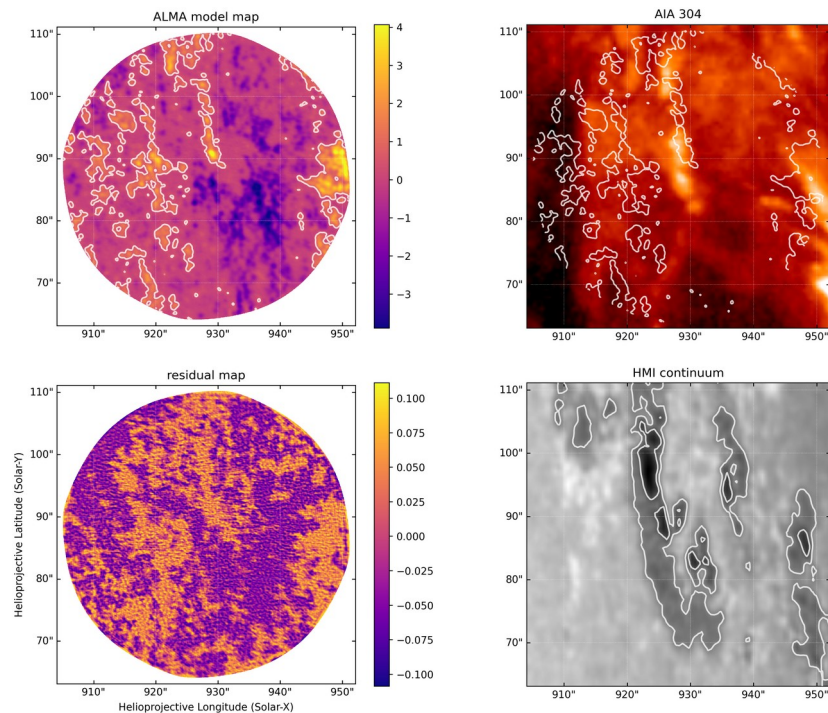


ESO ALMA Development Study

HIGH-RESOLUTION SOLAR OBSERVATIONS WITH ALMA

Final report



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1. Introduction

This report summarizes mostly the advances achieved during the 2nd half of the study (see the next Section 1.1), while the motivation, goals, suggested methodological approaches, and the early results have been already published in the *Mid-term-report* of the project. For the sake of just quick introduction, let us thus start with brief summary of the project aims and suggested ways how to reach them.

The main goal of the study is to investigate possibilities of extending ALMA array configurations for solar observations in order to achieve the higher spatial resolution, what is highly demanded by the community. The main obstacle to utilization of more extended configurations in solar mode has been identified in uncompensated fast phase errors during the target on-source observation between two phase-calibrator scans (there is no WVR pre-correction possible in solar mode). In order to overcome this issue, we study two approaches suggested in the project proposal:

1. Time-scale separation between the actual dynamics in the solar atmosphere and apparent time variations in the image caused by fast phase errors related to the turbulent terrestrial atmosphere. The goal is reached by self-calibration of the high-cadence series of solar interferometric images with hierarchical consecutive diminishing of the gain-cal solution interval. The high cadence (extra short integrations) is essential (i) for keeping coherence, and (ii), for sampling the phase variations on the time-scale much shorter than the actual dynamic timescale of studied phenomena at the Sun, so that we can separate both time-scales. In further text it is referred as *Approach 1*.
2. Using water-vapor signatures and their fast time variations in the spectra of science SPWs close to the water line in the Band 5 – this represents an alternative to the WVR measurement (which are impossible in solar observations) and subsequent `wvrgcal()` calibration. We derived a proxy to the variations of the water-vapor column based on science SPW data (auto-correlations and SQLDs), which is subsequently recalculated to the phase delays, and eventually applied for fast phase-error corrections (*Approach 2* in the following text)

The ultimate goal of the study is to provide a new suggested table of allowed array configurations for solar observations, an update to that, which is currently in charge in the [ALMA technical handbook for Cycle 12](#), at page 123. This final result is presented in the concluding section Error: Reference source not found

The new array-extension limits for each ALMA observing band are being found by:

- Simulations. We create simulated visibilities for various array configurations from the high-resolution and large field-of-view image (broad scale coverage), perturb these ideal visibilities by instrumental and – above all – atmosphere-related fast-varying errors, apply the self-cal procedure as like for actual observations according to *Approach 1*, and eventually compare the re-constructed images with the original model.

- In Band 5, the pre-correction similar to **wvrgcal()** according to *Approach 2* is applied before the calibration and self-calibration.
- In Bands 3 and 5 we have even test-observation data, whose analysis has verified outcomes of the simulation/self-calibration in those Bands and can be considered as a gauge to the entire procedure.
- For the ALMA observing bands, which are not yet commissioned for the solar observations (e.g., B1, B2, or B10), we provide at least estimation based on the scaling (the maximum baseline expressed in units of wavelengths) for the maximum acceptable extension of possible array configuration for solar observations.

We refer kind reader to find the details of scientific motivation and methodological approaches in the project *Mid-term report* (including its attachments), appended to this document as *Annex 1*.

1.1 Conceptual foreword to the *Final report*

In line with the contract and its *Statement of Work* (SoW) for this project, the *Final report* represents a key deliverable for the 2nd stage of the development study (DS2). Having this in mind, we have concentrated namely on the results achieved during the second half of the project when writing this document, avoiding as much as possible repeating presentation of the results reached in its 1st half. These are summarized in the *Mid-term report* (MTR), which has already passed its review in July 2024. The MTR is attached to this document and we frequently refer to it. We believe that this arrangement is more practical for reviewing the work done during the project and for clear assessment, whether the goals of the study has been achieved and results delivered. For the same purpose, we clearly refer to identifiers (DSx.x) of the deliverables as they are defined in the SoW, and action items (AIs) in *Recommendations for the 2nd half of the study* as they resulted from the mid-term review in the text of the current document.

On the other hand, we are highly internally motivated to see the results of our study being actually applied for enhancement of ALMA capabilities in the direction of improved resolution of solar ALMA imaging. For that sake, we are strongly committed to prepare a merged document, containing all important results in a single, more smoothly flowing text (e.g., with a bit different organization, no references to internal milestones of the study, etc.). This is going to be presented, after incorporation of recommendations from the Final review, as the deliverable DS3. Our clear intention is to formulate it as a reference material for the *ALMA ObsMode Committee* and the basis for its future decision about implementation of upgraded solar ALMA observing mode enhanced by high-resolution interferometry. A concise version of it will be highly likely also submitted for publication, as the results of this study may be interesting for the (not only solar) ALMA community, whether the enhanced solar ObsMode is eventually commissioned and implemented or not.

As already explained, we concentrate namely at topics not covered by the first half of the study and resulting MTR which are namely:

- Water-vapor line in Band 5, its emission and absorption features as captured in the *science* SPWs, and their application to the fast phase-error corrections as an alternative to WVRs and `wvrgcal()`, viable also for the solar observations. Insufficient elaboration of this topic has been actually the most serious concern of the expert panel during the mid-term review, so we have paid specific attention to it.
- Improved simulations of the atmospheric turbulence by the new *phase-screen* model.
- Presentation of the fully (self)calibrated test observations in Bands 3 and 5. As this is CPU-time demanding task, it was not possible to show the full-scale results already in the MTR.
- Improved quantitative metrics for the reconstructed-image quality plus new qualitative criteria for the credibility of the self-calibration loop.
- Summary of all the results, conclusion, and implementation recommendations.

In addition to the new tasks and deliverables specific for the 2nd stage of the study, there are also recommendations of the panel from the mid-term review for improvement of the results of its 1st half, formulated as a set of Action items (AI), appended as *Annex 2*. We wish to keep the flow of the text as smooth as possible and for that sake the special section (Section 3) is appended, which briefly describes resolution of each AI alongside with reference to the previous text, where it is handled in more detail.

Many results of the study, namely the full data products of the test observations in the form of self-calibrated high-cadence / high-resolution movies, are inherently of digital nature (and large in data extent) and in this report they will be presented by a link (URL) to the data repository directly in the text. For the limited space of the document, we also did not plot, e.g., the analytical results for all antennas and have selected just illustrative samples to be presented here. The rest is available on-line in the project repository http://wave.asu.cas.cz/shared/ESO/final_report.

The remaining relevant documents like Mid-term report (including its attachments), documentation to the SW developed in frame of the study, code listings, etc., are attached as annexes.

In spite the study should be concluded, and we believe that its goals have been achieved and clearly presented as deliverable documents, answers to the original questions – as usually in research – rise a bunch of the new ones. The very last Section 4 is therefore devoted to a brief indication of the future development in this direction. We are motivated and committed to carry out this research program even after the formal conclusion of the study, supported by our internal grants and institutional resources in order to improve the solar observing mode yet further.

1.2 Related documents

Annexes

- Complete *Mid-term report* after the last (accepted) revision – attached as *Annex 1*.
- Panel *Recommendations for the 2nd half of the study* – attached as *Annex 2*.

- Documentation for `sol_wvrgcal()` Python module – *Annex 3*.
- Listings of the scripts for simulation, calibration, imaging, and data analysis – *Annex 4*.
- On-line data & SW repository with results of the Study: Movies, datasets, XLS tables, etc.: http://wave.asu.cas.cz/shared/ESO/final_report/ – *online Annex 5*.
- *Directional phasor/Circular statistics* method for self-calibration quality assessment – *Annex 6*.

Other related documents

- General parent JIRA ticket for the 2023 test: <https://jira.alma.cl/browse/CSV-3831>
- Child-thread for the 2023 CSV observations: <https://jira.alma.cl/browse/CSV-3832>
- General parent JIRA ticket for the 2026 test: <https://jira.alma.cl/browse/CSV-4017>
- Child-thread for the 2026 CSV observations: <https://jira.alma.cl/browse/CSV-4018>

Further resources are available through the references/citations or as the functional hypertext links inserted directly in the text.

1.3 List of abbreviations

The standard and notorious abbreviations, used in the ALMA documents, are exploited also in this report without specific definition. Here we define just these less usual or ambiguous. A few others, which are not used too much repeatedly in the report, are defined in the appropriate place in the text.

AC	Auto-Correlations.
AI	Action Item, exceptionally also Artificial Intelligence
ASU	Astronomical Institute (of the Czech Academy of Sciences)
BP	Band-pass (calibrator, scan)
Cy <i>N</i>	(ALMA observing) Cycle no. <i>N</i>
EB	Execution block
INT	Interferometry/interferometric
FoV	Field of view
FR	Final report / Final review

MTR	Mid-term report / Mid-term review
PB	Primary beam
PM/PR	Progress meeting / progress report
PWV	Precipitable water vapor (a measure of, in mm)
QA2	Quality Assurance Level 2
(R)MHD	(Radiative) Magneto-Hydro Dynamics
SOT	Solar Optical Telescope (onboard Hinode satellite)
SoW	Statement of Work
SP	Single-pointing
SQLD	Square-Law Detector
SSG	Solar Script Generator
WV/WVR	(Earth atmospheric) water vapor / Water-vapor radiometer.

2. Results

We follow the chronological line of the project development and start with the incremental improvement to the results reached during the 1st part of the project and described in the MTR (*Annex 1*) in the Section 2.1. The next Section 2.2 handles the project conclusions, made based on those results of the particular tasks, generalization, and recommendation for the implementations of the enhanced Solar ALMA ObsMode.

2.1 Improved simulations and analysis of the phase-errors introduced by the atmospheric turbulence

The results presented here are related to Sections 4.1 – 4.3 of the MTR (deliverables DS1.1.1, DS1.1.2, and DS1.1.3) and represent significant improvement and extension of the preliminary outcomes reached in the 1st half of study. Most notably, the new, realistic model for the inclusion of the atmospheric turbulence increases credibility of our simulation results, what is essential for achieving the ultimate goal of the study – the table of maximum allowed configurations for solar observations presented in the Section Error: Reference source not found. In addition to improved simulations, also the analysis of the images re-constructed from the simulated visibilities is enhanced in order to quantitatively asses the ability of the self-calibration loop in removing the fast phase errors via fidelity between input-model and reconstructed images.

2.1.1 New model for the turbulent atmospheric refraction

Already the 1st half of the study, summarized in the MTR, has brought significant improvements in this direction to the earlier attempts in this topic, namely when compared to the previous solar development study (Wedemeyer et al., 2020). The features advancing the older approaches have been namely:

1. Using of flexible simulator tool instead of **simalma()** task for generating visibilities.
2. Having full ALMA heterogeneous array, including 7m-12m mixed baselines to capture the actual setup for solar observations,
3. Using high-resolution and, at the same moment, broad field-of-view image as an input model in order to reach broad span of the Fourier components in the input-model data.
4. Modeling the atmospheric turbulence via **ATMOS.py** (Ed Fomalont 2021) instead of CASA simulator tool and its methods **sm.settrop()** and **sm.setnoise()**, which are considered inappropriate for this task (L. Maud, personal communication).

Details of the simulation of visibilities, their perturbation by the atmosphere-generated errors via ATMOS, image reconstruction by means of the self-calibration loop suggested as the *Approach 1* for fast phase error corrections, and measuring the performance of the self-cal correction via calculating imaging fidelity between the input ideal model and resulting reconstructed image can be found in the MTR Section 4.1.

Nevertheless, in order to improve credibility of our simulations and asses the influence of the specific model of the turbulent atmospheric refraction to the entire simulation results, we have switched to the new, more realistic model for varying phase delays caused by the terrestrial atmosphere. The new model commonly known as *phase-screen* approach, is advanced namely in expressing influence of the

larger-scale disturbances. It has been kindly provided by Luke Maud and Yoshi Asaki (Maud et al., 2019) together with introduction, how to use it. The correct simulation of large-scale disturbances is an important feature of the model, as in the Band 3 we aim at observations in the array configuration C-7 with baselines longer than 3 km and the prospective estimation based on scaling of the density of visibilities in the uv space shows (see concluding Section 4) that it should be possible to use yet longer baselines in the (not yet commissioned for solar observations) Bands 1 and 2.

We use the same input model (high-resolution solar image from Hinode/SOT), simulation of the ideal (or just instrumentally-corrupted) visibilities using the **sm** CASA tool for heterogeneous 7m+12m array, and the same procedure (self-calibration loop) for subsequent interferometric image reconstruction as in Section 4.1 of the MTR. Only difference is in application of the atmospheric turbulence, now we use the phase-screen model by Maud et al. (2019).

First, because in our simulations we focus on the self-calibration, there is no phase-reference source (calibrator) in the model. In order to reflect it, the code for the phase-screen model has been modified in the following way:

```
#Use assignPhase to generate phase screen
#parameters
res = 1
integration = 2 #sec
wspeed = 10
scaleh = 1000
maxBl = 4000
targetid = '0'
#phaseid = '0'
targetrms = 30
assignOutput = assignPhase(caltab=cal_table,caltabout=cal_table_out,
                           res=res,integration=integration,
                           wspeed=wspeed,scaleh=scaleh,maxBl=maxBl,
                           targetid=targetid,phref=False,
                           targetrms=targetrms,applyit=True)
```

We present here results for simulated observation in Band 5 in the array configuration C-6. The observing conditions has been simulated at two levels, characterized by the phase RMS measured over the reference time interval of 1 minute. Figure 1 shows the results for phase RMS 15 deg (over the 1 minute reference interval), Figure 2 for 30 deg. The first value corresponds to rather good observing conditions, the latter represents a typical or rather worse weather (Maud et al., 2017a). The upper panel in both Figs shows the map of ‘clouds’ – i.e., the distribution of the optical path over the antenna array. This distribution is advected over the antennas in the horizontal direction (x-coordinate in the map) with a wind speed that can be selected in the model; here we use 10 m/s. Time resolution is set to 2 secs, which corresponds to ~10 integrations in the actual solar observations (later we switched to the actual duration of solar-data integration of 0.192 s, the results are presented below). Middle left panel shows the phase errors as a function of time expressed as the index of 2-secs integration for each of the 53 antennas in the combined array (12m + 7m). Middle right shows the phase RMS as measured over entire simulated duration of observation, which is a 10 minute science-target scan. It is plotted for each antenna (53 points), the antenna is characterized by its radial distance from the reference antenna selected in the middle of the array. Bottom two panels display simulated atmospheric structure function (SSF) for the given parameters of the phase-screen model. Left panel is for the actual duration of the scan, the right one displays also the parameters for the reference time of 1 minute.

