1.5 Observational cosmology

The contemporary physical theories admit a large variety of theoretical models of the Universe. It is thus necessary to verify them and to choose between them by comparison of the observed properties of the Universe with theoretical predictions of different models.

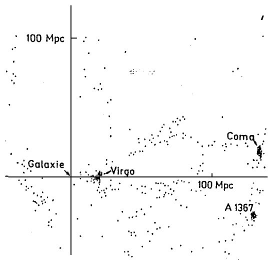

Figure 5: Large-scale structure of the Universe.

The basic observational test is the study of the distribution of discrete objects (e.g., galaxies and quasars). The statistics of the angular distribution (which is complicated due to partial screening by our Galaxy) as well as the distribution of objects at different distances r does not, generally, contradict the assumption of the homogeneity of the Universe. However, there are observed local condensations on different scales. Their existence imposes an additional constraint on the theoretical models. The largest formations presently known form the so-called *large-scale structure* resembling a foam with *large voids* with the size of the order of 100 Mpc – see Fig. 5. The luminous matter is concentrated into some walls (the so-called *pancakes*) between the voids and their cross-sections (threads) and their knots. These structures can be statistically described in terms of fractal dimensions

$$D = \frac{d\ln N}{d\ln r} = \frac{r}{N}\frac{dN}{dr} , \tag{18}$$

where r is the radius of a sphere around a chosen origin (the position of the Earth), and $N = N(r)$ is the number of the discrete sources contained in this sphere. Obviously, for the homogeneous distribution of sources in flat space $D = 3$, while in a planar structure $D = 2$, in a linear structure $D = 1$ and for an isolated condensation $D = 0$.[16]

The statistics of the distribution in r excludes the stationary models (both the static as well as the steady state). However, the non-stationarity of the Universe complicates the determination of the radial distances. For a relatively small neighbourhood of our observational site, the distance of a cosmic object can be found in the standard way of the Newtonian physics, e.g., from its angular size or its apparent brightness, provided that the proper size or intrinsic luminosity of the object are known (e.g., if they can be supposed to be equal in average for objects of the same kind – the objects are the so-called standard rods or standard candelas).[17]

At cosmological distances the relation of the magnitude of the observed quantities to the distance of the source is influenced by the geometry and dynamics of the Universe in a way which can be calculated depending on the cosmological model. It is obvious from Eq. (2) that the photons comming to the observer at $\chi = 0$ have $d\vartheta = 0 = d\varphi$ and hence the proper length D of a standard rod (oriented perpendicularly to the line of sight e.g. in direction of the coordinate ϑ) at event $\{t, \chi\}$ seen under the angle $\Delta\vartheta$ is

$$D = R(t)\Sigma(\chi)\Delta\vartheta , \tag{19}$$

[16] The results of most statistical studies indicate that D approaches the value of 3 at large scales, some observers argue that it is rather close to 2.

[17] In the static Euclidean space-time it would be the simple relation of inverse proportionality or inverse square dependence, respectively.

from where the distance can be found.[18]

In a simillar way the solid angle spanned by the source dicreases with the square of the angular distance. In addition, the energy of each photon radiated by the object is redshifted by the cosmological expansion by the factor $(1+z)$, as will be shown later, and in the same portion is increased the time-interval, during which the photons were emited. Consequently, the luminosity distance d_L defined by the ratio of the apparent luminosity L_{app} of the object and its intrinsic luminosity L_{int} (as seen from an unit distance) by $L_{app} = L_{int}/d_L^2$ is given by

$$d_L = d_A(1+z) = R(t)\Sigma(\chi)(1+z) \; . \tag{20}$$

Owing to the non-stationarity of the Universe there arises in practice a problem to find at different distances (and consequently in different ages of the Universe) mutually comparable sources and to exclude their evolutionary effects. More tricky physical processes must thus be used to set and verify the scale of radial distances (e.g. the gravitational lensing).

