FINE ORBIT TUNING TO INCREASE THE ACCURACY OF THE GRAVITY-FIELD MODELLING

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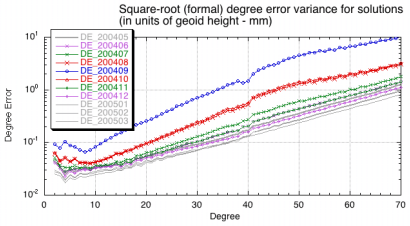
Abstract

Fine orbit tuning will be presented as a tool to enhance the accuracy of the gravity-field parameters based on the data from satellite missions around the Earth or other planetary bodies. A slight variation in the satellite altitude of a few hundred metres or kilometres may dramatically change the pattern and density of the groundtracks, thus leading to a significant difference in the quality of the derived gravity-field parameters. This aspect is important not only to missions dedicated to the gravity-field mapping, but it can be applied to any planetary mission, whose orbital data may yield useful information on the particular gravity field. The geometry of satellite groundtracks is closely connected with the term orbital resonance or repeat orbit, which was intensively studied by the satellite geodesy community since the 1970s. In a systematic way, fine orbit tuning was first applied to altimetry missions for oceanographic purposes in the early 1990s, when it became clear that small changes in the satellite altitude might substantially influence the utility of the data from the onboard instruments. The monthly geopotential solutions from the GRACE mission (in orbit since 2002) displayed apparently worse precision in August–September 2004, which was later found to be caused by a sparser groundtrack pattern due to the passage of the GRACE satellites through the 61/4 orbit resonance. The lessons learned from GRACE were applied by ESA to its gravity field mission GOCE (in orbit since 2009). Here, the situation is different, as the GOCE onboard thrusters are capable of maintaining the satellite at a constant altitude. In order to fully use the measurement potential of the first space gradiometer ever flown, in the GOCE mission planning the influence of orbit geometry was taken into account, and a minimum 2-month repeat period for the orbit was specified. We analysed several orbital configurations of GOCE, as possible candidates for the gravity mapping phases. We found that apart from the required minimum density of the groundtrack grid, also the regularity of the groundtrack pattern may be "tuned" better by small shifts of few hundred metres in the satellite altitude. Finally, we will show how the insight gained on the Earth may be applied to future orbiters around other celestial bodies. Here we found that contrary to the slowly rotating bodies like the Moon, Mercury or Venus, the methods of fine orbit tuning might be particularly useful for space probes orbiting Mars.

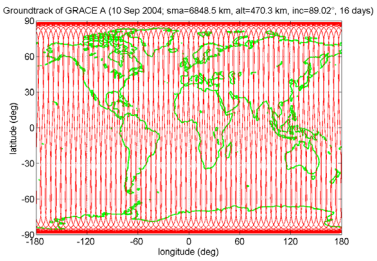
Motivation

(1) GRACE monthly solutions in 2004

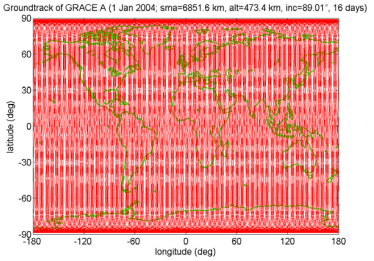
- Observed large degree error in gravity solutions in September 2004



- Explanation: Grid of groundtracks in Sep 2004 was rather sparse due to passage through 61:4 orbit resonance

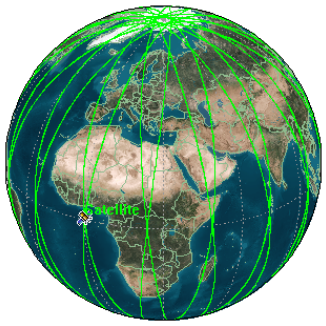


- In other months, grid of groundtracks is much denser (fig. for Jan 2004)

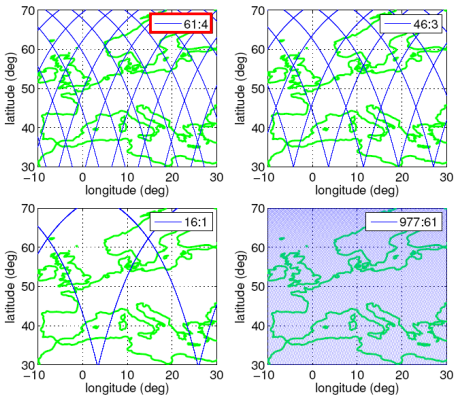


(2) Orbital resonance

- Orbital resonance R:D takes place, if groundtracks are exactly the same after R nodal revolutions and D nodal days
- Equivalent names: repeat orbit, resonant orbit, exact repeat mission
- Fig: Satellite in 15:1 orbital resonance