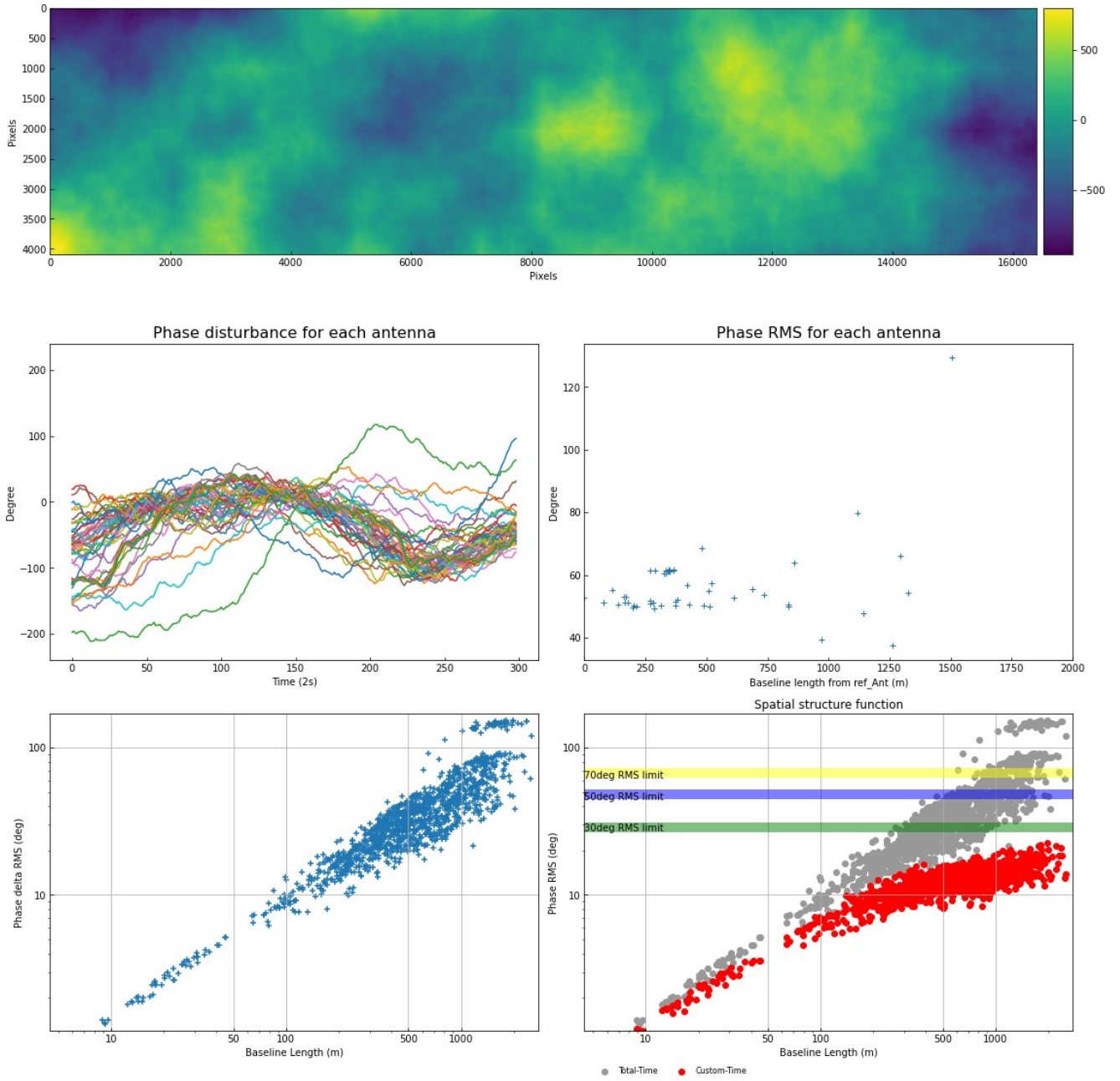


Figure 1: The phase-screen model for phase errors introduced by a plain advection of the turbulent structures in the terrestrial atmosphere over ALMA array calculated by modified code of Maud et al. (2019) for ALMA observations in Band 5 for the array configuration C-6. The reference phase RMS over the 1-minute interval is 15 deg, the actual duration of the simulated scan is 10 minutes. See text for explanation of the displayed panels.

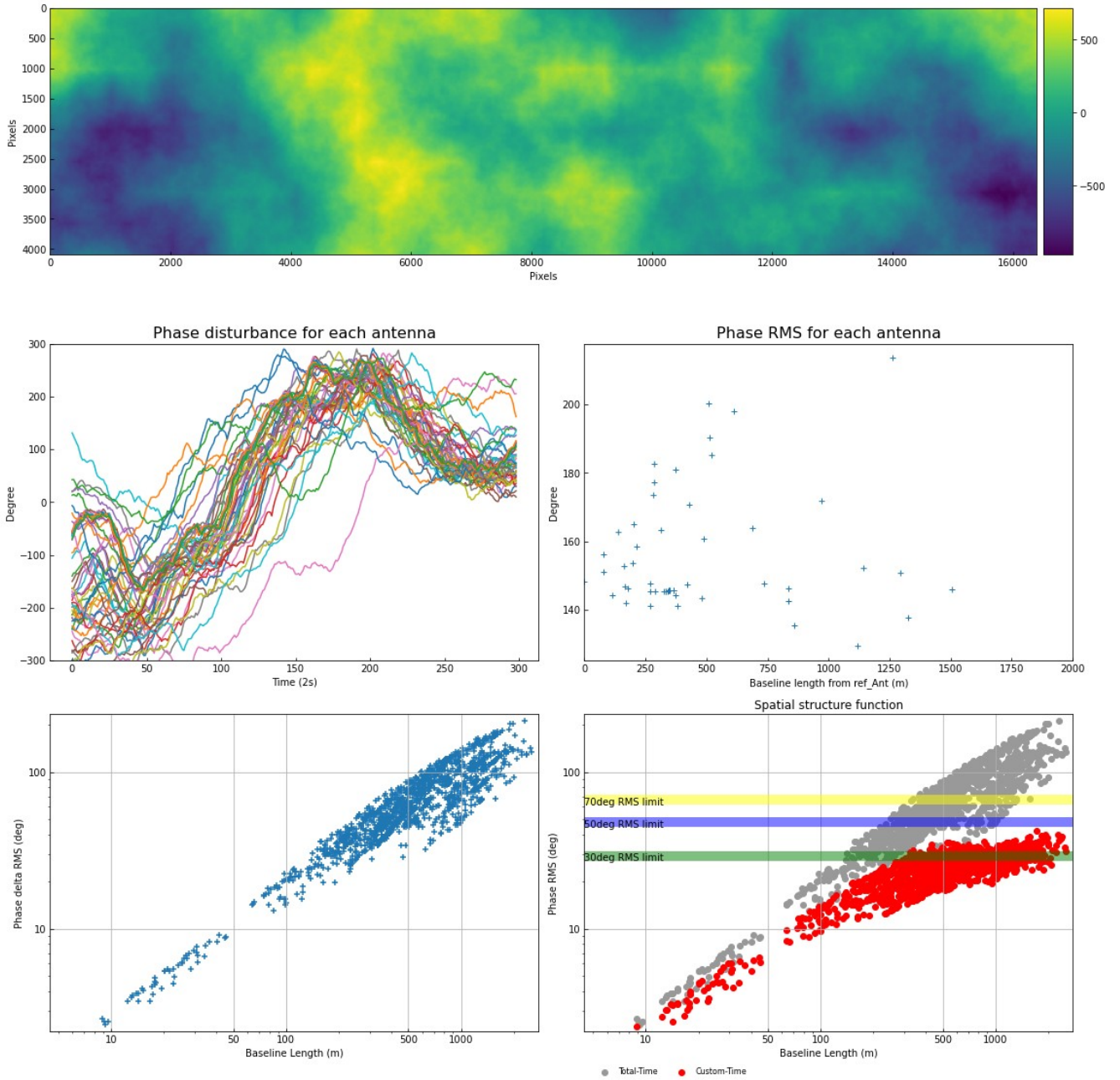


Figure 2: Like in Figure 1, now for the higher value of the phase RMS over the reference time interval of 1 minute. This correspond to typical / rather worse weather conditions. See details in the text.

In order to assess the impact of using different, more accurate model for phase errors, rooted in the atmospheric turbulence, to the validity of results found in the 1st half of the study with the ATMOS.py code, we applied the output of the new phase-screen model to perturbation of our simulated visibilities. These have been subsequently used for making the reconstructed interferometric image via the self-calibration procedure with consecutively hierarchically diminishing *solint* interval, described by the algorithm in Section 4.1.3 of the MTR (the full code listing is available in the MTR, too, as its *Annex 4*). Results of this comparison studies are presented in Figs. 3 and 4.

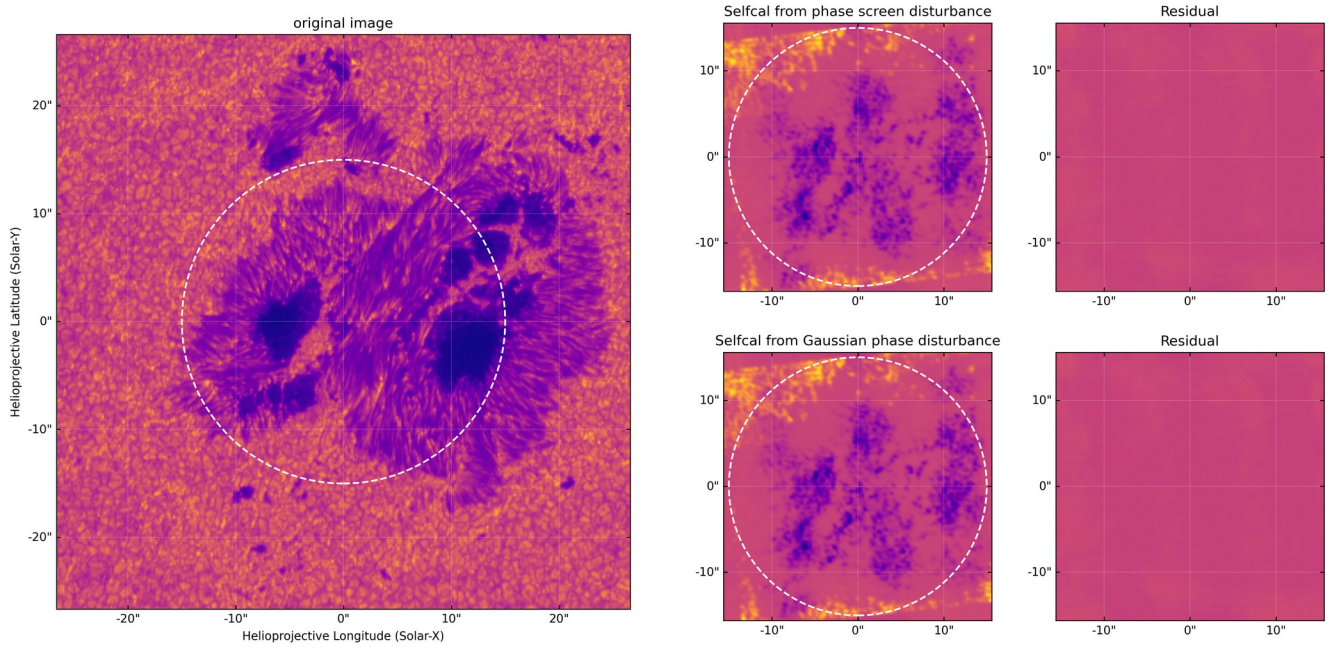


Figure 3: Comparison of images reconstructed by our hierarchical self-cal procedure (Approach 1) from simulated visibilities perturbed by (i) phase-screen model – two upper-right panels, and (ii) Gaussian-distribution scattering of the idealized atmospheric structure function (lower right panels). For both approaches the phase RMS over 1 minute reference interval was 30 deg. The input model image (Hinode/SOT) at the left panel is common for both methods. Simulated observation in Band 5, configuration C-6.

Figure 3 shows simulated observation in Band 5, array configuration C-6 (incl. 7m antennas) based on the same input model (optical solar image from Hinode/SOT) as described in the MTR Section 4.1.1 (the left panel). We then used the ATMOS.py and phase-screen models with the comparable structure function represented by phase RMS of 30 deg over 1 minute interval for perturbing the visibilities obtained by simulation. Both measurement sets has than been processed by the hierarchical self-cal procedure and resulting reconstructed images (and their residuals) compared. Results for the new phase-screen model are shown in upper-right panels, their counterparts for the phase errors with the means corresponding to the idealized atmospheric structure function with Gaussian scattering are shon below them. While the reconstructed images show differences up to 5%, the difference in residuals is negligible.

Figure 4 presents comparison between the two approaches to modeling of the atmospheric perturbations in its time evolution over a simulated scan (10 minutes). We show, how the qualitative performance metrics for the self-calibration procedure, defined in the MTR Section 4.2.4, vary with time. Apparently, for the more realistic model of the phase perturbations based on the phase-screen approach, our procedure reaches even better results, with much less frame-to-frame jumps in the image, that are present because of not being captured and fixed by the hierarchical self-calibration. Most likely the reason for that result is in the inappropriateness of the Gaussian perturbation method for the super-fast (sub-second) variations of the phase errors: The random choice of the phase error in the next integration is not in any way correlated to the phase delay (or optical path) just a little moment ago. In the real atmospheric flow, which is continuous, this is actually not possible. Hence, the ATMOS.py

model represents rather upper limit of the negative impact of the atmospheric turbulence to the phase delays.

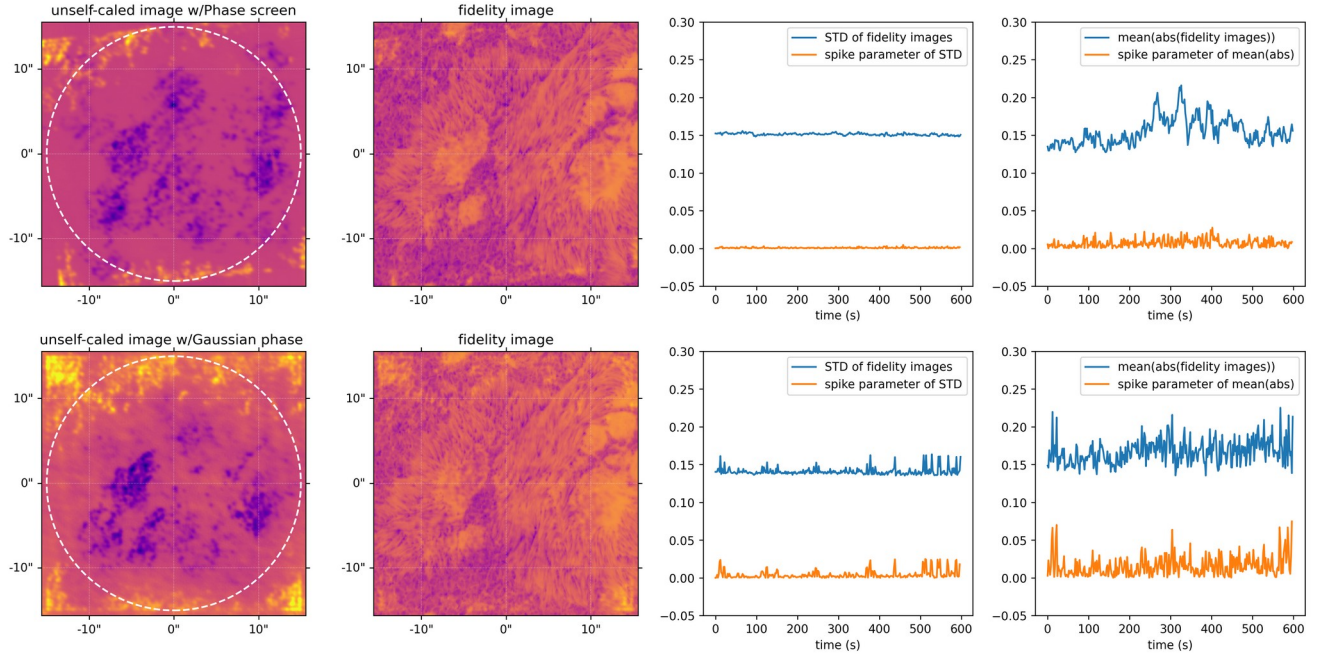


Figure 4: Comparison of quantitative metrics for the performance of the hierarchical self-calibration loop for the phase variations based on (i) the phase-screen model – upper row, and (ii) Gaussian perturbations (lower row). Presented are standard deviation of the fidelity images and the frame-to-frame sudden changes expressed as the spike-parameter (as defined in MTR Section 4.4.2).