The most important confirmation of the dynamics of the Universe was done by Hubble's discovery of the redshift of spectral lines in the light from distant galaxies. It gives the evidence of the radial motions of objects with velocity v (approximately) proportional to their distance r, according Eq. (10) where the Hubble constant is determined by the function $R(t)$,

$$H = \frac{\dot{R}(t)}{R(t)} \; , \tag{21}$$

and it has observationally found value $H \simeq 75$km s^{-1} Mpc^{-1}. It means that, in fact, it is also a function of time. The redshift z of the wavelengths of spectral lines,

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} \; , \tag{22}$$

where $\lambda_{\rm em}$ and $\lambda_{\rm obs}$ are the emitted and observed wavelengths, resp., is then the result of subsequent Doppler increase of the wavelength of a photon during its travel from the source to the observer. The final value of the redshift depends on the ratio of how much the radius R of the Universe has increased in the course of the travel time of the photon,[19]

$$1 + z = \frac{R(t_{\rm obs})}{R(t_{\rm em})} \; . \tag{23}$$

The redshift z is used as a measure of the distance (and thus also the measure of the 'antiquity') of the distant sources. Its relation to the real actual distance r or the radial coordinate χ is determined by the function $R(t)$,

$$r = R(t_{\rm obs})\chi = R(t_{\rm obs}) \int_{t_{\rm em}}^{t_{\rm obs}} R^{-1}(t) c dt \; . \tag{24}$$

The actual velocity Hr of the recession (which, however, does not have any direct physical meaning) can be greater than c for objects with large z (e.g., in the Einstein – de Sitter universe for $z > 3$), and cannot be calculated from z by the special-relativistic expression for the Doppler effect.

The limit $r_{\rm h}$ of the expression (24) for $t_{\rm em} \to 0$, i.e. $z \to \infty$ is called *particle horizon.* This defines the most distant point $(\chi = \chi_{\rm h})$ from which any information could reach us (i.e. to reach the event $\chi = 0, t = t_{\rm obs}$) in the course of time from the beginning $t = 0$ of the Universe. The horizon $r_{\rm h}$

[18] Note that the so-called angular distance $d_A \equiv D/\Delta\vartheta = R(t)\Sigma(\chi) = R(t)\rho$, where *rho* is the radial coordinate used in the expression (7) for the metric, gives the circumference of a circle subtending the rod at the epoch of emission of the observed photons. The present size of the circle is rescaled due to the expansion factor $R(t)$. Moreover, the distance measured radially differs from d_A due to the curvature of the space (e.g. for $k = +1$ and $\chi > \pi/2$ the angular size of the rod is increasing with χ, what can be understood as a focussing of the radiation by the curvature of space).

[19] This can be easily seen from the equation $d\eta^2 = d\chi^2$ of a radially moving photon, where η is the so-called conformal time, in which Eq. (2) has the form $ds^2 = R^2(\eta)[d\eta^2 - d\chi^2 - \Sigma^2(\chi)(d\vartheta^2 + \sin^2\vartheta d\varphi^2)]$. Two world-lines of photons (or maxima of an electromagnetic wave) have constant diference in η and thus diference in time $\Delta t \sim R(t)$.

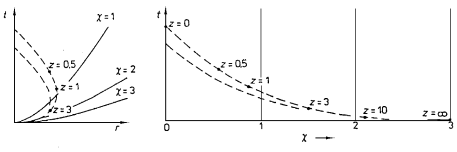

Figure 6: Past light-cone in physical and co-moving coordinates.

increases with the increasing time $t_{\rm obs}$. However, in some models exists a finite limit $\lim_{t_{\rm obs}\to\infty} r_{\rm h}$ (the so-called *event horizon*), hence the more distant regions of the Universe are causally separated forever. E.g. for a power-law expansion $R(t) = kt^{\kappa}$ (i.e. for both the matter- and radiation-dominated universe the comoving coordinate of the horizon reads[20]

$$\chi_{\rm h} = \frac{c}{k}\int t^{-\kappa}dt = \frac{ct}{(1-\kappa)R} \; , \tag{25}$$

while during an exponential expansion $R(t) = k\exp(Ht)$ between the epochs t_1 and t_2 the horizon increases for[21]

$$\Delta\chi_{\rm h} = \frac{c}{k}\int \exp(-Ht)dt = \frac{c}{H}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \; . \tag{26}$$

Consequently, if $R_2 \gg R_1$, the final horizon after an initial power-law and subsequent exponential expansion

$$\chi_{\rm h} = \frac{ct_2}{(1-\kappa)R_2}\left[\frac{R_2t_1}{R_1t_2} + \frac{1-\kappa}{\log(R_2/R_1)}\left(1-\frac{t_1}{t_2}\right)\left(\frac{R_2}{R_1}-1\right)\right] \; , \tag{27}$$

is much larger than it would be for the power-law only.

