Analysis of gamma-ray burst duration distribution using mixtures of skewed distributions

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Overview

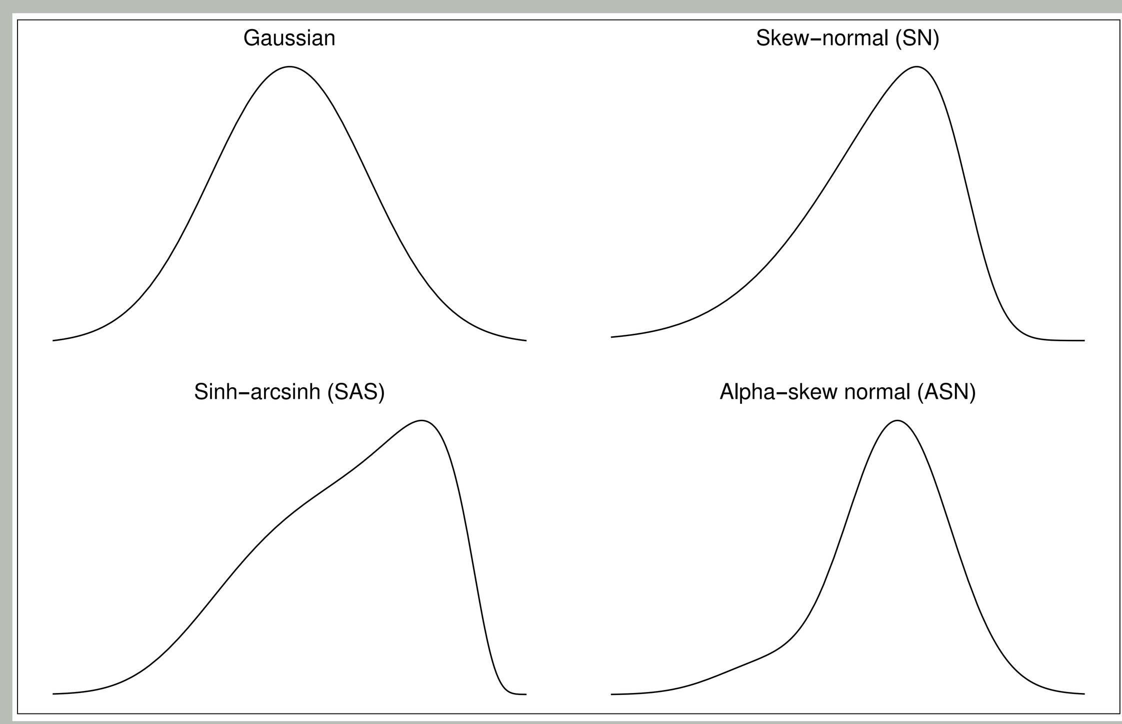
Context. Observed distributions of gamma-ray burst (GRB) durations (denoted T_{90}) have been thus far modeled with standard Gaussians only. The introduction of a third—intermediate—class of GRBs (besides short and long) was made based on such modeling, but its physical machanism remains unknown. Moreover, the data sets gathered by *Fermi*/GBM, *CGRO*/BATSE, and *Swift*/BAT are all bimodal, hence the existence of a third class is uncertain.

Results. It is found [1] that a 2-component mixture of skewed distributions is a better description of the data than a 3-Gaussian; this provides a much simpler explanation that does not require to introduce another physical phenomenon. Hence, the third class is not necessary. The asymmetry might come from a non-symmetric distribution of the envelope masses of the progenitors of the long GRBs [2].

Distributions

The following distributions are used, with p being the number of free parameters in a mixture of k components:

- Standard Gaussian (G) with $p = 3k - 1$ parameters. It is symmetric (non-skewed).
- Skew-normal (SN) distribution with $p = 4k - 1$ parameters [3, 4]. Its skewness is limited to the interval $(-1, 1)$.
- Sinh-arcsinh (SAS) distribution with $p = 5k - 1$ parameters [5]. Its kurtosis also can be varied.
- Alpha-skew normal (ASN) distribution with $p = 4k - 1$ parameters [6]. Its skewness is limited to the interval $(-0.811, 0.811)$. Depending on the value of the parameter governing the skewness, the ASN distribution can be unimodal or bimodal.