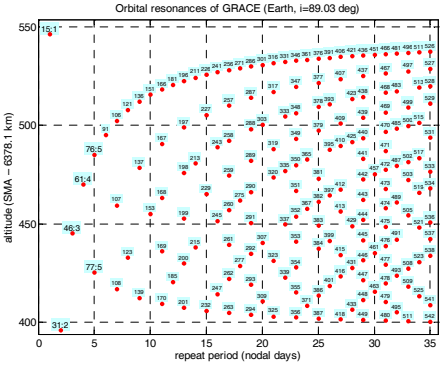


- Fig: Groundtrack grids of three low-order and one high-order resonances



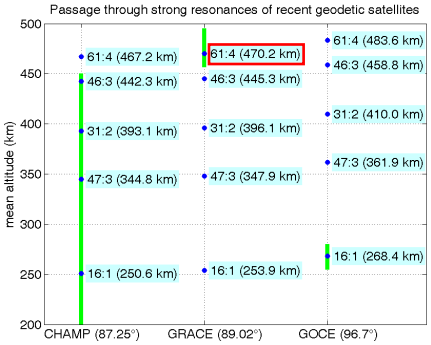
(3) Resonance diagram

- As time passes, satellite loses altitude due to atmospheric drag and descends through resonant configurations



(4) Recent gravity missions

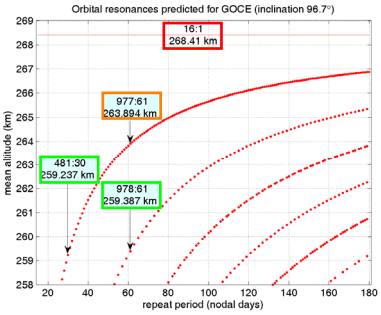
- Passage through strong resonances has impact on the gravity field products
- CHAMP: 46/3, 31/2, 47/3
- GRACE: 61/4
- GOCE: 16/1



GOCE and fine orbit tuning

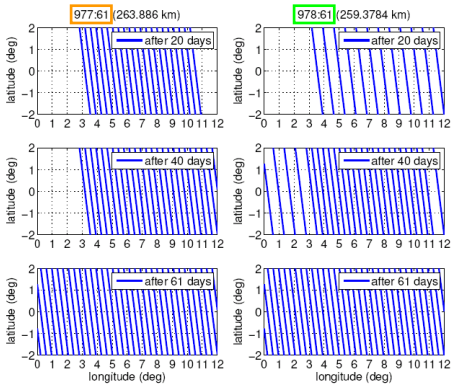
(5) Global gravity field model

- Resolution of 100 km → minimum repeat period of 2 months
- Altitude as low as possible
- Strong 16:1 resonance to be avoided for the measurement phases



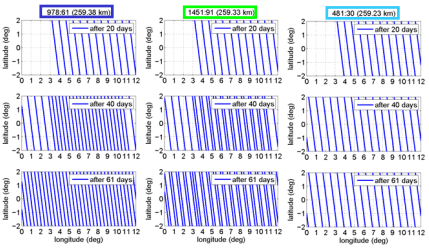
(6) Temporal evolution – subcycles

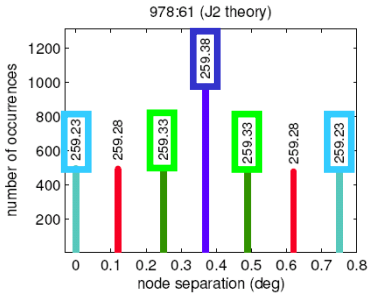
- Repeat orbit with no subcycles
  → gradually filling up the Earth surface
- Repeat orbit with a subcycle
  → two (or more) almost regular grids



(7) Small variations in altitude

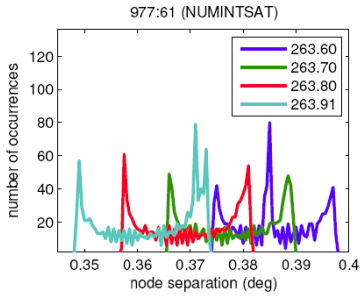
- Exact 61-day repeat orbit
  - one peak in histogram
- Height lower by 50 m
  - grid not regular after 61 days
- 30-day subcycle
  - mean height lower by 150 m
  - inacceptable for GOCE