The results in Figs. 3 and 4 are available also as the full-time cadence movies in the project data repository http://wave.asu.cas.cz/shared/ESO/final_report/phase_screen/. There are two files, one for the ATMOS-based and one for the phase-screen simulations. In this case, the time-cadence used for the modeling of the atmosphere has been set in line with our observation-data integration time to 0.192 s. In addition to the performance metrics displayed in the figures above, the comparison between series of the raw frames (without self-calibration) and self-calibrated is presented. One can clearly see that the phase-screen provides more realistic model for the image distortions: The raw (non-selfcal) movie exhibits clear drifts in position (shaking) as there is a tilt in the planar wavefront as the first term of the phase error. Similar features can be seen in non-corrected time series of images obtained from the real observational data. Contrary, the Gaussian perturbation model shows rather local deformations in the image accompanied by (apparent) localized changes in brightness.

Let us conclude this topic with the following main findings:

- The new phase-screen model for atmosphere-generated variations of the phase delay represents definitely more realistic approach to the ATMOS.py code (which is based on Gaussian perturbations with the means corresponding to the ideal structure function) that has been used in the 1st part of the study. The non-corrected (without the self-cal being applied) simulated interferometric movies exhibit similar features like these produced from the actual observations – namely wave-front tilt, and, vice versa, less artifacts like sudden jumps in phases, which we

can see in the Gaussian-perturbation model, which has essentially zero coherence time between two consecutive modeled phase perturbations.

- Albeit the phase-screen model is built on different backgrounds, the validity of the results found in the 1st half of the study using the ATMOS.py code remains untouched – difference in the resulting reconstructed interferometric images between both models is less than 5%. The array configuration limits found for each ALMA observing band via earlier ATMOS-based simulations are preserved in the sims involving the phase-screen method, too.
- With either method it was found that for non-extreme weather conditions (phase RMS 30deg over 1 min reference interval) we are able to fix the phase errors by hierarchical self-calibration loop up to the very limit of the maximum baseline length: If we use yet more extended configuration, we are not capable to produce image of reasonable quality even in the case when atmosphere is ‘switched off’. In other words, the actual limits for the array extension are given by the too much diluted *uv*-space coverage, which current CLEAN-based imaging algorithms are not capable to handle (see Fig. 6 in the MTR).
- As it can be seen in Section 2.4.2.1 Setup of the observation, rather than the regular atmosphere-generated phase perturbations or their specific model there can be another limiting factor for the maximum usable array extension – the number of available antennas. This is apparently related to the issue described right above – the low density of points in the *uv* space.

2.1.2 Additional metrics for assessment of the self-calibration performance

A question arises up, how credible is the procedure of hierarchical self-calibration for image reconstruction, i.e., how far is the high-cadence time sequence of the reconstructed interferometric images from its idealized counterpart, which we would have seen if we had the perfectly resolving telescope and no interleaving medium (namely the terrestrial atmosphere). In case of simulated images, the natural choice is the image fidelity *IMF*, defined as (Pety et al, 2001, ALMA Memo Series 398):

$$IMF = \frac{|(M_{ideal})|}{|(M_{ideal} - I_{rec})|},$$

where M_{ideal} is idealized input model (Hinode/SOT high-resolution solar image in our case, with possible re-gridding applied – see Section 4.1.1 in the MTR), and I_{rec} is one reconstructed image in the time series obtained by our self-calibration procedure. Hence, for the time sequence of images we obtain the IMF(t) also as the time series (see, e.g., Fig. 4, the right panels).

Another useful metric is the power spectrum of the Fourier components contained in the reconstructed image. Again, this could be compared directly with the same quantity for the idealized model and see, whether all scales have been actually reproduced or not, i.e., whether the procedure does not act as undesirable filter introducing the frequency-domain distortions.

We have used these metrics in the 1st part of the study quite intensively (e.g., Figs. 14 – 18, Fig. 22 in the MTR), displaying usually the inverse value of the fidelity 1/IMF, as we are interested rather in searching for the weak points of the procedure manifested by the discrepancy between the original model and the reconstructed image.

In the 3rd revision of the MTR, accepted in July 2024 we have introduced another new metric for the self-cal performance quality, as per one of the Action items in *Recommendation for the 2nd half of the study* (Annex 2). It is the *spike parameter* $SPP(t)$, again being defined with the time dependence as

$$SPP(t) = \frac{\left| \sum_{ij} [I_{rec}(i, j; t+1) - I_{rec}(i, j; t)] \right|}{\sum_{ij} I_{rec}(i, j; t)}$$

The spike parameter is measured in high-cadence series of the self-calibrated images only and represents a measure of normalized difference between two neighboring frames $t+1$ and t , where i and j have meaning of the pixel coordinates in the image. Ideally, its value should be zero because – in line with the basic idea of *Approach 1* – the frame-to-frame differences at the shortest time-scale should be caused solely by the phase errors and these are supposedly removed by the self-calibration procedure. Hence, any jump between two frames is detected by increased value of SPP. This metric is used, e.g., in Fig. 4 (right panel). It is not as robust as the first two criteria (IMF and Fourier scale analysis, which were actually used already in the previous solar study by Wedemeyer et al., 2020), on the other hand, it is not limited to the case, where we have apriori knowledge about the ideal model and can be thus utilized also in analysis of the image-series reconstructed from the observed data, when this procedure is going to be applied in practice. It can be complemented by the time series of the residual phase-error, the auxiliary metric of the self-calibration quality displayed, e.g., in Figs. 8 – 13 of the MTR.

During the 2nd half of the study we have been working mostly with the test-observation datasets and therefore we have focused on devising such additional metrics of the self-calibration/imaging loop performance that are independent of the apriori ideal model, as we have none in this case. One possibility is a stability of the phase solution over frequency channels. In the correct gain caltable one expects the phase solution to be only weak and smooth function of frequency – c.f., for example, the *phase vs. frequency* diagnostic plots created during manual QA2 for the band-pass calibrator – if the complex gain solution is not a smooth and weak function of the frequency channel, the band-pass calibration is deemed as failed. In the solar self-calibration procedure, which works in multiple – typically five – hierarchical steps with consecutively diminished *solint* down to the level of integration time, we have more ways how to assess the total phase solution over all steps

- Using **gaincal()** solution between the MODEL column of the very last self-calibrated MS and the DATA of the very initial, just standard-way calibrated (no self-cal) MS.
- Re-gridding the incremental phase solutions in gain caltables for each hierarchical step of the self-cal loop to the finest time resolution and accumulating the partial results to the total phase solution, and
- The phase delays may be simply assessed as a phase differences between the MODEL column of the final data and DATA column of the initial MS, keeping just the baselines where the first antenna in the pair is the reference antenna.

The last method provides the approximate single-baseline solution for each frequency channel separately and thus can be used for extraction of the *phase vs. channel* dependency. From the per-channel phase solutions we can make an average (here we neglect the small frequency dependence of the phase delay for the same optical path as the SPWs are as narrow as 0.5% of their nominal frequency), having thus just one representative point per integration and SPW (like in normal gain table) and, moreover, the standard deviation – measure of how the phase solutions in the specific time instant (integration) are spread over the frequencies. The per-channel phases has to be unwrap (by

+/-180 deg) before they are subtracted because the wrapping to the standard angle interval occurs in different moments of time for different channels. Results of this procedure are shown in Figs. 5 and 6. Figure 5 shows comparison between the standard `gaincal()` solution for accumulated (over the five steps of our hierarchical self-cal loop) phases and the phase difference (final MODEL – initial DATA) averaged per channel for antenna DA42 and our Band 5 test observation data (detailed description of the MS is in the MTR). The 1st panel (in the order left to right, top to bottom) represents the initial DATA column, the 2nd final MODEL, the 3rd is their difference averaged over channels. The 4th panel is the standard `gaincal()` solution over final MODEL and initial DATA columns and the 5th represents the difference between the approaches in panel 3 and panel 4. The last panel is the standard deviation corresponding to averages displayed in panel 3.

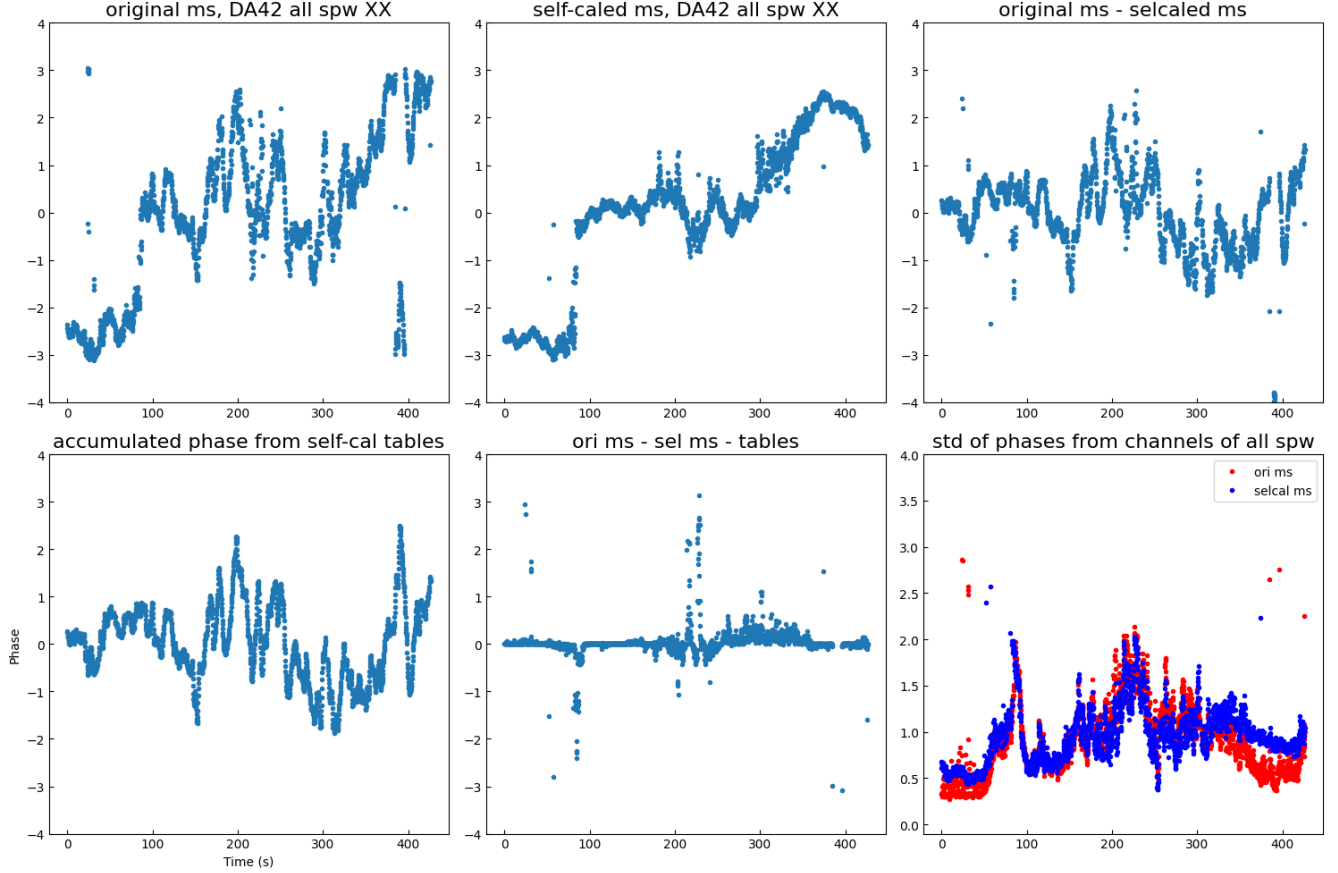


Figure 5: Comparison between phases obtained: (i) by the standard `gaincal()` solutions accumulated over all five steps of the self-calibration loop, (ii) as averaged per-channel phase differences between original and final self-calibrated MS. See detailed panel description in text.

As it can be seen in panel 5, at most time instants the phases inferred by both methods correspond each other. The little difference reflects more accurate `gaincal()` approach as it takes into account multiple baselines and minimizes the phase-closure error, while the per-channel approach uses just a single-baseline solution (between DA42 and the reference antenna, which is DV11 in our case). The few ‘singular’ time instants, where the difference is significantly higher, correspond to the integrations, for which the standard deviation displayed in panel 6 has a spike.

High standard deviation of per-channel phases may indicate failed self-calibration loop solution for the given frame, so we propose watching this quantity as one of the QA criteria of the entire hierarchical self-calibration procedure. The method has been further developed into *Directional phasor formulation/Circular statistics* and it is presented as *Annex 6* to this report.

Figure 6 presents closer look at the *phase vs channel* solution – its panels display per-channel phase solutions at selected time samples for different color-coded SPW.

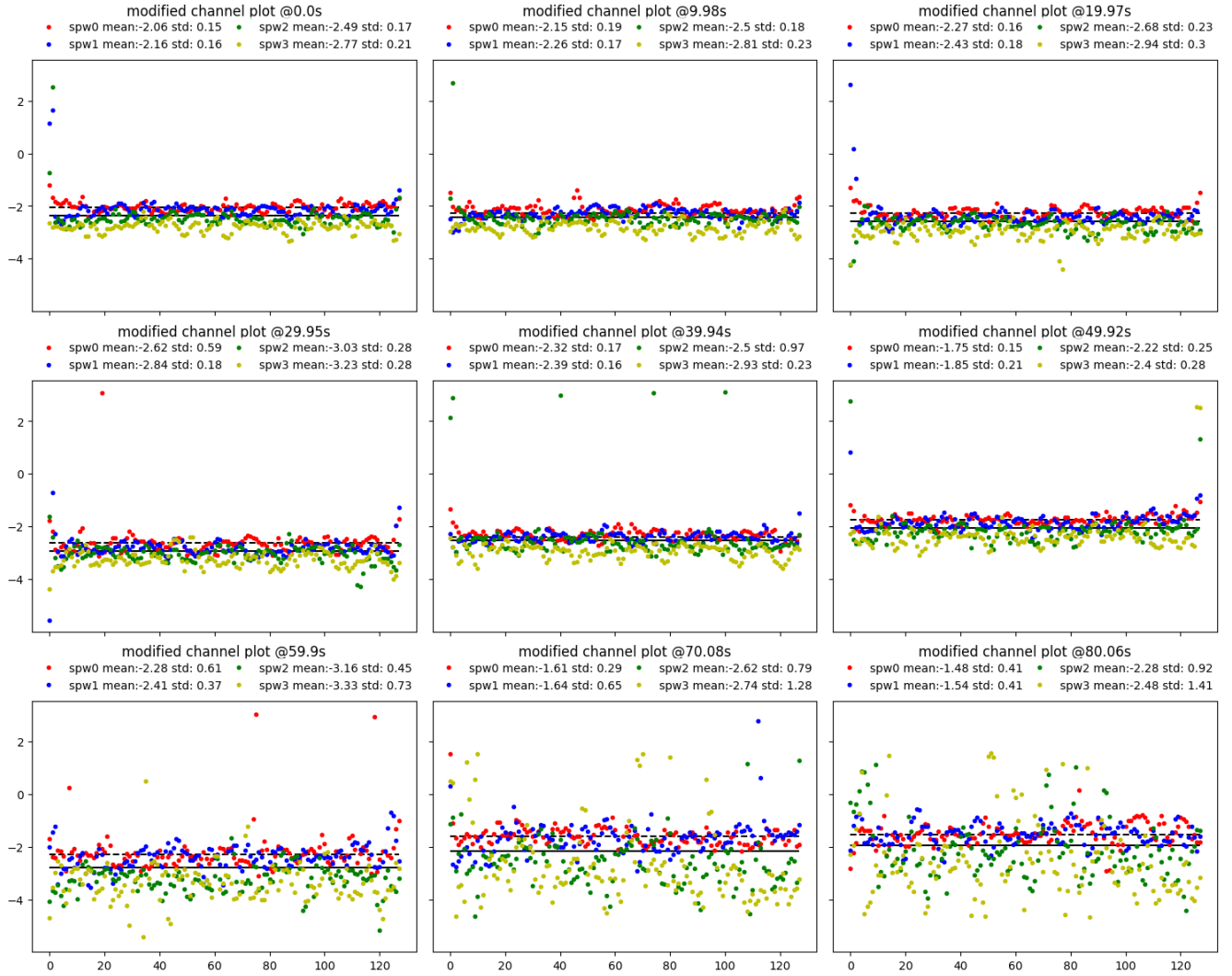


Figure 6: Variations of phase delays vs. frequency channel, the SPW number is color-coded in the legend. The data are for antenna DA42, each frame represents different time sample described in the respective panel title. The standard deviations computed for each time instant are plotted as panel 6 in the previous figure.