Akaike information criterion (AIC)

AIC [7] is employed as it can be applied to non-nested models (which is the case here; comparison of log-likelihoods $\mathcal{L}$ can be done for nested models only). It is given by

$$AIC = 2p - 2\mathcal{L},$$

where p is the number of parameters. The best model among the examined ones is that with the lowest AIC, denoted $AIC_{\min}$.

One compares the differences $\Delta_i = AIC_i - AIC_{\min}$. These are related to the relative probability that the i-th model minimizes AIC via:

$$Pr_i = \exp\left(-\frac{\Delta_i}{2}\right)$$

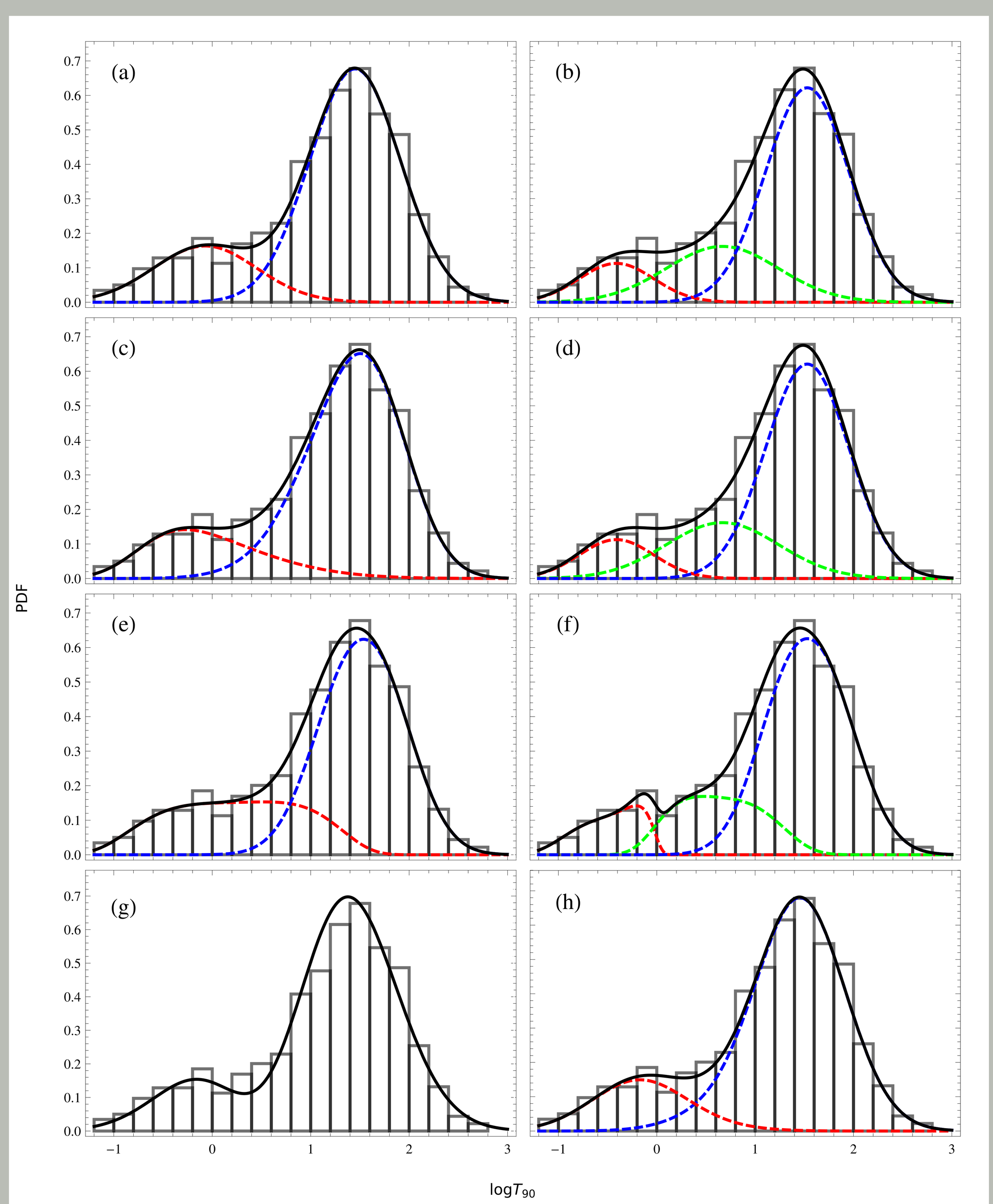
Rules of thumb [8]:

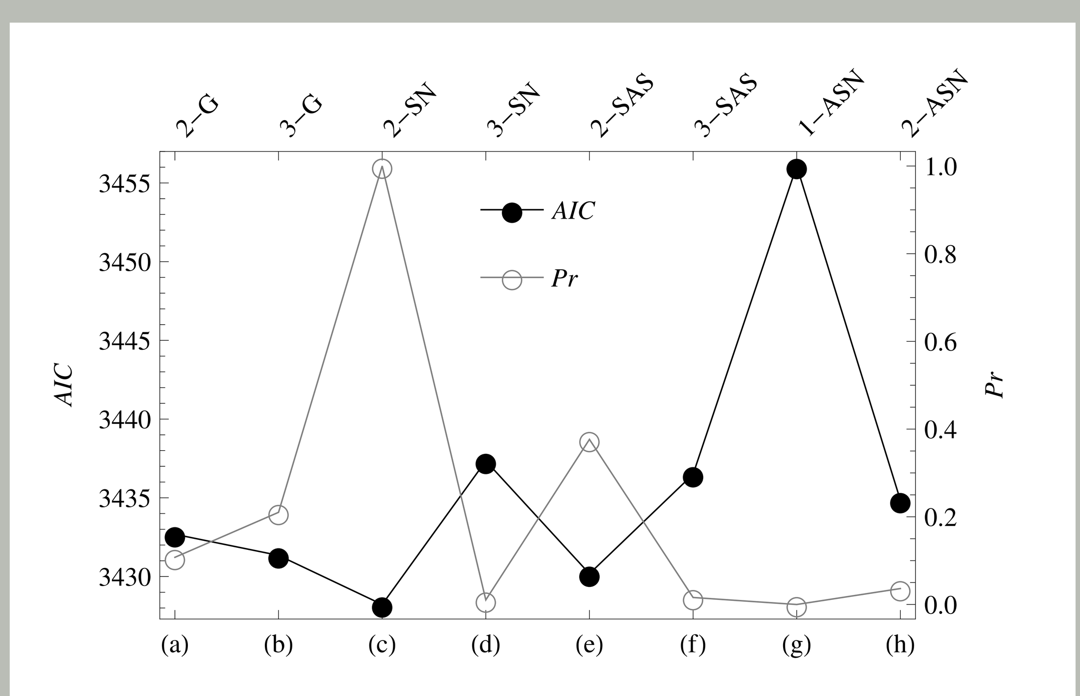
- $\Delta_i < 2$, then there is substantial support for the i-th model (or the evidence against it is worth only a bare mention);
- $2 < \Delta_i < 4$, then there is strong support for the i-th model;
- $4 < \Delta_i < 7$, there is considerably less support;
- models with $\Delta_i > 10$ have essentially no support.

Results [1]

The following mixtures of distributions are examined: a two- and three-component Gaussian (2-G and 3-G), a two- and three-SN (2-SN and 3-SN), a two- and three-SAS (2-SAS and 3-SAS), a one-, two- and three-ASN (1-ASN, 2-ASN and 3-ASN), for each of the three examined data sets. The models are evaluated based on their AIC (bottom pictures). The relative probabilities are also displayed.

