Global gravity field models from the GPS positions of CHAMP, GRACE and GOCE satellites

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Summary

Inversion method

- Data: kinematic orbits, SST high-low (CHAMP, GRACE, GOCE)
- Acceleration approach: Newton's second law relates the observed acceleration of the satellite with forces acting on it
- Observed accelerations: numerical 2nd derivative of GPS orbits
- Other forces: modelled (e.g. tides) or measured (nongrav. acc.)

Some original elements of our inversion method

- Aim: simple, straightforward and statistically correct model
- Model is linear in harmonic coefficients, no a priori gravity field model is needed, no regularization is applied
- Amplified noise due to numerical derivative mitigated by Generalized Least Squares (linear transformation)
- Decorrelation of errors in GPS positions significantly improves the accuracy of harmonic coefficients by a factor of 2-3
- Separately computed along-track, cross-track and radial solutions merged into combined solution using normal matrices

Results from real-world data of CHAMP, GRACE, GOCE

- Kinematic orbits of CHAMP and GRACE: 7 years (2003-2009)
  Kinematic orbits of GOCE: 2 months (Nov/Dec 2009)
- Long-term static gravity field models (CHAMP, GRACE, GOCE): similar or better quality compared to other published solutions
- Time variable gravity (CHAMP, GRACE): mean annual signal clearly shows important hydrological variations on continents
- Geocentre motion (GRACE): estimated degree-one coefficients display seasonal variations in accordance with results from other measurement techniques
- Computations performed on ordinary PC up to max. degree 120

Method of inversion

(1) Linear regression model

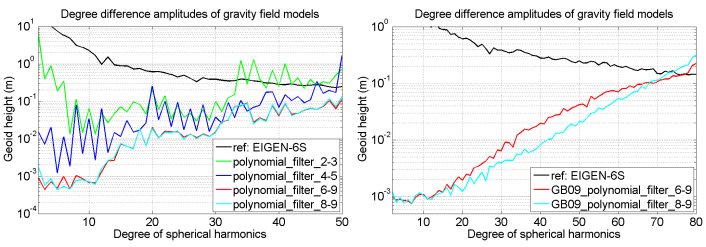
$$\sum \mathrm{SC} \times \nabla \mathrm{SSH}(r,\theta,\varphi) = d^2\mathbf{r}/dt^2 - (\mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{NG})$$

Stokes coefficients (SC) are obtained from observational equations, where:

∇SSH…gradient of solid spherical harmonics
$d^2\mathbf{r}/dt^2$…observed accelerations
$\mathbf{a}_{LS}$ … lunisolar effects; $\mathbf{a}_{TID}$ … solid Earth and ocean tides
$\mathbf{a}_{NG}$ … acc. of nongravitational origin (drag, radiation pressures)

(2) Numerical approximation to the second derivative

- Digital filter of the second derivative is an approximation to the analytical operation. Solutions may differ significantly depending on the particular choice of the filter parameters.

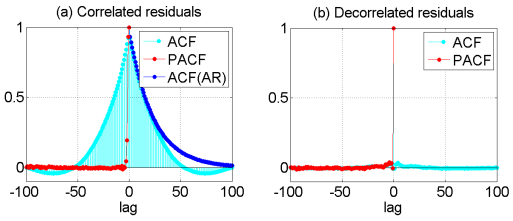


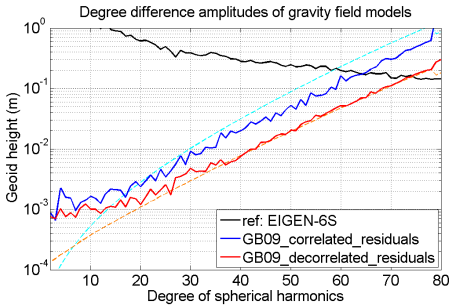
(3) Mitigation of noise amplification

- Problem: Numerical derivative amplifies noise in GPS positions
- Solution: Generalized least squares (GLS) → application of GLS leads to linear transformation of model (1)

(4) Decorrelation of noise in GPS positions

- Problem: GPS positions have correlated errors
- Solution: Sample autocorrelation function (ACF) and especially partial autocorrelation function (PACF) indicate suitability of autoregressive model (AR) to represent correlation structure
- Decorrelation of residuals improved the accuracy of harmonic coefficients by a factor of 2-3
- Decorrelation again defines a linear transformation of model (1)