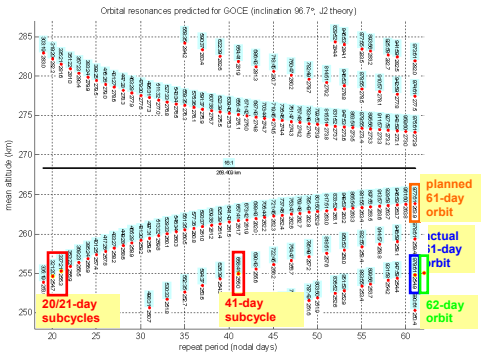
(8) Analytical vs. numerical modelling

- So far, graphs were based on simple theory with only the zonal term J_2=–C_{20} due to Earth flattening
- Our final calculations are made with all orbital perturbations (geopotential, lunisolar, tides, radiation, …), e.g.:



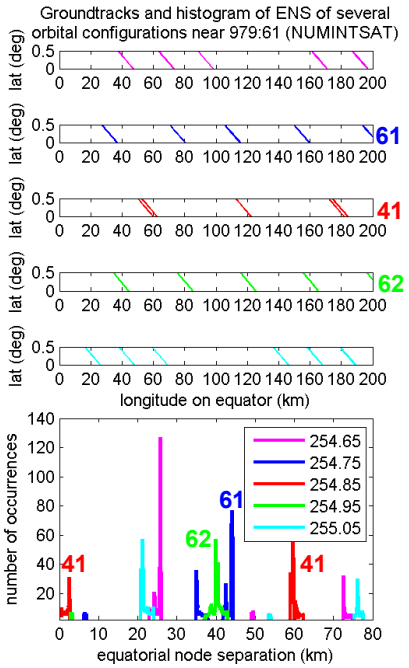
- Peaks in histograms widened
- Repeat character is kept

(9) Actual mapping orbit of GOCE



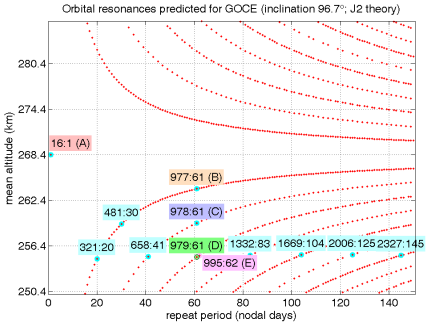
(10) Regularity of groundtrack pattern

- Equator with groundtracks after 65 days
  - orbits with different mean altitudes
- Repeat orbits:
  - 61-day (selected for actual mapping)
  - 41-day subcycle
- 62-day orbit compared with 61-day
  - has more regular groundtrack grid
  - is only by 200 m higher



(11) Possible 145-day repeat orbit

- 145 repeat orbit considered by ESA
  - node spacing ≈ 17.2 km
- Nearest subcycle repeat orbits
  - 62-day, lower by 30 m (40.3 km)
  - 83-day, higher by 23 m (30.1 km)
- Ion thruster performance ±50 m
  - 145-day repeat is a good choice

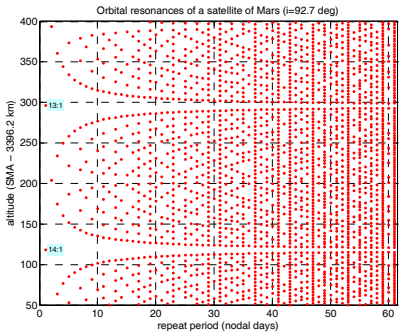


Planetary orbiters

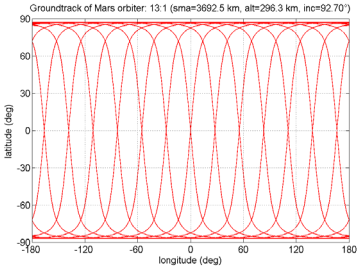
- Orbital data of any space probe moving around a celestial body → gravity model
- But repeated groundtracks reduce resolution of the obtained model
- What altitudes to avoid/prefer for a particular celestial body?