In principle, the deceleration parameter q, which is in agreement with Eq. (12) expressed in terms of $R(t)$ as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} \; , \tag{28}$$

can be determined by measuring the non-linearities of the Hubble-relation $z = z(r)$. Neglecting in Eq. (5) the terms with P and Λ (which apparently really are negligible at the present epoch of the evolution of the Universe) we arrive at the relation

$$kc^2 = R^2H^2(2q-1) \; . \tag{29}$$

The geometry of the whole Universe can thus be determined from the value of q. The critical value ($k = 0$) of the curvature corresponds to $q_{\rm crit} = \frac{1}{2}$. The observed value of q coincides – within quite large observational errors – with $q_{\rm crit}$.

Similarly, according to Eq. (4), k could be determined from the present mean density of the Universe comparing it with the critical value

$$\rho_{\rm crit} = \frac{3H^2}{8\pi G} \simeq 10^{-26}{\rm kg.m}^{-3} \; . \tag{30}$$

The observed luminous matter of the galaxies gives a density approximately one or two orders of magnitude lower. However, it follows from the inner dynamics of galaxies as well as from the dynamics

[20]The horizon $R\chi$ is thus of the order of Gpc.

[21]Note that in the case of exponential expansion lasting forever the above defined horizon as the limit of the past light-cone is at infinity, because the model was asymptotically static. However, the distance c/H defines the most distant observers which can be reached by a future light-cone. It means that any two particles will be causally disconnected after they separate behind this limit.

of clusters of galaxies that there must be present also a great deal of the so called *dark matter* (or the hidden matter) which holds the galaxies bounded by its gravity. Up to now, the substance of this dark matter is unknown. It could consist of neutrinos if their rest mass would be of the order of a few eV or it could consist of some still unknown particles. From the point of view of observability of this matter can not be excluded some forms of non-luminous bodies from the normal baryonic matter either. The estimates of the total amount of the dark matter in the Universe are so uncertain, that we cannot determine the value of k.

The value of the deceleration parameter can be restricted by comparison of the observed age t of the Universe with the Friedmann models according to which it should be

$$t = \frac{1}{(2q-1)H}\left[\frac{q}{\sqrt{|2q-1|}} f\left(\frac{1-q}{q}\right) - 1\right], \quad \text{where } f(x) = \begin{cases} \operatorname{arccos} x & \text{for } x \leq 1 \\ \operatorname{arccosh} x & \text{for } x > 1 \end{cases}. \tag{31}$$

In the case $q = \frac{1}{2}$, i.e. $k = 0$, $t = \frac{2}{3}H^{-1} \simeq 8.7 \times 10^9$years (for $H = 75$km s^{-1} Mpc^{-1}), for $q > \frac{1}{2}$ the age t is even smaller. Following the radiometric chronology of meteorites and Earth and Moon rocks the lower limit of the age of the Solar system is about 4 or 5×10^9 years. Comparing the theoretical models of the stellar evolution with the observed HR-diagrams the age of some stellar clusters can be estimated to be up to 10 or 15×10^9years. It is an evidence in favour of either $k = -1$ or a smaller value of H.

Exercise 6 *Find the mean fractal dimension of the 3-dimensional Cantor discontinuum (cf. Fig 1).*