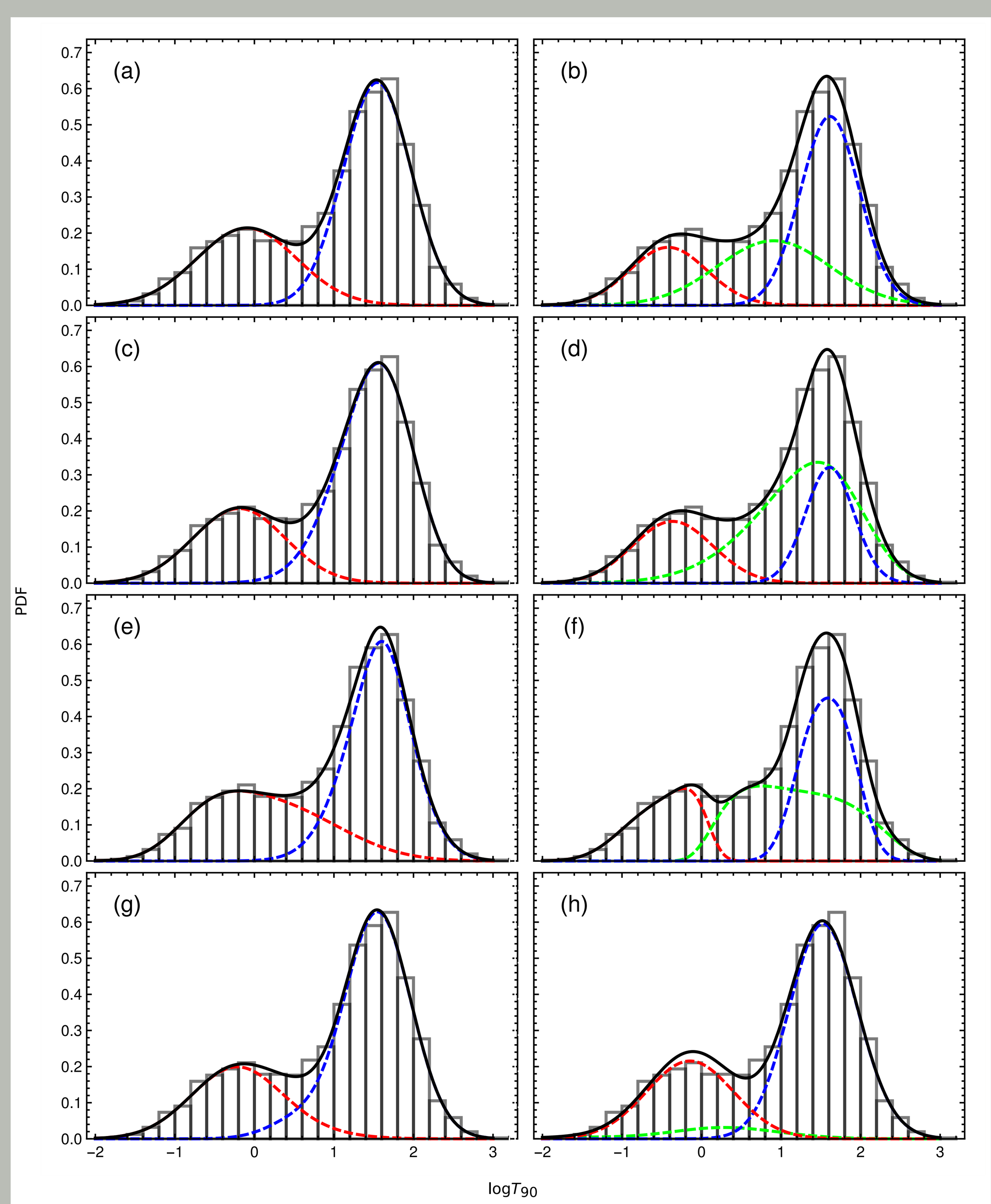
Fermi (1596 GRBs)