Identifying possible issues with self-calibration loop solution for specific time instants, we have turned our attention to another check of the procedure – we wish to make sure that imaging for the particular time instant actually converges. As it has been explained in the MTR, for the high resolution imaging

the scale separation between the *cellsize* and *imgsize* is large even in the case of single-pointing observations (typical image has 2kpx squared). Such a large span of scales in the image requires multi-

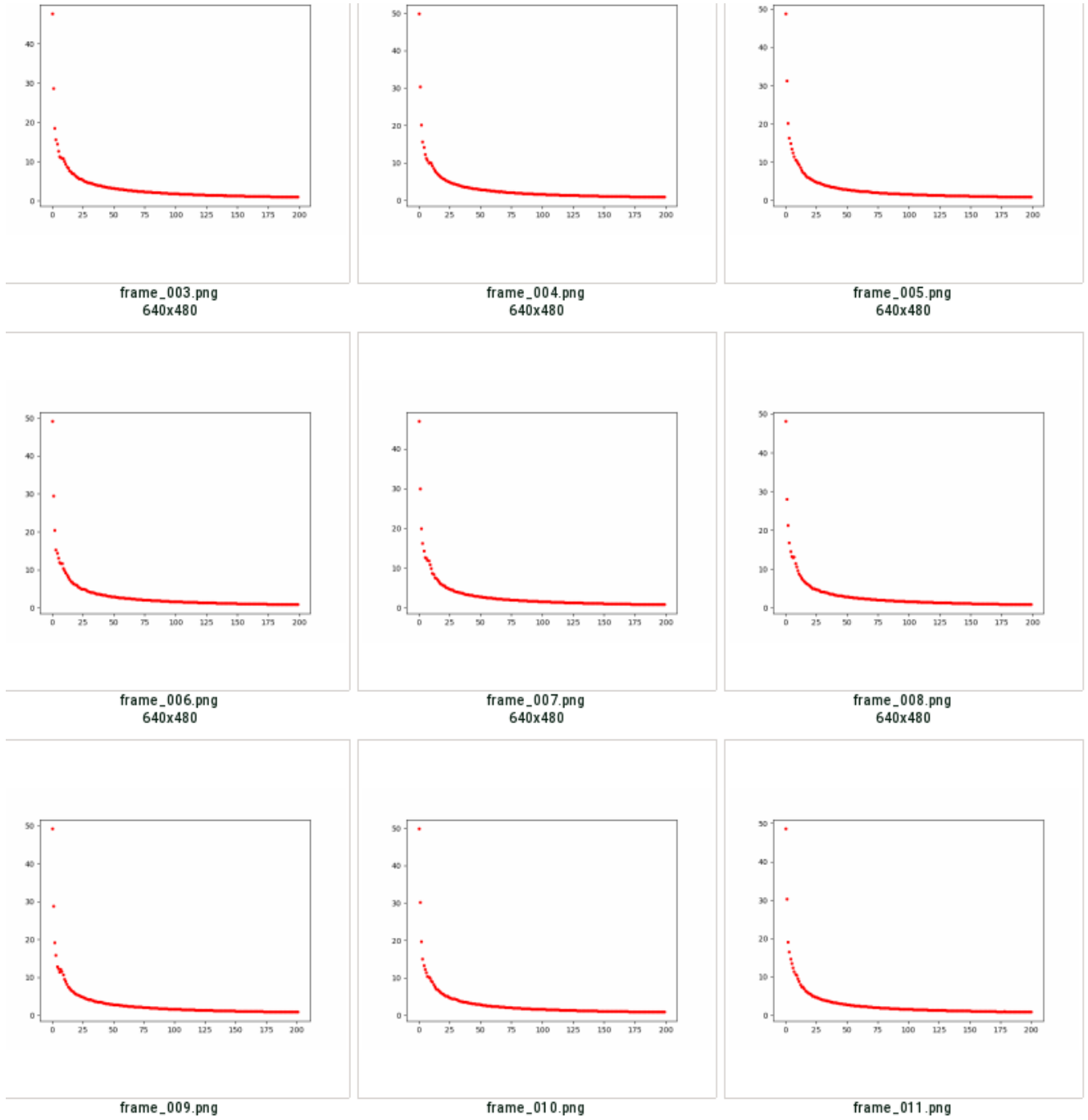


Figure 7: Convergence plots for the multi-scale *tclean()* used in our self-calibration/imaging loop. For each movie frame, that has been formed from the time-range corresponding to solint interval in *gaincal()*, the peak residual flux is displayed vs. the major cycle count.

scale cleaning. The multi-scale *tclean()* has, nevertheless, sometimes issues with convergence. Indeed, practical experimentation with the *tclean()* settings has shown, that the maximum range of the scale

vector in *tclean()*, which yields stable solution is **scales = [0,7,15]**. Moreover, we have to limit the number of iterations in the minor cycle to 100, and set the gain limited to 0.07. The *timerange* parameters selects the integration(s) used for the particular *solint* time interval in corresponding preceding *gaincal()* call in the self-cal loop:

```
converg = tclean(vis = msc, imagename=imgName, timerange=tRange,  
                scales = [0,7,15],cycleniter=100, gain=0.07,..., fullsummary=True)
```

We use *fullsummary* parameter set to ‘true’ and save the convergence-tracing dictionary in the output variable *converg*. We extract peak-residuals for each major cycle

```
res=converg['summaryminor'][0][0][0]['peakRes']
```

and plot them for each frame of the self-calibrated movie, i.e. for the time of the self-cal solution interval defined by the *timerange* parameter. Small sample of the grid of such convergence plots is shown in Fig. 7. This is another QA diagnostic for the self-calibration/imaging procedure that should be observed during the processing of high-resolution solar data.

To sum up this section – we propose the following QA metrics for measuring performance of processing of the high-resolution solar datasets via *Approach 1*:

- The spike parameter and residual phase errors.
- Standard deviation of the distribution of self-calibration phase solutions over frequency channels.
- Convergence of the multi-scale *tclean()* measured by residuum peak flux.
- Span of the scales in the image assessed by Fourier power spectra.

In case of the simulated MSs, where the ‘ground truth’, i.e. the ideal model is known, these metrics can be supplemented by the image fidelity and frequency-domain distortion expressed as normalized difference between Fourier power spectra of the model and reconstructed image.

2.1.3 Results of the 2023 test data processing

Because the hierarchical, multi-step self-calibration/imaging loop for the high-resolution (each image is 2kpx square) and high-cadence (~5 frames per second) solar data is very CPU-time demanding, the test data from the end of 2023 has been only partly processed by the time of the Mid-term review. The MTR report thus contains just preliminary results for the Band 3 MS, while the Band 5 data had passed just the standard QA2-like processing by the time of the MTR.

During the 2nd half of the project we have managed to apply our Approach 1 (hierarchical self-cal/imaging loop described in the MTR Section 4) to both datasets. The Band 3 data has been processed up to the level of one image per integration, with the Band 5 MS we have finished with one image per 10 integrations, albeit the *solint* parameter in self-calibration *gaincal()* has been consecutively diminished down to the integration level, in order to remove the fast phase variations induced by the atmospheric turbulence.

The fully processed datasets are inherently digital and therefore we provide here URLs to accessing the movies in the project data on-line repository

- Band 3 dataset: http://wave.asu.cas.cz/shared/ESO/final_report/test_data/Band3/

- Band 5 dataset: http://wave.asu.cas.cz/sharedESO/final_report/test_data/Band5/

Along with the set of quite large movies (the integration-cadence based movie has 300GB) there are descriptions of each file directly in the same directory of the data repository.

We have performed analysis of the typical dynamic scale in the Band 3 movie by measuring the light-curve at the specific location in the image, followed by frequency decomposition using periodogram and Welch's methods. Results are shown in Fig. 8.

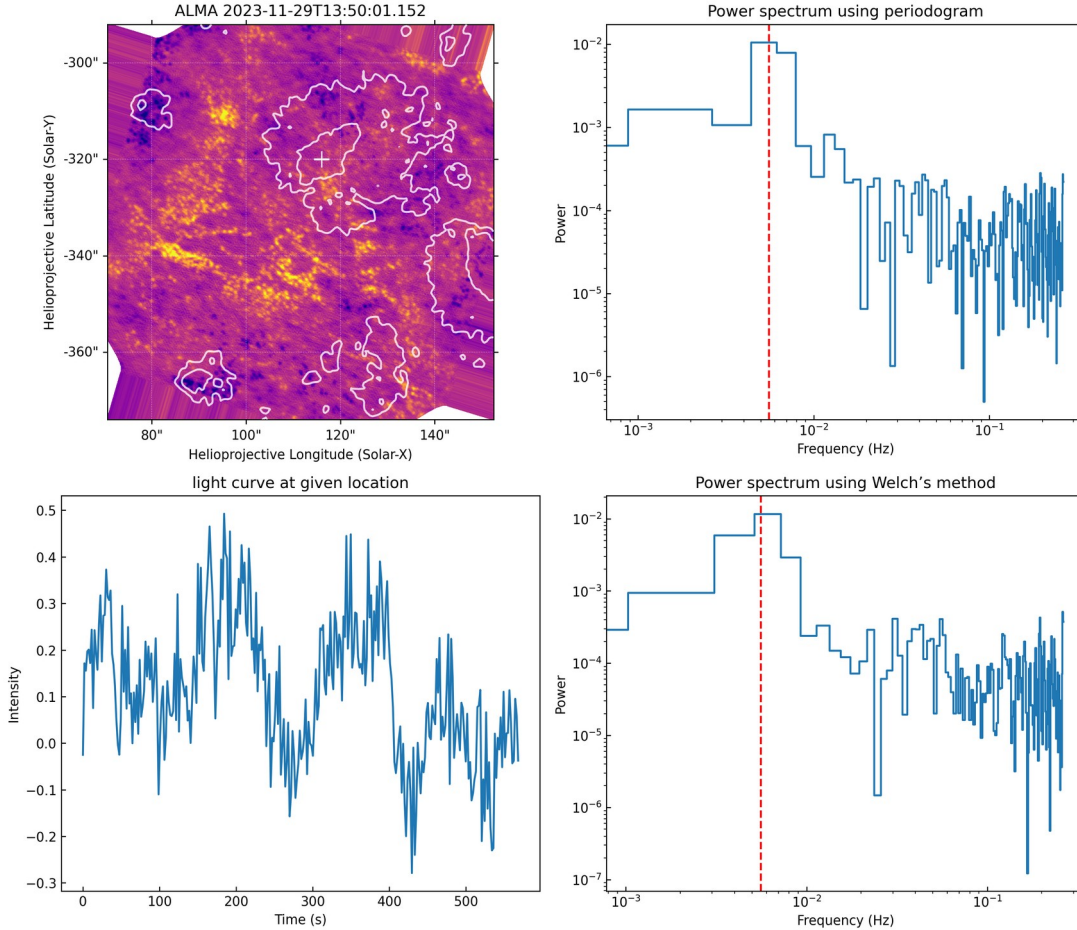


Figure 8: ALMA Band 3 map with indicated point (the white cross) selected for dynamical analysis (top left), the light-curve determined in that point (bottom left), and frequency analysis of the LC constructed via two different methods (right).

Basically, there is no real motion visible on the time-scales less than 2 secs for this specific dataset displaying essentially quiet part of the solar chromosphere. This is in line with our estimation based on the signal propagation with the typical sound speed (0.2 arsecs, which is our current resolution, corresponds to 140 km at the Sun, having the sound speed in the 10000 K warm chromosphere about 10 km/s and the Mach number in the convective-generated shocks ~ 3 , the typical transit time is ~ 5 secs – see detail in the MTR). However, we would like to point out that this need not to be true in general – the active regions at the Sun, most notably the emission kernels in the solar flares, may show faster changes as their propagation – in case of the flare kernels – is determined by the apparent motion of the

anchor of the field line currently reconnecting much higher up in the solar corona. Nevertheless, for the quiet chromosphere it justifies decreased cadence of imaging, still keeping the full time-resolution for *gaincal()/applycal()*, which we have utilized in the Band 5 dataset.

As a sample of the fully-processed datasets we present here just two snapshots in Figs. 9 and 10.

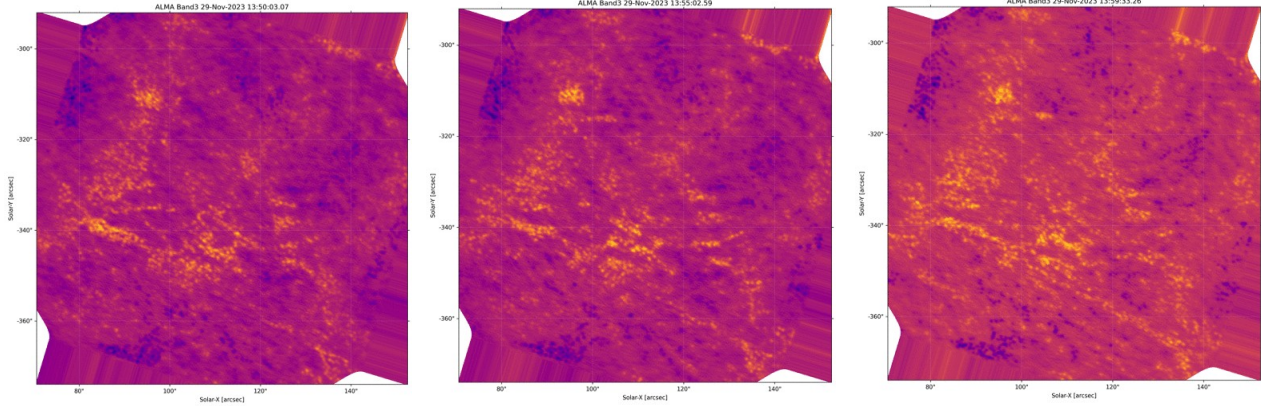


Figure 9: Three frames of the full movie of the Band 3 test data, taken at 13:50, 13:55, and 13:59 UT, consecutively from left to right. Time evolution at the very small, in ALMA images never before resolved scales, is obvious.

Figure 9 shows the ALMA Band 3 interferometric maps in three consecutive times. The presence of very small scale structures, never seen in the ALMA images before, is apparent, together with their dynamics.

Fig. 10 displays one snapshot of the Band 5 ALMA interferometric map (and its residuum) accompanied by the AIA 304Å image and HMI continuum image. The AIA wavelength is sensitive to the solar chromosphere, in the Band 5 we are looking a bit deeper. The HMI optical continuum shows the underlying photospheric structure and serves as the context image. The AIA image is overlaid with the ALMA brightness contours in order to see correspondence between the bright emission features in both images.

As a brief summary of this part we can say

- Our previous simulations identified the configuration C-7 and C-6 as upper limits for solar observations in Band 3 and Band 5, respectively. The test observations confirmed this statement as we are able to produce high-resolution solar images, which pass the QA criteria defined in Section 2.1.2. Additional metrics for assessment of the self-calibration performance in both observing bands. To that extent, the test observations provide us with a ‘gauge’ to the more general simulation approach.
- The Band 5 data has been actually observed in quite worse weather conditions (see the MTR for detailed observation description and the phase plots in the next section) than we used for simulations, yet we were able to reconstruct the images solely using *Approach 1* (the hierarchical self-calibration loop).
- For quiet solar chromosphere the decreased-cadence imaging is acceptable, yet using the full cadence data for self-calibration. This may not be true for the active regions, specifically flares.

