1.4 Dynamics of the Universe

From the non-relativistic point of view, the dynamics of the Universe, i.e. the function $R = R(t)$ in the metric (2) should not be influenced by any value of the pressure P, because, due to the homogeneity of the space, there is a zero gradient of P, and it thus cannot exert any force on the motion of the mass. However, from the point of view of general relativity, the pressure as a component of the stress-energy tensor is also a source of a gravitational field which according to the difference between Eqs. (4) and (5)

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + \frac{3}{c^2}P) + \Lambda\frac{c^2}{3} , \tag{13}$$

will contribute to the acceleration of $R(t)$.[14] The dynamics of the Universe is thus influenced by the matter (and energy) content of the Universe and its corresponding equation of state.

For example in the case of a coherent dust[15] which agrees with the above given Newtonian cosmology

$$P_{\mathrm{mat}} = 0 \qquad \Rightarrow \qquad \rho_{\mathrm{mat}} \sim R^{-3} \qquad (R \sim t^{\frac{2}{3}}) . \tag{14}$$

However, if the Universe was filled by a photon gas (i.e. radiation) only, or by any other ultrarelativistic gas, then

$$P_{\mathrm{rad}} = \frac{1}{3}c^2\rho_{\mathrm{rad}} \qquad \Rightarrow \qquad \rho_{\mathrm{rad}} \sim R^{-4} \qquad (R \sim t^{\frac{1}{2}}) . \tag{15}$$

In both cases ρ can be integrated from Eq. (6) in the form of a power dependence on R. The terms with k and Λ are thus negligible for $R \to 0$ compared to the right hand side of Eq. (4) and the solution $R(t)$ can then be asymptotically approximated by the simple power-law expressions given in the parentheses. For a mixture of both gases, the Universe is dominated by the radiation ($\rho_{\mathrm{rad}} > \rho_{\mathrm{mat}}$) for a sufficiently small R, and its dynamics thus obeys Eq. (15). For larger values of R at a later epoch, the Universe is dominated by mass ($\rho_{\mathrm{mat}} > \rho_{\mathrm{rad}}$), and its dynamics thus satisfies (14). At even later epochs the term with the curvature becomes important, and the dynamics of the Universe depends on the value of k as can be seen in Tab. 1 and Fig. 4. These solutions start from a singularity ($R = 0$, $\rho = \infty$) and either re-collapse again in the case of a closed Universe or expand forever in the case of an opened Universe.

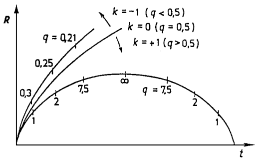

Figure 4: The dynamics of Friedmann models with zero pressure and cosmological constant.

It is important to note that the presence of a relativistic matter in the Universe dominates the equation of state and also the dynamics of the Universe near the singularity. Following the Stefan-Boltzmann law, the temperature for the radiation is determined by the energy density, which varies with the radius of the Universe,

$$\rho_{\mathrm{rad}}c^2 = \sigma T^4 \sim R^{-4} . \tag{16}$$

It thus means that the temperature of the radiation $T \sim R^{-1}$ and consequently $T \to \infty$ near the singularity. This is why the singularity is called the *hot big bang*.

[14]The influence of the pressure on the dynamics of the Universe can also be understood as a result of the conservation laws (6), which are a consequence of the Einstein field equations, because, due to the equivalence of mass and energy, the work exerted by the pressure will change the dependence $\rho = \rho(R)$.

[15]And approximately also for any classical non-relativistic gas because then $P \ll \rho c^2$.

$k = +1$	$R = R_0(1 - \cos\eta)$ $t = t_0(\eta - \sin\eta)$
$k = 0$	$R = R_0(t/t_0)^{2/3}$
$k = -1$	$R = R_0(\cosh\eta - 1)$ $t = t_0(\sinh\eta - \eta)$

Table 1: The dynamics of the Friedmann models with zero pressure and cosmological constant.

Owing to the strangeness of the physical conception of the initial singularity, it became temporarily popular to apply the stationary cosmological models (the so-called steady state models by H. Bondi, T. Gold and F. Hoyle), according to which equation (6) of continuity of mass is violated by a spontaneous creation of mass everywhere in the Universe, with the rate just needed to balance the decreasing mass density due to the cosmological expansion. However, these models which are homogeneous also in time, do not agree with the results of the statistics of distribution of galaxies at large distances (i.e., with their density in the past), and they were completely abandoned after the discovery of the *cosmic background radiation* which also confirms a high temperature in the earlier epochs of the evolution of the Universe.

Following the Grand-unification theories (GUT), the equation of state $P = -\rho c^2$ could be valid during the very early epoch (the so-called *inflationary epoch*) of the evolution of the Universe. Consequently, following Eq. (6),

$$P = -\rho c^2 \qquad \Rightarrow \qquad \rho = constant \qquad (R \sim \mathrm{e}^{Ht}) \; . \tag{17}$$

The work exerted by the expansion of the Universe against the negative pressure thus could in these circumstances really produce a mass balancing the decrease of its density due to the expansion, and it thus could temporarily mimic the effect of the cosmological constant.

Exercise 5 *Verify that the solutions given in Tab. 1 satisfy Eqs. (4) and (5) and find the proper values of the free parameters.*