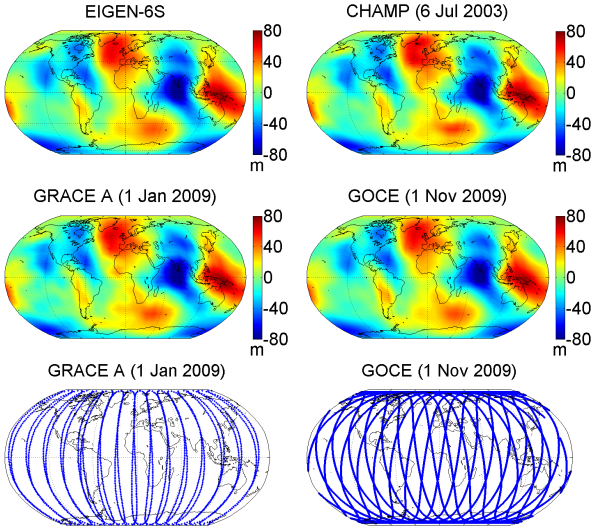




Results for real orbits

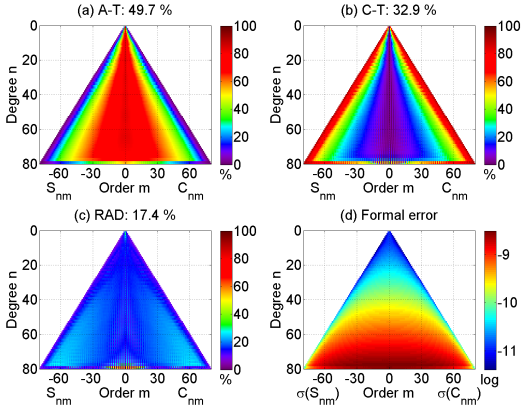
(5) Gravity field from one day of real data

- Model (1) is linear in SC, no a priori gravity field model is used. After applying two linear transformations (3) and (4), SC are obtained directly in one step using the ordinary least squares.
- Graphs show reasonable results for real data (max. degree 10)



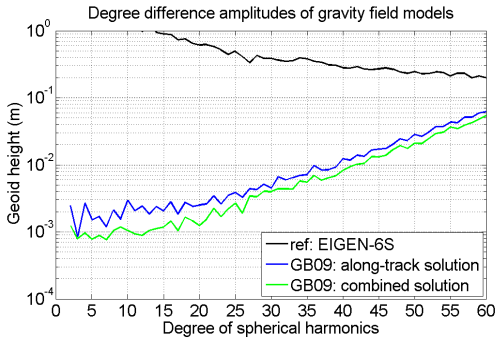
(6) Along-track, cross-track and radial solutions combine

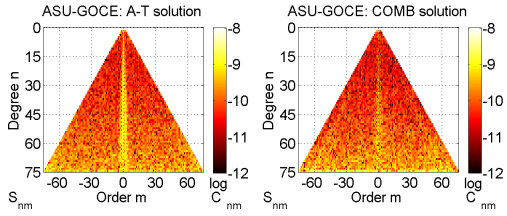
- We found it better first to compute individual solutions for along-track (A-T), cross-track (C-T) and radial (RAD) directions.
- Then we obtained a combined solution using normal matrices.
- Relative contribution of the along-track component to the combined solution is 50 percent on average.



(7) Along-track solution vs. combined solution

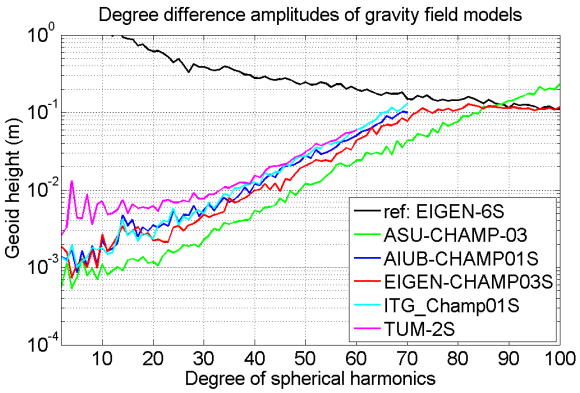
- Systematically, individual along-track solutions give worse results compared to combined solutions.
- Polar gap of GOCE: the combined solutions give better near-zonal coefficients than along-track ones.