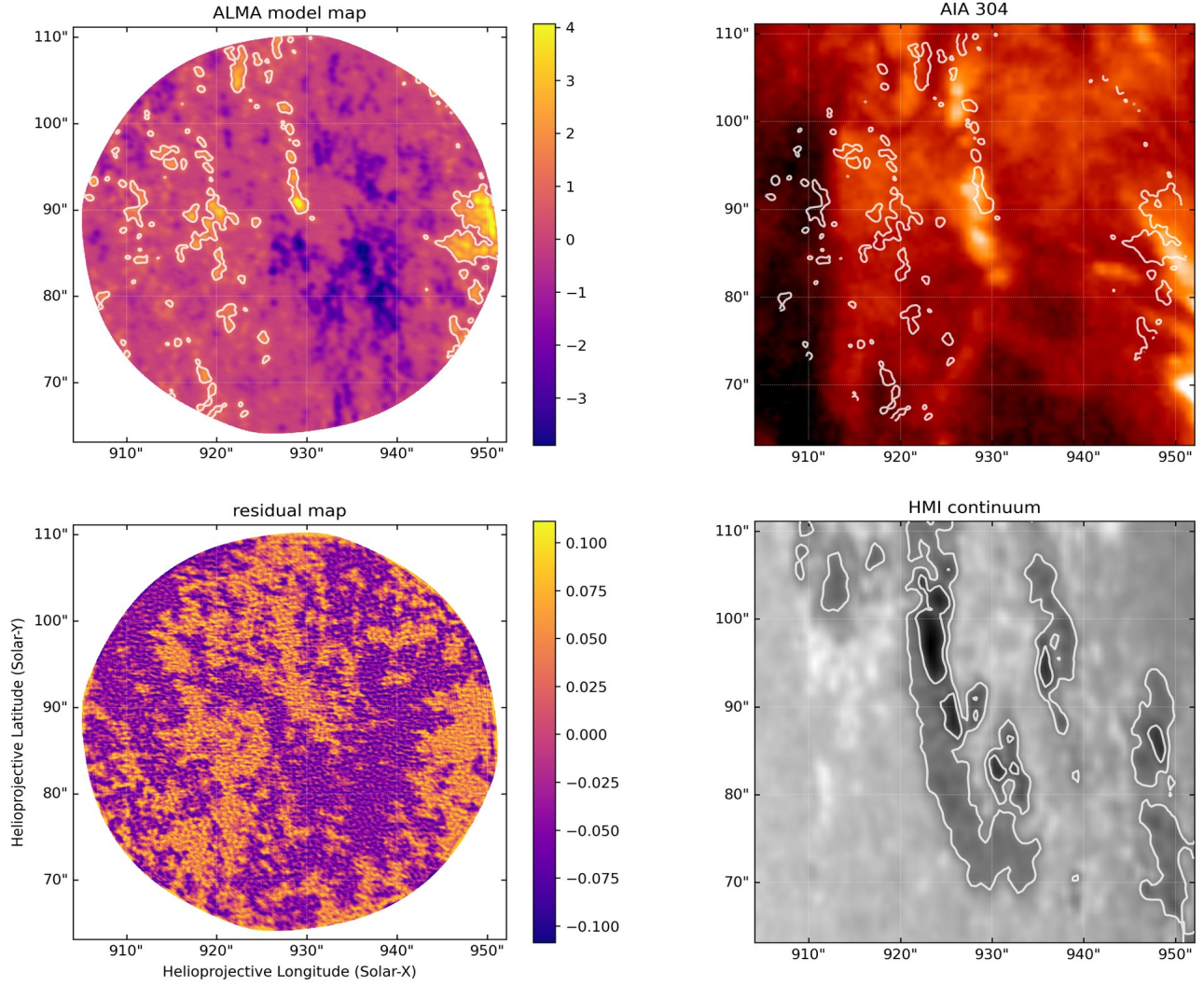


Figure 10: ALMA Band 5 interferometry map (top left), its corresponding residuum (bottom left - note different color scale in this case), and comparison of structures seen in co-observation by SDO/AIA (top right) and HMI. ALMA brightness contours are overlaid over AIA raster map for comparison of locations of the bright features in both images. The small displacement can be explained by a parallax (different points of observations for ALMA and SDO).

2.2 Water-vapor signal in the science SPWs in Band 5 applied to phase-delays corrections

This topic has remained under-developed during the 1st half of the study and thus we focused on it during the 2nd part of the project. The goal of this *Approach 2* (DS.1.2) is to provide an alternative to `WVR/wvrgcal()` that would be viable also for the solar observations, where WVRs are saturated in the on-source scans. We have essentially developed, extended, and tested the legacy idea of Richard Hills (2019), which is presented as the *Annex 3* to the MTR.

Here, after developing basic theoretical concept on how to calculate PWV proxy from the raw data (measured powers) in the science SPWs and its implementation as a collection of Python routines, we apply the methodology first to the band-pass calibrator scan. This is for three sakes: (i) Clear and easy comparison between PWV-based correction of phases with the exact *gaincal()* phase solution for the calibrator, (ii) clear and easy comparison between our *PWV proxy* and directly measured (by WVRs) PWV, and (iii), because we need not care about how much the variations of the signal in the science SPWs is influenced by variability of the target source – the calibrator is so weak that in auto-correlation (AC) signal we essentially see just the intrinsic emission of the terrestrial atmosphere (+ obvious instrumental noise). The more complicated case of the solar data deserves an extra chapter and it is handled in detail in the next part, in Chapter 2.3.

2.2.1 Theoretical background

We are going to use the *science* SPWs for extracting emission (in case of the weak source, typically band-pass or phase calibrator) or absorption (in case of the solar on-source scans) features connected with the atmospheric WV line at 183 GHz in Band 5. Fast variations of those features are supposedly connected with varying PWV and used to calculate a proxy quantity to the differential (i.e., the time-dependent differences to the time-averaged PWV) water-vapor column measure. We call this “*proxy dPWV*”, in the following analytical derivation it is usually denoted just as *dw*. Eventually, this *dPWV proxy* is transformed to the phase delays, similarly as in the *wvrqcal()*, and used for corrections of the fast-varying phase errors.

We have multiple possibilities for choosing the science SPWs for analysis – it can be auto-correlations (ACs) either in the channel-resolved SPWs – i.e. SPWs 5,7,9, and 11 in our Band 5 MS data, or their channel-averaged counterparts (SPWs 6,8,10, and 12). Alternatively, the SQLDs in SPWs 0,1,2,3 may be utilized for the goal. Anyway, even in the case of channel-resolved SPWs, we have to perform integration at least over part of the SPW channels, in order to reach sufficient signal to noise ratio. In the following we start with general radiative transfer over the terrestrial atmosphere with wet and dry components and arrive to a general expression for the dPWV proxy expressed in terms of variations of the brightness temperature (as a function of both time and frequency). Because we are in fact measuring voltages/powers, we have to transform the brightness temperature to the directly measured quantities using calibration equations. After doing that, we provide specific expressions for the dPWV proxy in the case of channel-resolved AC SPWs, and channel-averaged ACs/SQLDs.

Brightness temperature T_{sky} , which represents a signal contributed by both the terrestrial atmosphere and the celestial source, can be expressed as (see Richard Hills original analysis):

$$T_{sky} = \frac{1}{BW} \int_{spw} [\eta T_{src}(f) \cdot tr(f) + T_{atm}(1 - tr(f))] df \quad (2.2.1)$$

where η is the effective filling factor for the antenna primary beam (for the Sun observed on the disc it is 1.0), BW is the bandwidth over which we collect the signal (~2 GHz for channel-integrated ACs and SQLDs), and $tr(f)$ is the transmissivity of the terrestrial atmosphere

$$tr = \exp[-(w \cdot \tau_1 + \tau_0)] \quad (2.2.2)$$

In (2.2.2), w is the water-vapour column (PWV), $\tau_1(f)$ is the (frequency-dependent) opacity for PVW=1.0, and $\tau_0(f)$ is the ‘dry’ component of the opacity.

Here we ignored the forward and back-ward antenna efficiencies described in White et al. (2017) because, if needed, these essentially instrument-determined parameters (T_{spill} in Eq. (5) in White (2017) is approx. the ground temperature in the location of antenna, perhaps not changing during the scan) can be incorporated into factor inverse gain and the additive constant in calibration equations below, which are instrument-based parameters, too.

Now, we assume that the water-vapor content w as a function of time during the scan can be decomposed into the constant (during the scan) mean part $\langle w \rangle$ and a small fluctuation dw (c.f. Richard’s analysis; the dw is a mathematical symbol for the quantity that we refer as dPWV in the text)

$$w(t) = \langle w \rangle + dw(t) \quad (2.2.3)$$

where quantities in $\langle \rangle$ ’s mean time-averaging over the scan duration. With this, one can express the transmissivity (2.2.2) as

$$tr(f, t) = \exp[-(\langle w \rangle \tau_1(f) + \tau_0(f))] \cdot \exp(-dw(t) \tau_1(f)) \approx \langle tr(f) \rangle \cdot (1 - dw(t) \cdot \tau_1(f)) \quad (2.2.4)$$

Now, the source itself may have its own brightness variability, say its brightness temperature can be split into time-averaged (over the scan) part and a small fluctuation as

$$T_{src}(f, t) = \langle T_{src}(f) \rangle + \delta T(f, t) \quad (2.2.5)$$

After inserting the fluctuating transmissivity (2.2.4) and source temperature (2.2.5) into Eq. (2.2.1), we get

$$T_{sky}(t) = \frac{1}{BW} \int_{spw} (\langle T_{src}(f) \rangle + \delta T_{src}(f, t)) \cdot \langle tr \rangle \cdot (1 - \tau_1 dw) + T_{atm}(1 - \langle tr \rangle) + T_{atm} \langle tr \rangle \tau_1 dw(t) df \quad (2.2.6)$$

The integrated expression can be re-arranged as

$$\delta T_{src} \langle tr \rangle + \langle T_{src}(f) \rangle \cdot \langle tr \rangle + T_{atm}(1 - \langle tr \rangle) - dw(t) \tau_1 (\langle tr \rangle \cdot \langle T_{src} \rangle - \langle tr \rangle T_{atm} + T_{atm} - T_{atm}), \quad (2.2.7)$$

where the last two terms $+T_{atm} - T_{atm}$ have been added in order to combine the expression in the parentheses into more meaningful form (see Eq. (2.2.9) below). We have also neglected the term $\delta T_{src} \cdot dw$, which is second order in small fluctuations. Because the fluctuations of the intrinsic source brightness and fluctuations in the terrestrial-atmosphere transmissivity are uncorrelated, we can rewrite their time-average product as

$$\langle T_{src} \rangle \cdot \langle tr \rangle \approx \langle T_{src} \cdot tr \rangle$$

When we use the expression for T_{sky} in Eq. (2.2.1) to find its time-average (over a scan)

$$\langle T_{sky} \rangle = \frac{1}{BW} \int_{spw} [\langle T_{src} \cdot tr \rangle + T_{atm} (1 - \langle tr \rangle)] df, \quad (2.2.8)$$

the Eq. (2.2.6) can be rewritten as

$$T_{sky} = \langle T_{sky} \rangle - dw(t) \left(\frac{1}{BW} \int_{spw} \tau_1(f) \langle T_{sky}(f) \rangle df - \frac{T_{atm}}{BW} \int_{spw} \tau_1(f) df \right) + \frac{1}{BW} \int_{spw} \delta T(f, t) \langle tr(f) \rangle df \quad (2.2.9)$$

The intrinsic source-brightness fluctuations (the last integral) are unknown, so we neglect them for now (and append the corresponding term to the final result for completeness). We, however, return back to this term in the following Chapter 2.3, which handles the case of solar data. In order to explicitly distinguish between the quantities that are averaged over the frequency channels with total bandwidth BW , we add the mean index to them, while the time-averaging over the scan is designated by the angular brackets. With this, we eventually arrive to expression

$$T_{sky}^{mean}(t) = \langle T_{sky}^{mean} \rangle - dw(t) \frac{1}{BW} \int_{spw} \tau_1(f) (\langle T_{sky}(f) \rangle - T_{atm}) df \quad (2.2.10)$$

which relates the variations of the brightness temperature in the pass-band BW with the variations of the water-vapor column $dw(t)$ – i.e. our dPWV proxy.

2.2.2 From brightness temperatures to measured powers

We follow here the general solar ALMA calibration procedure as it is described in White et al. (2017) and Shimojo et al. (2017).

The sky brightness temperature is related to the received power via Eq. (5) in White et al. (2017):

$$P_{sky} = G \cdot (T_{RX} + T_{sky}) + P_0 \quad (2.2.11)$$

where G is the (generally channel-dependent) gain, T_{RX} is the instrumental receiver temperature and P_0 (in alternative notation P_{zero}) is the instrumental offset, which is zero in the case of auto-correlations and may have non-zero value for SQLDs.

In order to find the unknowns G and T_{RX} , ALMA antennas are equipped with the front-end ACD calibration device. This blocks (for a while, during the atmosphere / ATM scans) the sky view by the black-body obstacles at two different temperature levels – so called ambient and hot loads. The relation between temperature and corresponding power in the case of ambient, respective hot load thus reads

$$P_{amb} = G \{ T_{rec} + T_{amb} \} + P_{zero}$$

$$P_{hot} = G \{ T_{rec} + T_{hot} \} + P_{zero}.$$

From those, the gain G the effective receiver temperature (T_{rx} / T_{rec}) can be expressed as

$$G = \frac{P_{hot} - P_{amb}}{T_{hot} - T_{amb}} \quad (2.2.12a)$$

$$T_{rec} = \frac{T_{hot} (P_{amb} - P_{zero}) - T_{amb} (P_{hot} - P_{zero})}{P_{hot} - P_{amb}} \quad (2.2.12b)$$

Once G and T_{rx} are established for a given antenna, SPW, and ATM scan, they may be used for calibration of the AC power for the target – the BP calibrator or the Sun. With the BP calibrator the situation is simpler because it is observed with standard attenuation settings. So its corrected signal is

$$T_{sky} = (1 - e^{-\tau})\eta_l T_{atm} + e^{-\tau}\eta_l T_{CMB} + (1 - \eta_l) T_{spill} = \frac{P_{sky} - P_{zero}}{G} - T_{rec}$$

where P_{sky} is the measured (raw) AC or SQLD signal, and T_{sky} is the sum of contributions of the terrestrial atmosphere (the vastly dominant one) and the emission of the BP calibrator. This is what we are interested in – its variations follow the WV contents and can be re-calculated into phase shifts.

For the Sun, the situation is more complex: It is observed with enhanced attenuation and therefore yet another ATM sub-scans (off-source) are performed – these are the very first and very last sub-scans of the science-target scan – see the following scheme (Shimojo et al., 2017). These ATM sky-like sub-scans are performed with the same attenuation level as the Sun is observed.

M. Shimojo *et al.*

Standard Observing Sequence for non-solar objects



Observing Sequence for the Sun



Therefore (because of higher attenuation), we have a different gain factor G_{src} for the ‘true’-solar and solar-off scans:

$$P_{src} = G_{src} \{ T_{rec} + e^{-\tau} \eta_l T_{src}^* + (1 - e^{-\tau}) \eta_l T_{atm} + (1 - \eta_l) T_{spill} \} + P_{zero}$$

$$P_{off} = G_{src} \{ T_{rec} + (1 - e^{-\tau}) \eta_l T_{atm} + (1 - \eta_l) T_{spill} + e^{-\tau} \eta_l T_{CMB} \} + P_{zero}$$

In combination with the expressions above, this modified gain can be expressed by means of that ‘standard’ gain as

$$G_{src} = G \frac{P_{off} - P_{zero}}{P_{sky} - P_{zero}} \quad (2.2.13)$$

Example of the dual-load calibration results, showing the standard and reduced (solar) gains according to Eqs. (2.2.12a) and (2.2.13) are displayed in Fig. 11, full set being available online in the project repository. In spite some peculiarities, like that the attenuated solar gain is sometimes higher than the non-attenuated one and for some antennas both gain and T_{rx} somewhat oscillate with frequency, there is no visible excess in this procedure. However, as the final result dw is quite sensitive to errors in G and T_{inst} – see Eq. (2.2.14) below – even smaller inconsistencies in the ACD calibration may impact the outcomes.

From Eq. (2.2.11) the sky brightness temperature can be expressed as

$$T_{sky}(f) = \frac{1}{G(f)} \cdot P(f) + \left(-\frac{P_0}{G(f)} - T_{RX}(f) \right)$$

and after inserting it into the Eq. (2.2.10), relating the brightness-changes to water-vapor column variations, one can express our searched dPWV proxy as

$$dw(t) = - \frac{\int_{BW} \frac{1}{G(f)} \cdot (P(t, f) - \langle P(f) \rangle) df}{\int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \tau_1(f) df} \quad (2.2.14)$$

In this universal relation all quantities at the RHS are in principle known – the power $P(t, f)$ and its averaged (either in over pass-band channels or time of the scan) values are found in the science SPWs of the MS, gains and the total ‘instrumental’ temperature

$$T_{inst}(f) \equiv \frac{P_0}{G(f)} + T_{RX}(f)$$

can be calculated from the Eqs. (2.2.12b) and (2.2.13) above, where P_o required for SQLDs is found in the ‘zero’ ATM scan of the MS. The unit opacity, temperature of the atmosphere and scaling air-mass (secant of the zenith distance) can be found using the CASA **atmosphere** tool.

