere corresponding to $z = 1000$ in the same ratio. In addition to the isotropy of the Universe, the inflationary models explain also its proximity to the critical model (i.e., $q = 0.5$) and some other problems resulting from the theories of grand unification (e.g. the problem of unobservable density of monopoles). This fine tuning of conditions which are satisfied by the observed Universe, despite they are not trivial from the point of view of physics, is necessary for the origin of stars, the nucleosynthesis in their interiors, and thus also for the rise of life. It can thus be explained also by the anthropic principle.