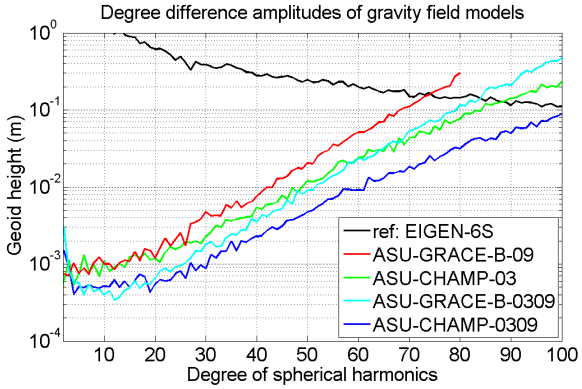
(8) Gravity field model from orbits of CHAMP in 2003

- This CHAMP solution was computed by many groups (see http://icgem.gfz-potsdam.de/)
- Satisfactory behaviour of our solution (ASU-CHAMP-03) is also due to improved processing of kinematic orbits by AIUB



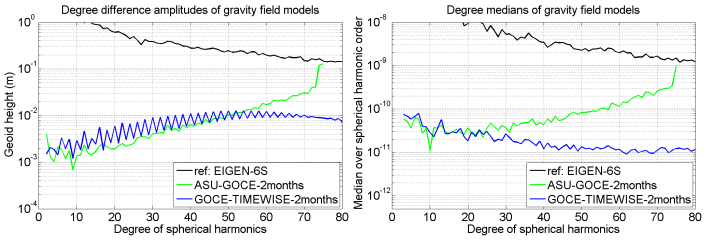
(9) Seven-year solutions from CHAMP and GRACE orbits

- Apart from physical causes of difference (e.g. mean altitude: CHAMP…400 km, GRACE…500 km), also the quality of GPS data and parameters of the method may play the role (e.g. (2))



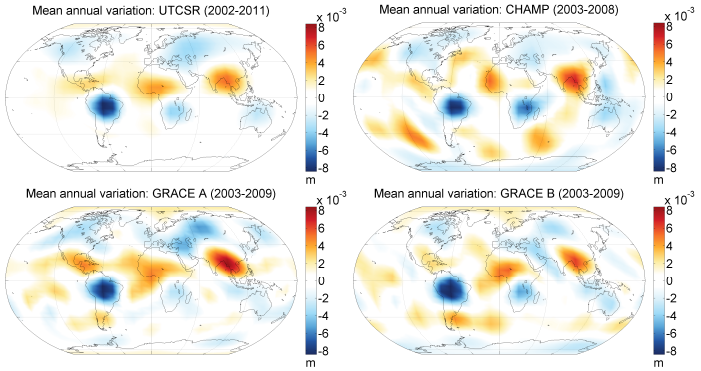
(10) First results for GOCE

- This is our preliminary result compared to the ESA solution, whose long-wave part is supposed to be not regularized
- In the ESA solution, SGG data start to dominate from degree 25



(11) Time variable gravity from GPS orbits (CHAMP, GRACE)

- The acquired average annual signal shows clearly well-known continental areas with important hydrological variations



(12) Geocentre motion

- We tried to estimate the degree-one harmonic coefficients, which correspond to the geocentre motion
- Our orbit-based seasonal variations are in accordance with results from other measurement techniques

Data set	x		y		z		time span
	amp(mm)	phase(deg)	amp(mm)	phase(deg)	amp(mm)	phase(deg)	
SLR: UTCSR	3.0 ± 1.2	53.9 ± 22.3	2.5 ± 1.0	-39.7 ± 21.7	4.6 ± 1.9	35.4 ± 23.4	2005–2011
Rietbroek et al 2011	2.0 ± 0.5	58.9 ± 14.5	3.6 ± 0.4	-40.6 ± 6.3	3.6 ± 0.7	16.1 ± 11.6	2005–2009
Swenson et al 2008	1.6 ± 0.3	99.7 ± 11.1	1.2 ± 0.4	-88.8 ± 16.8	2.1 ± 0.3	96.0 ± 7.2	2005–2010
ASU: GA0509	1.2 ± 0.8	103.3 ± 37.1	1.3 ± 0.8	-21.6 ± 36.4	4.0 ± 2.6	11.7 ± 37.3	2005–2009
ASU: GB0509	1.1 ± 0.7	68.0 ± 39.1	1.4 ± 0.9	-19.7 ± 35.6	4.6 ± 2.9	7.7 ± 36.2	2005–2009

Acknowledgments

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