In case of the channel resolved windows, the integrals in the expression (2.2.14) can be calculated directly. For the averaged ACs (SPW 6,8,10, and 12) and SQLDs we have just the SPW-averaged values for P , G , and T_{inst} , which cannot be directly inserted into (2.2.14). Yet we can perform the integration over entire frequency range of those single-channel SPWs at least approximately: We start with the Eq. (2.2.10) and consider the functions $\tau_i(f)$ and $\langle T_{sky}(f) \rangle$ as linear over the integrated interval; for the SPW bandwidth of 2 GHz this is quite well satisfied (verified using the ATM model in the *atmosphere* CASA tool):

DV20

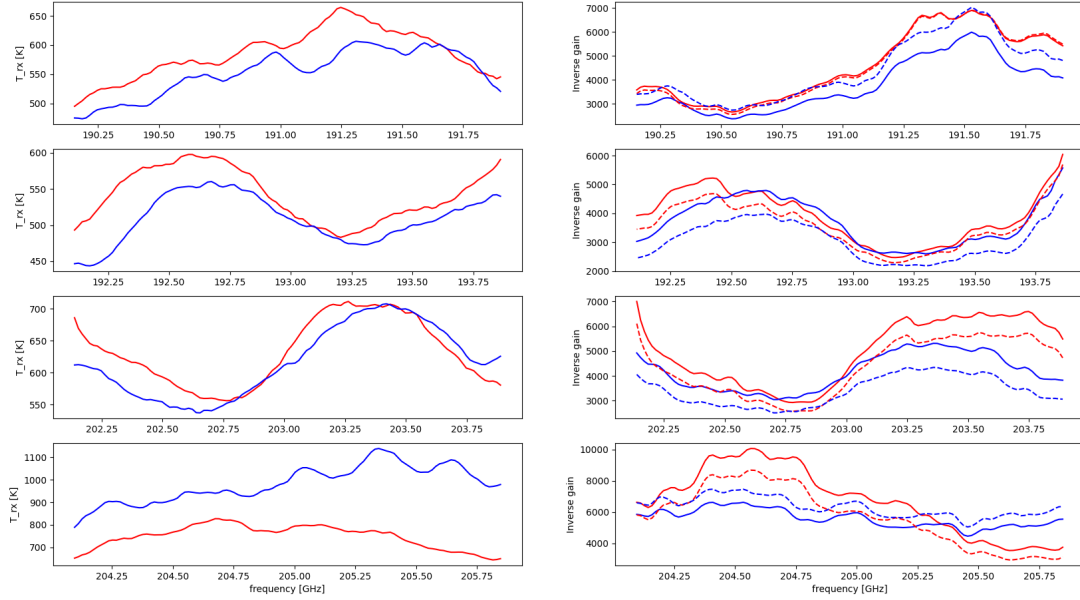


Figure 11: Receiver temperature T_{rx} (left part) and the inverse gain $1/G$ (right part) of antenna DV20 for SPWs 5,7,9, and 11 (from top to bottom) as a function of frequency. Red: XX, blue YY. Calculated from the ACD dual-load ATM scan preceding the Sun in-source target. The Sun is being observed with increased attenuation, so we need to calculate also the modified gain G^* (referred as G_{src} in Shimojo et al., 2017, see Eq. 2.2.13), which reflects the increased attenuation and is relevant for the Sun. It is calculated using the very first and very last sub-scans of the solar scan, which are done off-source – see `acd_caltabs()` in Annex 4. Here, the non-attenuated inverse gain $1/G$ is marked by dashed lines, the actual effective inverse gain $1/G^*$, is plotted solid by line.

$$(f(x) \cdot g(x))_{mean} \equiv \frac{1}{b-a} \int_a^b f(x) g(x) dx \approx f_{mean}(x) g_{mean}(x) + \frac{1}{12} (f(b) - f(a)) \cdot (g(b) - g(a))$$

In this approximation, Eq. (2.2.10) can be rewritten as

$$T_{sky}(t) = \langle T_{sky} \rangle - dw(t) \left(\tau_1^{mean} \langle T_{sky} \rangle - T_{atm} \tau_1^{mean} \right) + \frac{1}{12} \left(\tau_1(f_{hi}) - \tau_1(f_{lo}) \right) \cdot BW \cdot \frac{d\langle T_{sky} \rangle}{df} \quad (2.2.15)$$

Here, the index ‘mean’ means averaging in frequencies over the entire spectral window and we do not put it explicitly to T_{sky} , as it is clear that this quantity is both time and frequency averaged. Frequencies f_{lo} and f_{hi} are the edge frequencies of the SPW, and the (linear) spectral slope of the $\langle T_{sky} \rangle$ is determined by linear regression of the corresponding channel-resolved SPW (e.g., spw 5 in case of extracting the powers for brightness-temperature calculations from the spw 6), example of which is displayed for the Sun in Fig. 12 below.

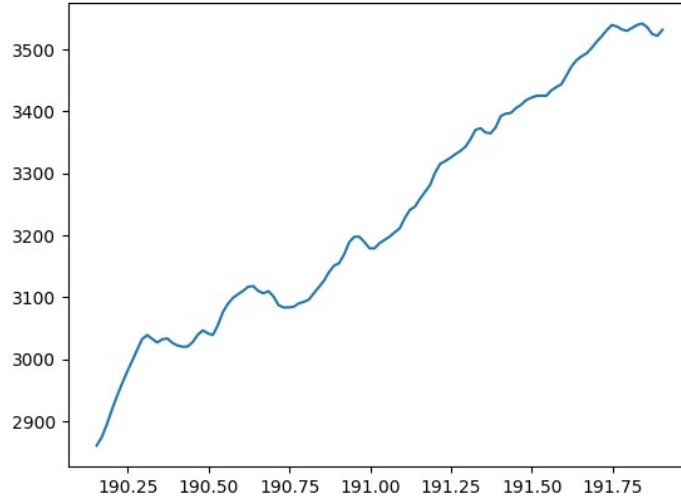


Figure 12: Time-averaged (over a scan) spectrum (frequency in GHz) of the sky brightness temperature when pointing to the Sun in the SPW 5 (in Kelvins). The T_{sky} has been calculated by Eq. (2.2.15) using the ACD dual-load calibration.

Eventually, if we express $T_{sky}(t)$ and its time-averaged value $\langle T_{sky} \rangle$ from the dual-load calibration Eq. (2.2.11) by means of directly measured power $P(t)$ (and its time-average over the scan), and insert it into Eq. (2.2.15), we get

$$P(t) - \langle P \rangle = S_{ant}^{spw} \cdot dw(t) + \delta P_{src}^{tr}(t) \quad (2.2.16)$$

Here, the last term corresponds to (unknown) intrinsic source variability as transmitted through the terrestrial atmosphere, and the *sensitivity factor* S_{ant}^{spw} , which represents how much are the changes in PWV sensitive to the variations of the power measured by given antenna in a given SPW, is expressed as

$$S_{ant}^{spw} = -G \tau_1^{mean} \cdot \left(\frac{\langle P \rangle}{G} - T_{inst} - T_{atm} \right) - \frac{G \cdot BW}{12} \cdot \left(\tau_1(f_{hi}) - \tau_1(f_{lo}) \right) \cdot \frac{d\langle T_{sky} \rangle}{df} \quad (2.2.17)$$

To sum up, we use directly Eq. (2.2.14), when expressing our dPWV proxy from the channel-resolved AC SPWs, while Eqs. (2.2.16) and (2.2.17) are suitable for the single-channel SPWs (averaged ACs or SQLDs).

2.2.3 Implementation

The above formulae and evaluation of all the necessary quantities are implemented – in addition to further methods for recalculation of the dPWV to phase delays, as a set of routines collected in the Python module *sol_wvrgcal.py*. Taking advantage of stability of the dual-load calibration in the channel-integrated SPWs, we use the ACs in single-channel spectral windows 6,8,10, and 12, by default, albeit alternatives using the channel-resolved auto-correlations and SQLDs have been tried, too, and comparison of all SPW choices is presented below. Namely we use the SPW 6, which is closest to the WV line center and the sensitivity factor (2.2.17) is thus the highest. In order to get water-vapor variations dPWV (represented by the symbol $dw(t)$ in Eq. 2.2.17) we need to:

1. Extract the raw measured power for the source as a function of time (per integration) from the MS for selected scan and spw – this is achieved by means of `tgt_pow()`. We then calculate also its time average $\langle P \rangle$ and subtract it from the raw data (see Eq. 13).
2. In order to calculate the sensitivity factor S , we need:
 - G (or G^* for the Sun) – obtained via `acd_cal_tabs()`
 - T_{inst} (which needs P_0 and T_{RX} in addition to G) – achieved via `acd_cal_tabs()`
 - τ_1 – the wet opacity for PWV=1.0 – its mean value over the SPW and values for the edge channels of the SPW. This is achieved via ATM model – we set the atmosphere tool in `setAtmTool()` similar way like `au.plotAtmosphere()` does and then extract the necessary ATM characteristics (spectral dependence of τ_1 and the mean atmospheric temperature T_{atm}) using `wopacity()`. The spectral slope $d\langle T_{\text{sky}} \rangle / df$ is obtained from time-averaged spectrum of the brightness temperature (source + atmosphere) – the power data in 128-channel spw (spw 5 in our case) are calibrated by the dual-load (ACD) calibration as implemented in `tgt_Tb()`; standard `np.polyfit()` is used for linear regression then.
 - Eventually, we divide the difference $(P(t) - \langle P \rangle)$ by the calculated factor S and get the $dw(t)$, i.e., dPWW as a function of time per antenna and SPW (neglecting the intrinsic source variability, of which we have no clue). Entire combined procedure is assembled in the routine `dPWV()` at the end of the file.
3. In interferometry, we are not measuring the phase delays w.r.t. the source in vacuum, but always w.r.t. the reference antenna. The last step therefore is that we calculate dPWV for a given antenna, the same quantity for the reference antenna (DV10 = id 37 in our specific dataset) and subtract the latter from the former. This quantity should be already proportional to the phase delays (related in any gain calibration also w.r.t the reference antenna).
4. In order to make actual comparison, an auxiliary method `getPhase()` is contained in our collection, which extracts the fast phase variations from the calibration table found as a fast (integration-based) `gaincal(type='p')` solution for the band-pass (BP) calibrator (this calibration is always done before `bandpass()` is called).

2.2.4 Correlations between dPWV proxy and the actual phase delays in BP calibrator

We try to compare the data for the BP calibrator first, where we can trust the phase solution (it is a self-cal for the point source, i.e., straightforward). The case of the solar scan is handled in the separate Chapter 2.3. Figs. 13 and 14 show comparison between our dPWV proxy obtained by the procedure described above (Fig. 13), and the phase delays as per BP-calibrator phase gaintable (Fig. 14), for the antenna DA45 (one of the antennas far from the array center). Actually we are plotting $-dPWV$, because the coefficient between dPWV and phase delay is negative and the obvious similarity is better visible in the positive correlation.

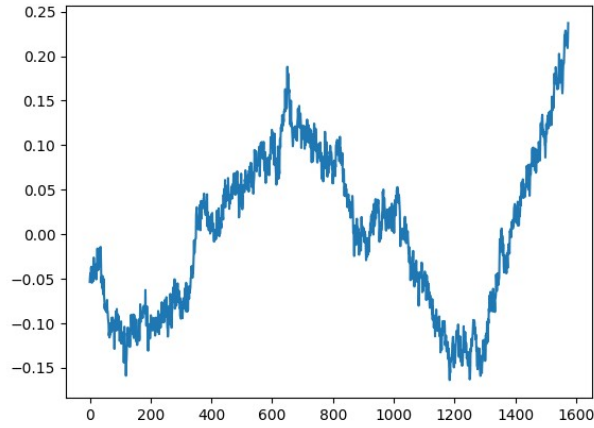


Figure 13: Negative (i.e. multiplied by factor -1.0) dPWV proxy calculated by Eq. (17) from the channel-integrated AC (SPW 6) in the BP-calibrator scan. Time is expressed as an integration number (integ. duration is 0.192s), dPWV value is in mm (of PWV).

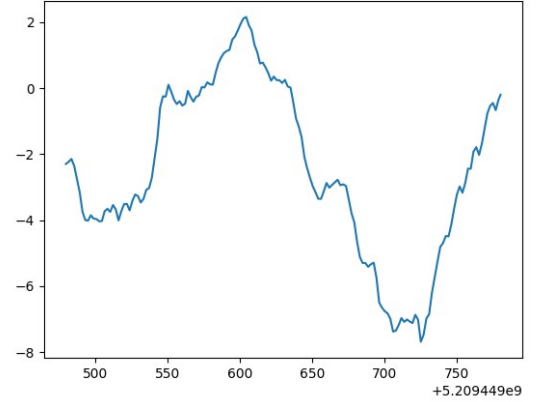


Figure 14: Integration-based phase solution for the BP calibrator in spw 5, the same scan as in left figure. Time is expressed in secs as the MJD, phase variations have been un-wrapped and are displayed in radians, i.e. the displayed phase range is over 540 degs!

Similarities in both curves are apparent. We can go further and make cross-correlation plot between both quantities (after rebinning the dPWV to the same cadence of 1.92 sec). This is shown in Fig. 15.

We have deliberately used the line connecting the individual scatter-plot points for the same time instants, the line thus basically represents the time path or a ‘history’. From that point of view, there is a strong impression that the slope (the coefficient that relates changes in dPWV and corresponding changes in phase) is more or less constant but the phases (their PWV-independent part) are systematically decreasing with time. We return to that effect later, now let us find the coefficient of proportionality between dPWV proxy and corresponding phase error.

We can start with the theoretical relation (R. Hills, 2019 – see MTR)

$$d\phi = 2\pi/\lambda \cdot r_{\text{eff}} \cdot dw \quad (2.2.18)$$

If we assume average wavelength 1.6mm for the solar Band 5 observations in the low-frequency SPW 5 (the left spw in the LSB; as much close to the WV line as possible) and the effective refractive index (R. Hills, 2019) of value of $r_{\text{eff}}=6.3$, the coefficient of proportionality is going to be -25.0 rad/mm.

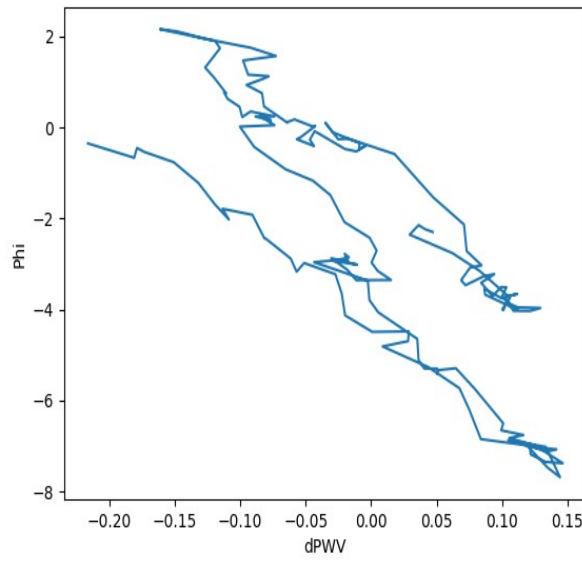


Figure 15: Cross-correlation between $dPWV$. Intentionally we use continuous line in the plot, which somehow helps to track also the time dimension. Correlation is apparent but there is a strong impression that the absolute value of the phase is decreasing also with time.

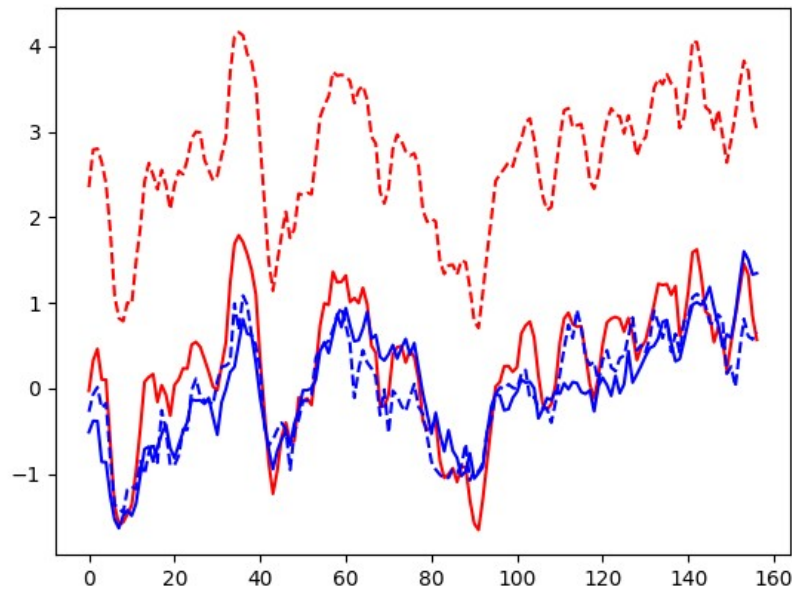


Figure 16: Actual phase delays as found from `gaincal()` for the BP calibrator (red curves; solid represents XX and dashed YY pol, respectively) in comparison with our prediction from the $dPWV$ proxy, scaled by the theoretical factor -25.0 (blue) for antenna DV14. X-axis shows time expressed as integration number, integration duration is 1.92 sec. Y-axis is phase in radians.