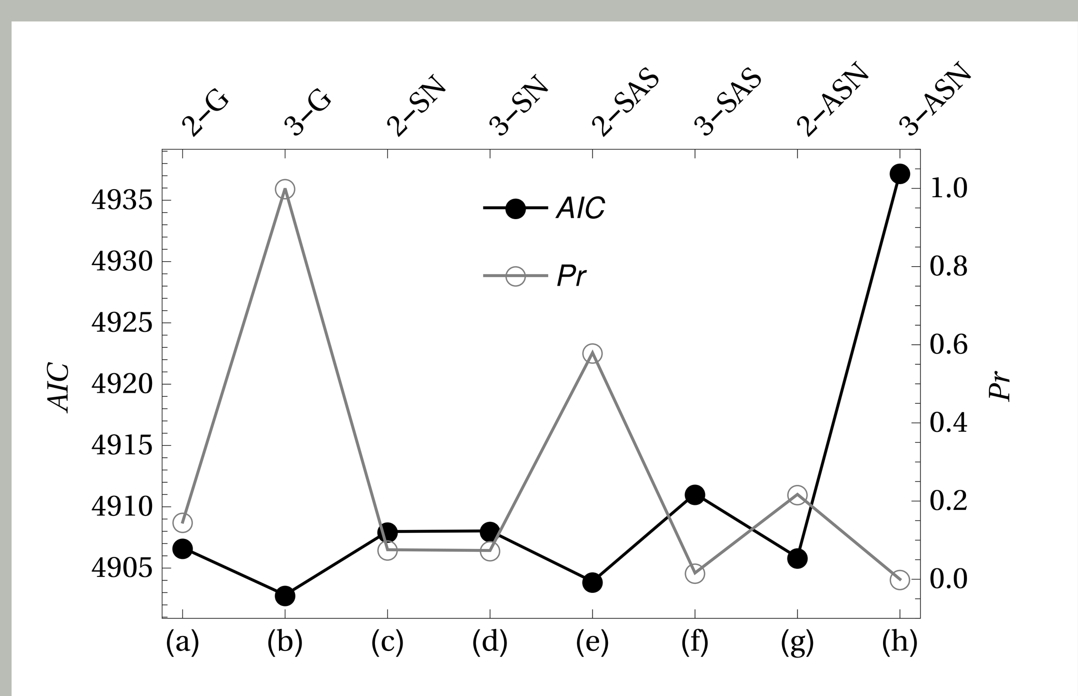




- Best model: 2-SN ($p = 7$)
- 2-SAS ($p = 9$; $Pr = 37.7\%$ and $\Delta_i = 1.953$)
- 3-G ($p = 8$; $Pr = 21\%$ and $\Delta_i = 3.119$)

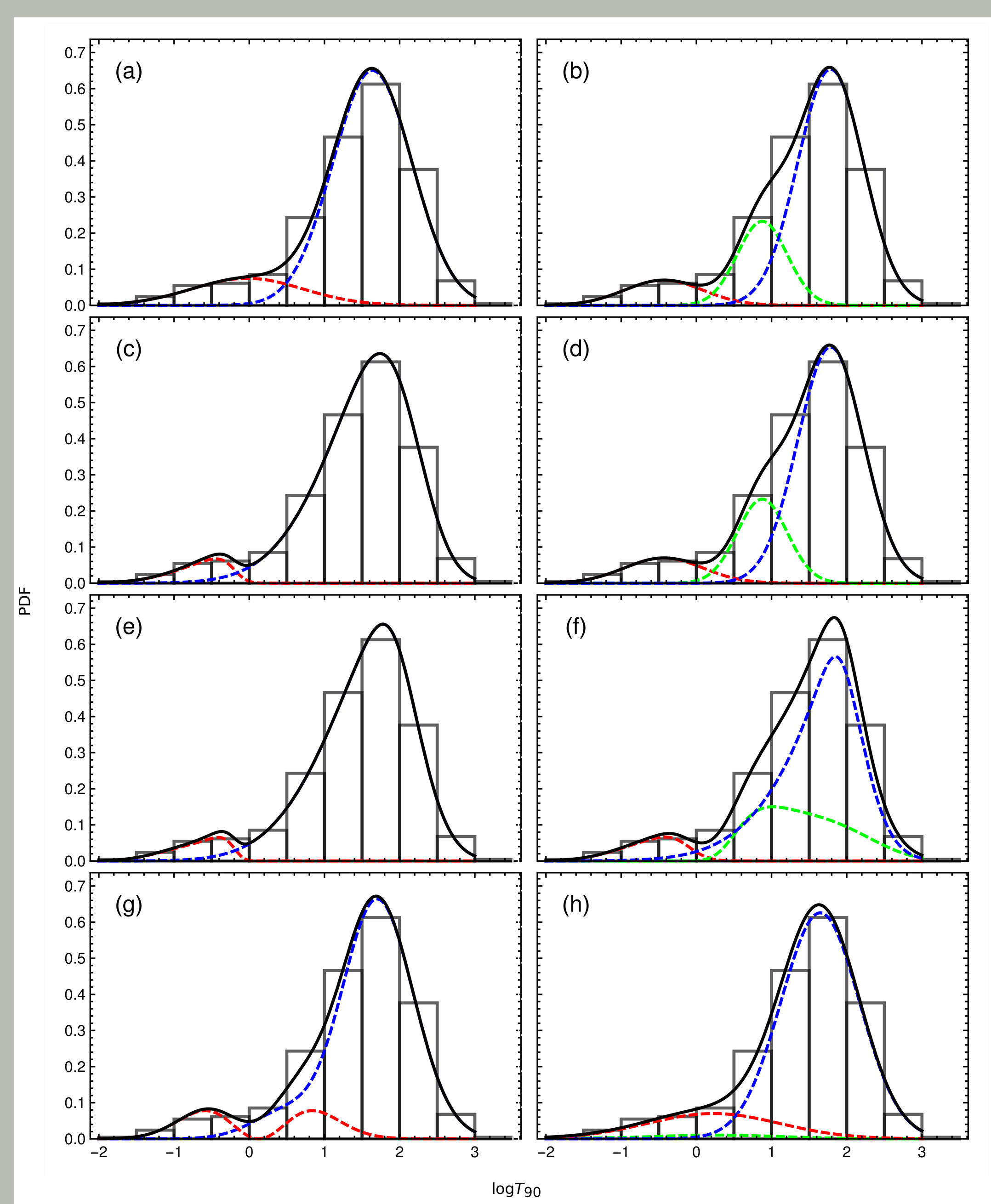
BATSE (2041 GRBs)