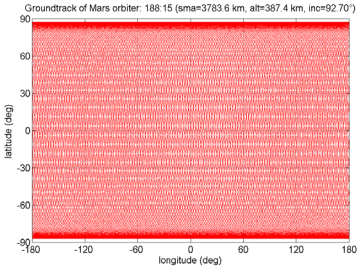
(12) Mars

- Resonance graphs similar to Earth
- The same problems
  → low-order resonances to be avoided



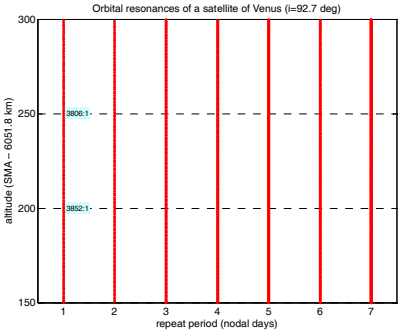
- Fig: Mars orbiter in low-order and high-order repeat orbit

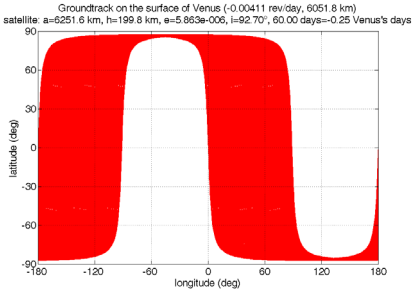




(13) Moon, Mercury, Venus

- Slow rotation → satellite has to do many revolutions for the groundtrack to repeat
- No similar problems in gravity sampling due to repeatability: needed much more orbital revolutions per a nodal day than is the actual maximum degree of the particular gravity model
- E.g. Venus: 3852 rev. ≫180 max. d/o





(14) Rationale behind fine orbit tuning & conclusions

- Without "good" coverage even the most sophisticated space instrument would not produce "good" geopotential coefficients!
- A small shift in altitude may considerably affect the full utilization of the accuracy of the instrument
- Series of resonant orbits in Fig **(11)**:
  - ◊ heights within only ±180 m
  - ◊ groundtrack density extremely different
  - ◊ differences in coverage regularity

(15) References

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[2] Klokočník J, Wagner CA, Kostelecký J, Bezděk A, Novák P, McAdoo D, 2008. Variations in the accuracy of gravity recovery due to groundtrack variability: GRACE, CHAMP, and GOCE. *J Geod* 82, 917–927

[3] Bezděk A, Klokočník J, Kostelecký J, Floberghagen R, Gruber C, 2009. Simulation of free fall and resonances in the GOCE mission. *J Geodyn* 48, 47–53

[4] Klokočník J, Bezděk A, Kostelecký J, Sebera J, 2010. Orbit tuning of planetary orbiters for accuracy gain in gravity-field mapping. *J guid, contr, and dyn* 33, 853–861

[5] Bezděk A, Klokočník J, Kostelecký J, Floberghagen R, Sebera J, 2010. Some aspects of the orbit selection for the measurement phases of GOCE. Proc. of ESA Living Planet Symposium, ESA SP-686

Appendix: Gravity field from kinematic positions using the GLS: GRACE, GOCE

(16) GPS-based accelerations

- $\mathbf{a}_{GPS}(\mathbf{r})$ obtained thru second-derivative digital filter of kinematic positions

(17) Stokes parameters (SP)

- Obtained directly from linear system:

$$\mathbf{a}_{geop} = \mathbf{a}_{GPS} - (\mathbf{a}_{LS} + \mathbf{a}_{TID} + \mathbf{a}_{NG})$$

where $\mathbf{a}_{geop}(\mathbf{r}) \equiv \sum \mathrm{SP} \times \mathrm{SSH}(r,\theta,\varphi)$, geopotential in spherical harmonics

(18) Generalized least squares (GLS)

- Linear transformation of Eq. **(17)** to decorrelate the covariance matrix of obs. → GPS noise is not amplified

(19) Resulting gravity parameters

- No preliminary gravity model needed
- Huge normal system built up sequentially
- Direct solution of linear system: estimated SP & their covariance matrix
- Three independent solutions can be optimally combined using covariance matrices → deficiencies of along-track solution only are mitigated

Fig. GLS inversion of gravity parameters: GRACE A (1 day; max. degree 15)