Using this value, we can overplot the expected (by measuring the dPWV and using the above scaling) and actually measured (determined by integration-based *gaincal()* solution for the BP calibrator) phase delays. This is implemented in *dwXphi_plots()* and a sample result for the antenna DV14 is shown in Fig. 16. Again, the similarity of predicted to actual phase delays is obvious.

2.2.5 Time-detrended correlation between the phase delays and dPWV

In spite the ‘by eye’ comparison between the predicted and actually determined phase delays for the BP calibrator shows high level of similarity, the scatter plot shown in Fig. 15 reveals that there may be some slow dynamics (a time trend) in phases, not correlated to the changes in PWV. In order to study this phenomenon systematically, we have adopted the following approach: We are trying to do linear regression between dPWV and phase-delay, but we, moreover, allow the phase variation to be depending also on time, which for the equidistantly-spaced time series can be expressed as an index i . Formally speaking, we are trying to minimize the functional (basically χ^2)

$$F(a, b, t) = \sum_{i=1}^N (y_i - t \cdot i - a \cdot x_i - b)^2, \quad (2.2.19)$$

where x_i values represent our independent variable dPWV, y_i are corresponding actual phase delays, a is the coefficient of linear regression between those two (we expect it ideally at its theoretical predicted value of -25.0), and t is the linear time trend, as the index i in the series represents homogeneously growing time. The absolute term b represents the constant offset between the actual phases and those predicted by the dPWV. This is not relevant, as we have no possibility to detect anything above the PWV variations over its scan-averaged value (which can be found in the MS using the *atmosphere* tool).

We minimize the functional via standard procedure, i.e., finding the stagnation point for its all three parameters:

$$\frac{\partial F}{\partial a} = 0, \frac{\partial F}{\partial b} = 0, \frac{\partial F}{\partial t} = 0 \quad (2.2.20)$$

which leads to the system of linear equations for unknown minimizing parameters a , b , and t :

$$\begin{aligned} a \sum x_i^2 + b \sum x_i + t \sum i \cdot x_i &= \sum x_i \cdot y_i \\ a \sum x_i + b \cdot N + t \cdot N \frac{(N+1)}{2} &= \sum y_i \\ a \sum i \cdot x_i + b N \frac{(N+1)}{2} + t \cdot \frac{2N^3 + 3N^2 + N}{6} &= \sum i \cdot y_i \end{aligned}$$

Here we have omitted the obvious sum-limits ($i=1..N$) and replaced sums of i and i^2 by their analytical expressions. The system may be further easily reduced to system of two equations by expressing b from the 2nd equation and inserting it into 1st and 3rd.

$$\begin{aligned}
a \cdot \left(\sum x_i^2 - N \cdot \bar{x}^2 \right) + t \cdot \left(\sum i \cdot x_i - N \frac{(N+1)}{2} \bar{x} \right) &= \sum x_i \cdot y_i - N \cdot \bar{x} \cdot \bar{y} \\
a \cdot \left(\sum i \cdot x_i - N \frac{(N+1)}{2} \bar{x} \right) + t \cdot \left(\frac{N(N-1)(N+1)}{12} \right) &= \sum i \cdot y_i - N \frac{(N+1)}{2} \cdot \bar{y}
\end{aligned} \tag{2.2.21}$$

Here, the bar over a quantity means its regular averaging (i.e., the sum divided by N). This linear 2x2 system is then easily solved by Cramer's rule. Not showing the algebra details here, it is straightforwardly implemented in helper routine *paregression()* in the attached collection `sol_wvgcal.py`. We have tested this routine on an artificial signal before using it for our data. Complete results of this analysis are summarized in the XLS table available online at http://wave.asu.cas.cz/shared/ESO/final_report/solar_wvr/, the meaning of the table columns is following:

Col A: Antenna id_name.

Cols B – L: Individual terms for dPWV calculation according Eq. (14) in *Progress report no. 11*.

Cols M – R: Coefficients of parameterized linear regression (routine *paregression()* in Annex 1) – linear time-trend, correlation coefficient, and offset, respectively – all extra for XX and YY pols.

Cols S – V: Positions of the antenna in the array and its distance from the reference antenna DV10.

Cols W – AA: Measures of the correlation quality – Xi-square and ‘by-eye’ quality mark for XX and YY separately, and better value of them. We have kept only antennas for whose the value in column AA is at least 6.

Cols AB and AC: Time-trend and correlation coefficient for the better polarization (XX or YY) – basically the copied value from the column M or N depending whether XX or YY pol has better qualitative mark for t-trend and, analogically, O or P-column value for the correlation coefficient.

Col AD: Position of the antenna (cols S and T) projected to the unit vector with azimuth defined by value (in degs) in the cell V41.

For some antennas, like DV17, whose case is displayed in Fig 17, the time-detrending helps a lot to make the scatter plot very narrow, having perfect correlation between the dPWV and actual phase delays. However, there are also cases, where no significant improvement is reached – see Fig. 18 displaying antenna DA41. Some antennas exhibit yet weaker correlations.

We first focus on the coefficients a of the linear correlation between the dPWV proxy and the phase delay (see Eqs. (2.2.19) – (2.2.21)) – we are namely interested, how close they are to the theoretical prediction based on Eq. (2.2.18), which foresees the value close to -25.0 rad/mm.

First, we would like to compare the regression coefficient only for the data with good Tsys, and consequently, good cross-correlation. To that end, two parameters are measured: The Xi-square –

exactly the value of F from the Eq. (2.2.19) above, and a subjective ‘by-eye’ quality mark which evaluates the Figs. like 17 and 18 for each antenna and polarization and attributes each result by value from 0 (no correlation) to 10 (perfect match). As some antennas have good T_{sys} in one polarization and bad in another one, it projects to the results here similarly and there are antennas with good quality mark in one polarization and bad in the other. Eventually, we have kept that polarization, which shows better quality to have a single number per antenna. Why we do not use just Xi-square? Because it should be most likely properly normalized: A good cross-correlation of the phase variations (those actual vs. those predicted by dPWV) may have rather large Xi-square just because both quantities exhibit large fluctuations (typically for distant antennas). So the value of chi-square itself is not really addressing the quality of correlation. Hence, we kept in our ensemble only such antennas, which have at least in one polarization the by-eye quality mark above 5 and reasonably low Xi-square at the same moment. This keeps 37 of total 52 (and of 50 unflagged in further calibration) antennas in the processing. As can be seen from the column AC (with title “K better”) in the on-line XLS table, the correlation coefficient $d\phi/d\text{PWV}$ between dPWV and measured phase variation, indeed, varies around its average value -25 rad/mm with its standard deviation ~ 8 rad/mm.

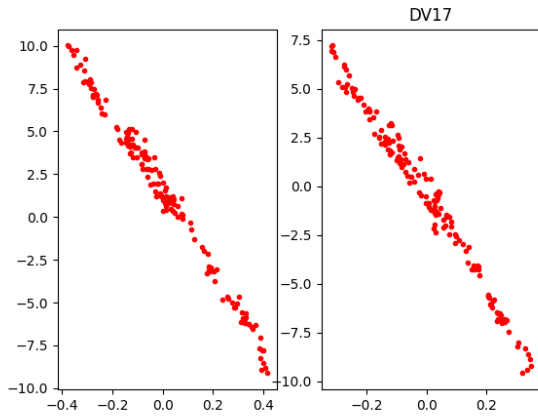


Figure 17: Phase vs dPWV for antenna DV17 – like in Fig. 15, but now after time detrending.

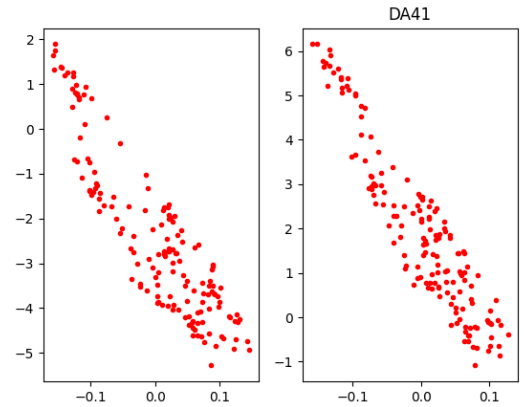


Figure 18: The same like in left figure, now for the antenna DA41.

In the next analysis we have been investigating the nature the slow phase-delay dynamics that is independent of the dPWV variations. Namely, we are interested in (i) whether and how the linear trend/drift in the residual phases (i.e., after the dPWV-correction is applied) is related to antenna position in the array (indication of whether the residual phase is related rather to instrumental or astronomical influences) and, (ii) whether the residual phase variations actually can be approximated as linear trend/drift or this slow dynamics is more complex.

Actually, easier answer is to the item (ii): No, as it can be seen from the overplots between the actual phases and their prediction made by dPWV scaled properly by the correlation coefficient (approx. -25.0 rad/mm) the difference between the two phases (i.e. phase residua) only sometimes can be approximated as a linear drift. In many other cases they show slow-time-scale variations that, however, cannot be interpreted as linear trend.

In this view, it is understandable that also answer to question (i) will be negative – we did not found any significant relation between the linear time-trend/drift and position of the antenna in the array. In many cases it is just because the linear trend is not appropriate description of the slow phase variations, as explained above. Anyway, for addressing this question we have characterized position of the antenna in the array by two quantities: (1) Distance from the reference antenna, (2) Position in the antenna in the array projected to the unit vector of selected direction (azimuth). The latter has been investigated for the sake to find possible correlation either with the source hour-angle or with the direction of the prevailing wind (that transports the PWV across the array). To that end, antenna positions in the array has been added into XLS table (referred by the URL above) and two columns of derived quantities: Distance from ref. antenna (column V), and antenna position projected to the unit direction vector at azimuth defined in the cell V41 (column AD). These two columns were compared with the linear trend/drift expressed for each antenna in the column AB (“T-trend better”).

Results are shown in the on-line available XLS table and for convenience partly also here as Fig. 19. apparently in none case the relation between the coefficient of linear frequency drift and measure of the antenna position has been found. The current result shown in the on-line available material are displayed for the projected antenna position corresponding to projection direction (azimuth) of 75 deg, the other attempted azimuths give even more chaotic (i.e. with no systematic dependence) results.

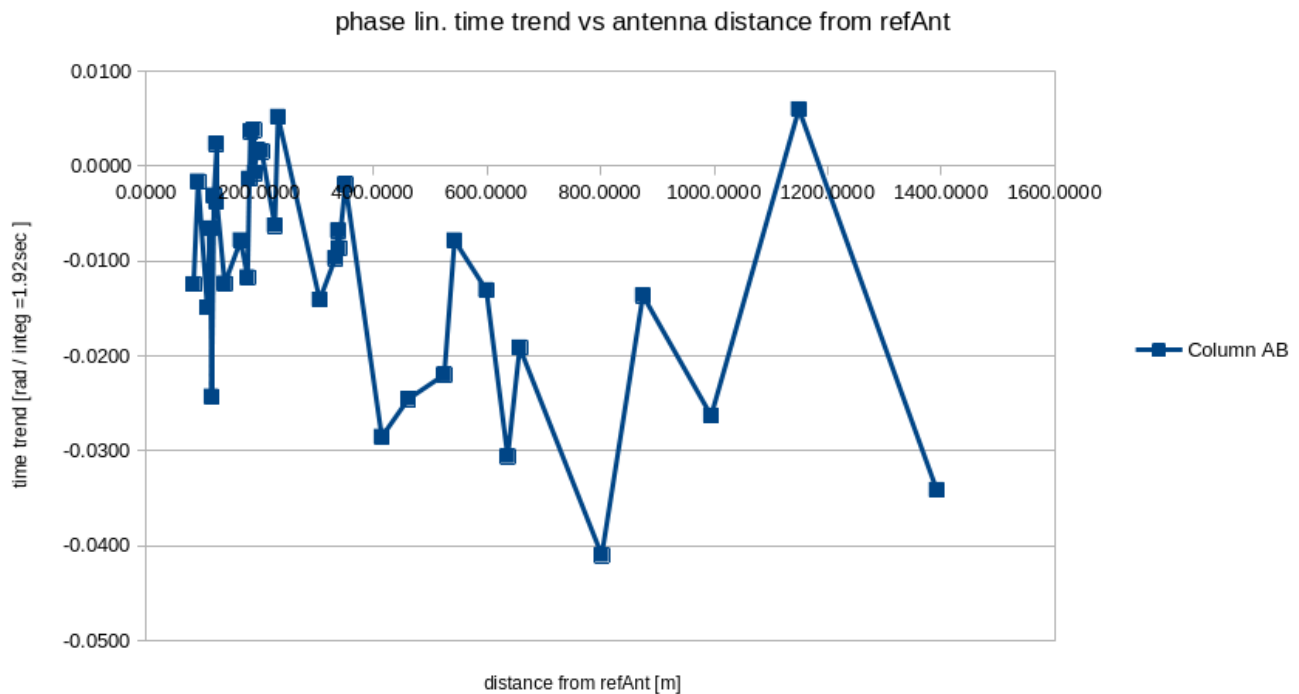


Figure 19: Linear time-trends/drifts of the frequency residuals in dependence on the antenna position in the array expressed by distance from the reference antenna. In the on-line XLS table there is moreover similar plot, where the slow phase dynamics expressed as the time trend is checked vs the antenna position projected to the specific direction vector. The results, however, show the same little or no systematic dependency.

2.2.6 Using the dPWV proxy to fixing the phase errors

From the plots like that in Fig. 16 we are already close to direct application of the dPWV proxy to apriori correction of the phase errors. Since it looks that for the good antennas (with good Tsys) the procedure works quite fine, we decided to come to the correction of the BP phases by dPWV. Two approaches were compared: (i) Blind correction, with the theoretical (and – basically – also experimentally confirmed by the analysis summarized in the on-line XLS table) correlation factor of -25rad/mm , and (ii) with the correlation factor experimentally found by solving Eqs. (2.2.19) – (2.2.21) – i.e. optimized for each antenna and with at least linear time trend in the slow dynamics of phases removed. The latter is studied just for comparison – in the future real applications to the solar data we will not have the actual phases to do the (even de-trended) regression with, only possibility is to study the stability of the correlation coefficient not just for the BP calibrator but also over the subsequent phase-calibrator scans, and use the same value also for the unknown target, the Sun. This approach mimics that used in `wvrgal()` calculations of dT/dL coefficients between the variations of WVR brightness temperature and corresponding fluctuations in optical path.

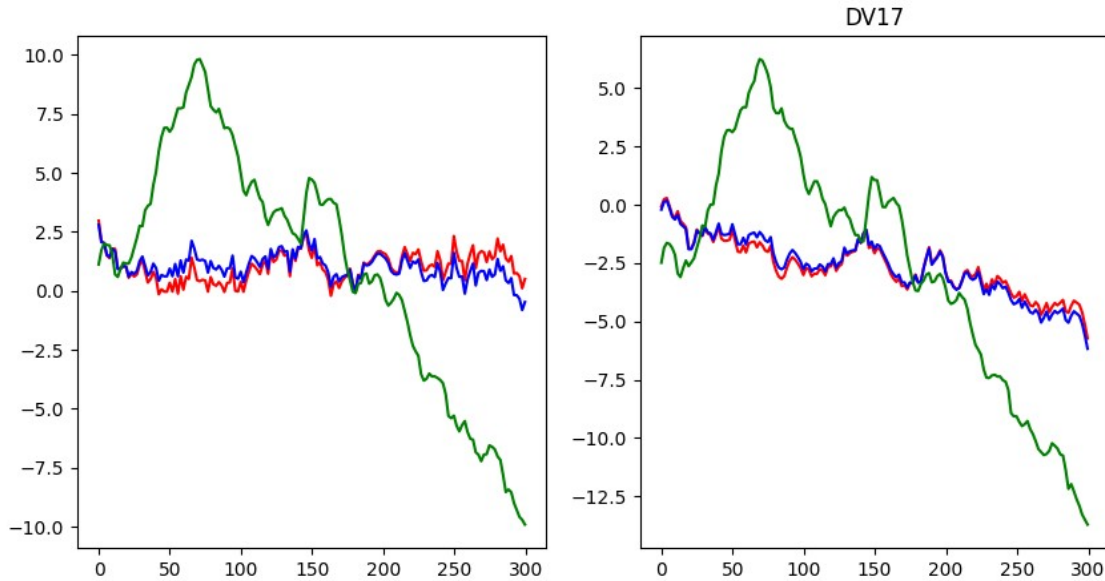


Figure 20: Antenna DV17, the BP-calibrator scan (no. 5) in SPW 5. Original phase variations (green) and after correction by dPWV-based calibration – a blind correction with the factor of -25rad/mm (red), correction by the optimised correlation factor found by linear regression with time de-trend (blue). Left and right panels are for XX and YY polarizations, respectively, the x-axis denotes time in seconds since the beginning of the scan. The y-axis is in radians (as all similar figures here).