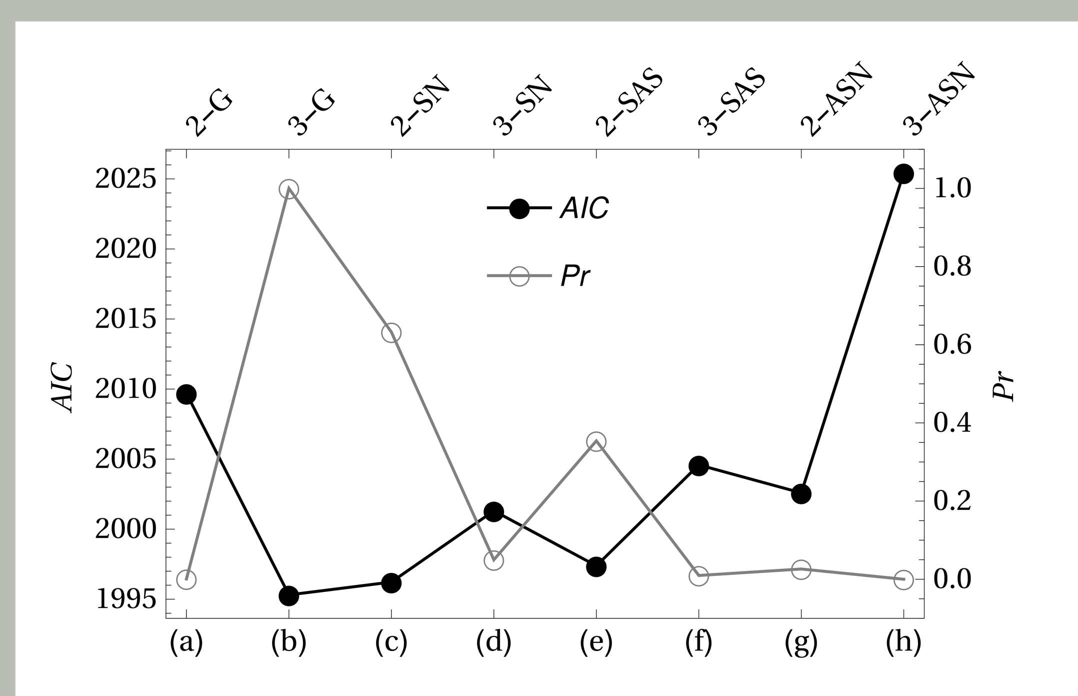




- 3-G ($p = 8$)
- 2-SAS ($p = 9$; $Pr = 57.9\%$ and $\Delta_i = 1.091$)
- 2-ASN ($p = 7$; $Pr = 21.7\%$ and $\Delta_i = 3.054$)

Swift (914 GRBs)





- 3-G ($p = 8$)
- 2-SN ($p = 7$; $Pr = 63.2\%$ and $\Delta_i = 1.040$)
- 2-SAS ($p = 9$; $Pr = 35.4\%$ and $\Delta_i = 2.077$)

Conclusions

Skewed distributions with 2 components describe the observed T_{90} data better (*Fermi*) or at least as good (BATSE, *Swift*) as a mixture of 3 standard Gaussians. Therefore, there is no need to introduce a third, intermediate in duration, class of GRBs. Additionally, similar conclusions were drawn for a sample of GRBs with measured redshifts [9, 10].

Bibliography

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