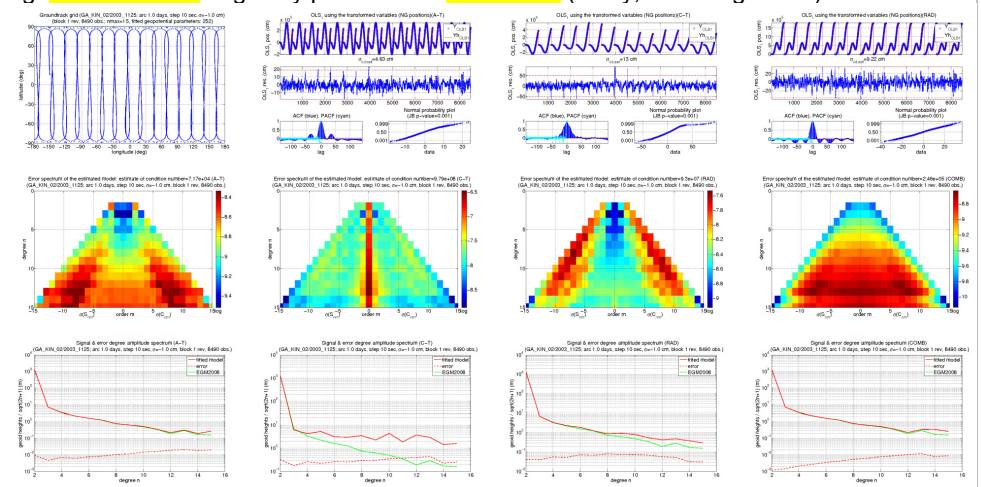

Fig. GLS inversion of gravity parameters: GOCE (1 day; max. degree 15)

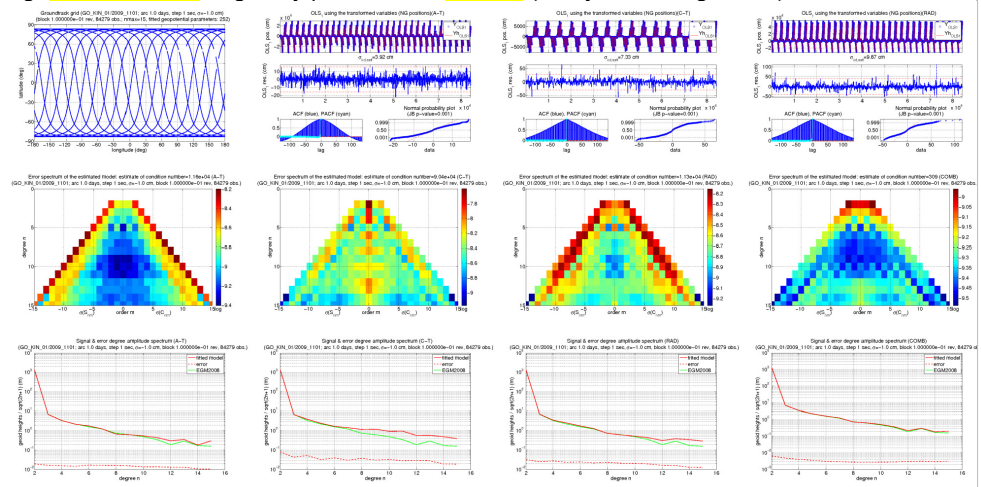

Fig. Geoid heights of the combined GLS solution for GRACE, GOCE vs. EGM08

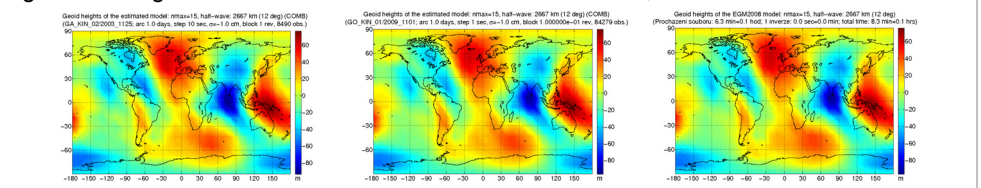
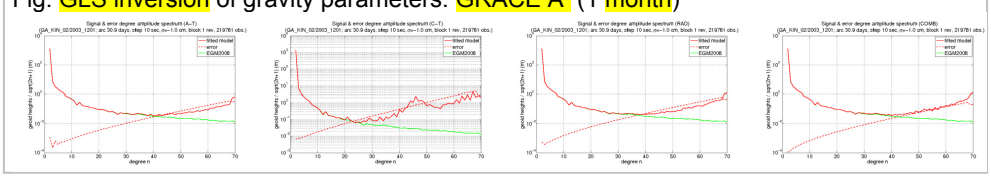
Fig. GLS inversion of gravity parameters: GRACE A (1 month)