Exercise 7 *Find the distance to SMC by comparing the observed periods P (in days) and the magnitudes m of cepheids in the SMC, given in the following table:*

star	*P*	*m*	*star*	*P*	*m*	*star*	*P*	*m*
HV 2019	*1.62*	*16.8*	*HV 1825*	*4.27*	*15.6*	*HV 847*	*27.5*	*13.8*
HV 2035	*2.00*	*16.7*	*HV 1903*	*5.13*	*15.6*	*HV 840*	*33.1*	*13.4*
HV 844	*2.24*	*16.3*	*HV 1945*	*6.46*	*15.2*	*HV 11182*	*39.8*	*13.6*
HV 2046	*2.57*	*16.0*	*HV 2060*	*10.2*	*14.3*	*HV 1837*	*42.7*	*13.1*
HV 1809	*2.82*	*16.1*	*HV 1873*	*12.9*	*14.7*	*HV 1877*	*50.1*	*13.1*
HV 1987	*3.16*	*16.0*	*HV 1954*	*16.6*	*13.8*			

with the periods and absolute magnitudes M of cepheids in the Galaxy:

star	*logP*	*M*	*star*	*logP*	*M*	*star*	*logP*	*M*
SU Cas	*0.29*	*-1.7*	*V 350 Sgr*	*0.71*	*-3.0*	*S Nor*	*0.99*	*-3.7*
EV Sct	*0.49*	*-2.4*	*CV Mon*	*0.73*	*-3.0*	*Z Lac*	*1.04*	*-4.1*
SS Sct	*0.56*	*-2.4*	*RR Lac*	*0.81*	*-3.4*	*RW Cas*	*1.17*	*-4.5*
SU Cyg	*0.58*	*-2.8*	*U Sgr*	*0.83*	*-3.5*	*Y Oph*	*1.23*	*-5.3*
Y Lac	*0.64*	*-2.8*	*η Aql*	*0.86*	*-3.5*	*T Mon*	*1.34*	*-5.6*
FF Aql	*0.65*	*-3.1*	*RX Cam*	*0.90*	*-3.7*	*SV Vul*	*1.65*	*-6.4*
CF Cas	*0.69*	*-3.4*	*DL Cas*	*0.90*	*-3.7*			

Cf. Passachoff J.M., Goebel R.V. 1979: Sky and Tel. (March) 241

Exercise 8 *Find the value of the Hubble constant using the magnitudes m and redshifts z of galaxies in the following table, assuming that the absolute magnitudes of these galaxies are $M = -22$.*

m	*z*	*m*	*z*	*m*	*z*	*m*	*z*
7.00	*0.00086*	*13.20*	*0.0188*	*14.50*	*0.0590*	*16.00*	*0.1343*
8.90	*0.00577*	*13.62*	*0.0241*	*14.80*	*0.0216*	*16.70*	*0.0497*
8.91	*0.00344*	*13.95*	*0.0302*	*14.86*	*0.0326*	*17.00*	*0.1300*
11.87	*0.0176*	*14.00*	*0.0163*	*15.00*	*0.0209*	*17.54*	*0.1959*
12.14	*0.0169*	*14.00*	*0.0203*	*15.00*	*0.0677*	*17.94*	*0.2106*
12.28	*0.0177*	*14.07*	*0.0450*	*15.19*	*0.0600*	*18.00*	*0.0728*
12.50	*0.0181*	*14.20*	*0.030*	*15.50*	*0.0437*	*18.02*	*0.2201*
12.84	*0.0289*	*14.30*	*0.0145*	*15.80*	*0.0518*	*19.00*	*0.18*
12.90	*0.0215*	*14.50*	*0.0374*	*16.00*	*0.0718*	*19.00*	*0.2561*

Cf. Evans A. 1978: Sky and Tel. 55, 299

Exercise 9 *Due to gravitational lensing by a galaxy G at $z_G = 0.39$, the quasar PKC 0957+561 at $z_Q = 1.4136$ is imaged as a multiple source with the brightest components A and B on the opposite sides of G at angular distances from it equal to $4.95''$ and $0.75''$, resp.*

(a) Calculate the distance of the quasar and the galaxy from the Earth, the apparent linear separation of components A, B, and the distances between the galaxy and the rays from A and B.

(b) Using the simplified model of a point-like mass M of the galaxy, according to which the deflecting angle of the light reads $\varphi = 4Gc^{-2}M/r$, find M and the true angular separation between the quasar and the galaxy.

(c) Estimate the Hubble constant using the time delay $\Delta\tau_{AB} \simeq 417$ days in the variability of the images A and B.

(d) Find the theoretical ratio of the brightness of the images A and B, and compare it with its value $\simeq 1.7$ observed in the radio wavelengths.