The procedure is implemented in the routine `blindPhiFix()`. We, again, did the study systematically for all antennas and both polarizations, just two illustrative examples are shown in Figures 20 and 21, where we present the original phase variations as found by `gaincal()` in green, the phase-variation residuum after subtracting the blind correction from the original variations according to relation $\varphi_{\text{residual}} = \varphi_{\text{orig}} + 25.0 * \text{dPWV}$ (in red), and optimized correction with cross-correlation coefficient and time-detrend found by `paregression()` routine (blue curve). The first figure for antenna DV17 shows

really good reduction of the phase variations, with a small linear trend in the residuum of pol YY (right panel), the latter figure (antenna DA41) presents still good reduction of the original phase variations, but it is clear, that the residual phase error can not be interpreted as a linear trend (this is related to our investigation above). Generally speaking, even the blind correction reduces the RMS of phases by factor at least 2.

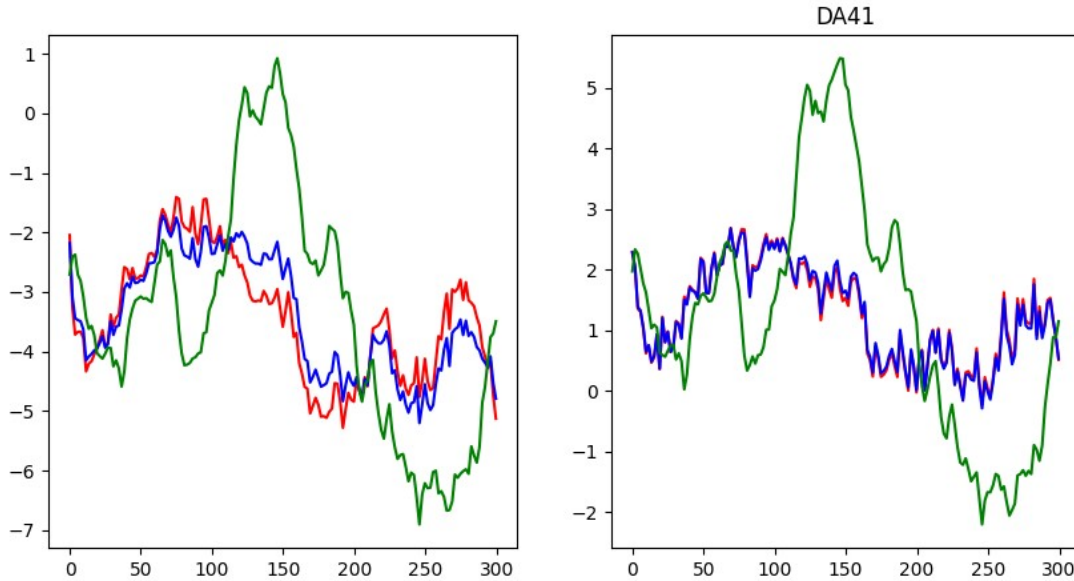


Figure 21: Like the figure above, now for the antenna DA41. Apparently, the residual (phase after correction) phase variations for DA41 exhibits the slow-time-scale variations which, however, cannot be approximated as a linear trend.

2.2.7 Comparison of dPWV proxy with WVR measurements

Clearly, if our approach to fixing the fast phase variations based on the varying WV signal in the science SPWs (the methodological *Approach 2* of the project) should represent an alternative to the standard WVR and `wvrscal()`, both methods should give comparable results for the cases, where we can compare them. In our solar dataset this can be done for calibrators, namely for the BP/flux calibrator, whose scan no. 5 is long enough (~ 5 min). In this paragraph we aim at such a comparison.

After inspection of our MS, we have found four resources related to the direct WV measurements:

- ASDM_CALWVR in MS
- ASDM_CALATMOSPHERE in MS
- in original ASDM dir: CalWvr.xml and CalAtmosphere.xml
- SPW 4 in the MS.

AnalysisUtils offer a couple of related utilities for accessing the data (actually those in the items 1 – 3 above), most relevant for us being *readPWVFromMS()* and its ‘graphical representation’ *plotPWV()*. Quick inspection of the data in *ASDM_CALWVR* shows that we have actually quite sparse time coverage: This table contains 468 points for 52 antennas, giving 9 time points for entire measurement set. The *ASDM_CALATMOSPHERE* table is only slightly better with 12 time points over the entire MS and the XML files in the original ASDM dir seem to be just formatted representations of those above mentioned tables. The situation is illustrated by the output of *plotPWV()*, which is shown in Fig. 22: We have (spline-interpolated) variations of the PWV for all antennas only on the slow temporal scales, based on the *ASDM_CALWVR* table data.

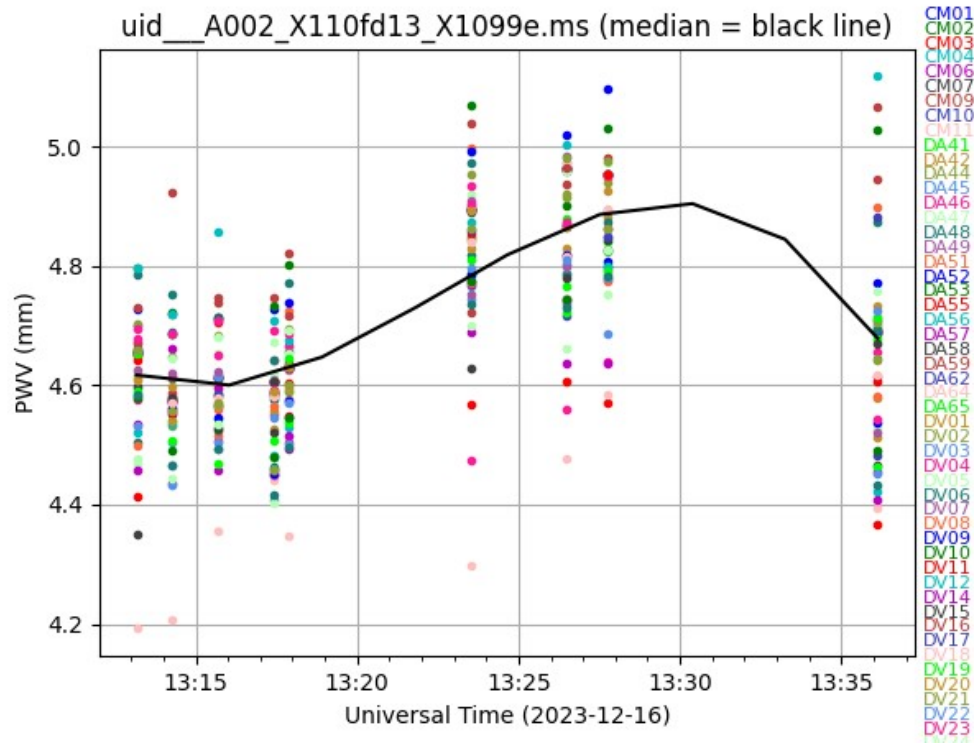


Figure 22: Output of *plotPWV()* for our MS.

Consequently, we have just a single point per scan and we thus cannot use the data for our purpose. The only usable high-cadence WV data are thus the direct temperatures in the four WVR channels in the SPW 4. These values are, on the other hand, far from the PWV derived from them. For this reason, we decided to make not the direct comparison between our dPWV-proxy and the directly measured PWV, but – also in view of the ultimate goal – compare the *impacts* of both quantities to the phase delays. Namely, we compare our dPWV proxy, scaled by the already several times mentioned factor $d\phi/dPWV = -25.0\text{rad/mm}$, with the phases found by call to *wvrscal()* routine. In order to prevent damage of the WVR data by the saturated solar scan, we have created the WVR gain table by a call

```
wvrgcal(vis=vis,caltable='wvr_gcal.W',segsource=True,
sourceflag=['Sun'],statsource='J1256-0547,refant='DV10',maxdistm=1.0,
minnumants=3)
```

which flags the solar data and calculates the stats from the BP calibrator. We have used the same reference antenna as for *gaincal()* in the calibration script. The last two parameters prevent making the *wvrgcal()* solution by interpolation for those antennas, which have no or corrupted WVR data. Namely, we prevent generating a ‘fake’ wvr gain caltable for 7m antennas, which are not equipped with WVRs.

The *wvr_gcal.W* table by the call above is accessed via *wvrPhases()* routine in our collection *sol_wvrgcal.py* and extracts the phases from the WVR-gaintable by applying the data selectors similarly as we do in *getPhases()* for the phases found by *gaincal()*. All antennas has been analyzed, here we present two randomly selected cases as an illustration – antenna DV10 (the reference antenna) and antenna DV17 – see Figs. 23 and 24.

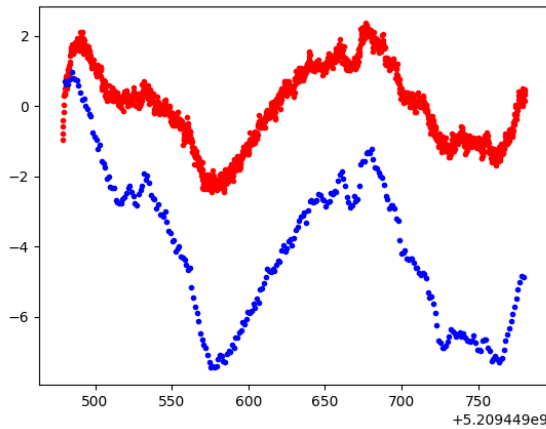


Figure 23: Comparison of phase delays derived from our dPWV proxy, as calculated from the differential absorption in the science SPWs (red), and those obtained via *wvrgcal()* from the original WVR data in SPW 4 for antenna DV10 (the reference antenna). X-axis is time in MJD seconds, phases at Y-axis in rads.

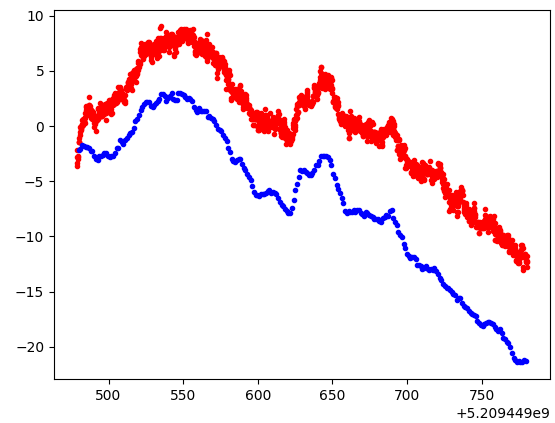


Figure 24: Like the figure on the left, now for antenna DV17.

In addition to comparing merely phase solutions as produced by our dPWV and *wvrgcal()* (the figures above) we would like to compare efficiency of both approaches in the phase correction. To that end we plot (i) the uncorrected phases as found by *gaincal()*, (ii) phases corrected by the *wvrgcal()* gaintable, and (iii) phases corrected via our dPWV proxy in a single figure. We prepare the corrected datasets as follows: (a) WVR-based correction of the BP-calibrator data (via *applycal()* of the *wvr-caltable*) prior running *gaincal()* on the same source, and (b) our dPWV-based correction to the uncorrected phases (we have no gaintable so far) by the phase subtraction. The procedure is implemented by routine *phiFixCompare()*.

In order to perform the WVR correction to the raw MS (solar MSs are *not* WVR corrected in the standard calibration procedure), we have performed the following steps prior call to *phiFixCompare()*:

```
clearcal('uid__A002_X110fd13_X1099e.ms',field='1')

wvrgcal(vis=vis,caltable='wvr_gcal.W',      segsource=True,      sourceflag=['Sun'],
statsource='J1256-0547',refant='DV10',maxdistm=1.0,minnumants=3)

from casarecipes.almahelpers import tsyssspwmap

tsysmap = tsyssspwmap(vis = mso, tsystable = mso+'.tsys', tsysChanTol = 1)

gaincal(vis=mso,caltable=mso+'.ap_pre_bandpass/wvrFix',      field='1',      scan='5',
nsolint='1.92s',      refant='DV10',      calmode='p',      gaintable=[mso+'.antpos',
mso+'.tsys','wvr_gcal.W'], gainfield=['','1','1'], interp=['','linear,spline',
'nearest'], spwmap=[[[]],tsysmap,[[]])
```

As usually, just a sample of the results is presented in Figs. 25 and 26 for two selected antennas, the full set is available on-line in the project repository at http://wave.asu.cas.cz/shared/ESO/final_report/.

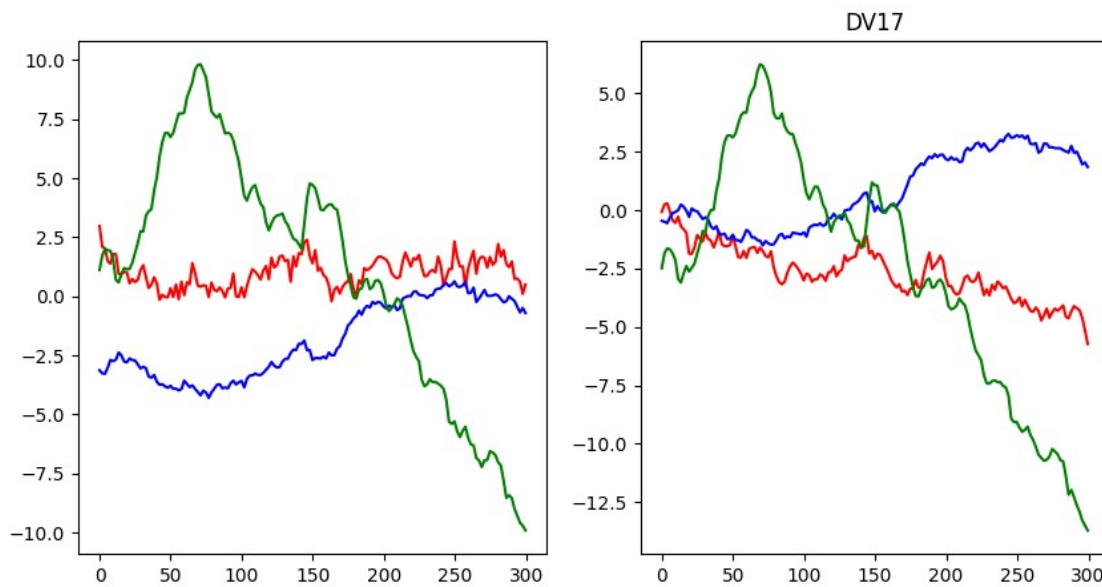


Figure 25: Comparison among (i) Raw phases for the band-pass calibrator – green, (ii) the same phase after being corrected by our proxy PWV method – red, and (iii) the phases after being corrected by applying *wvrgcal()* table – blue. This is example of well-working antenna DV17. It can be seen that the WVR-corrected phases may also exhibit long-term trends. Left panel: XX, right: YY. Time in secs from the BP-cal scan beginning.

What is interesting, and in line with the findings from our later analysis of another, low-PWV dataset (see below), is the relation between the raw phases (green) and their corrected residuals via PWV (red) and WVR (blue) methods in Fig. 26, left panel. Clearly, the PWV-corrected curve still exhibits features visible in the original (green) phases, so it means that we have subtracted a bit less than desirable. By

other words, our theoretical sensitivity coefficient, which relates dPWV proxy with the phase error ($\sim 25\text{rad/mm}$ for SPW 6) should be actually higher in order to do better job. And vice versa – the WVR corrected phase (the blue curve), which is in apparent anti-correlation with the dPWV-based correction exhibits ‘over-correction’ - i.e., the dT/dL coefficient for a given antenna, estimated in `wvrgcal()`, has been apparently a bit overestimated. We return back to this in the discussion of the low-PWV set in further paragraph.

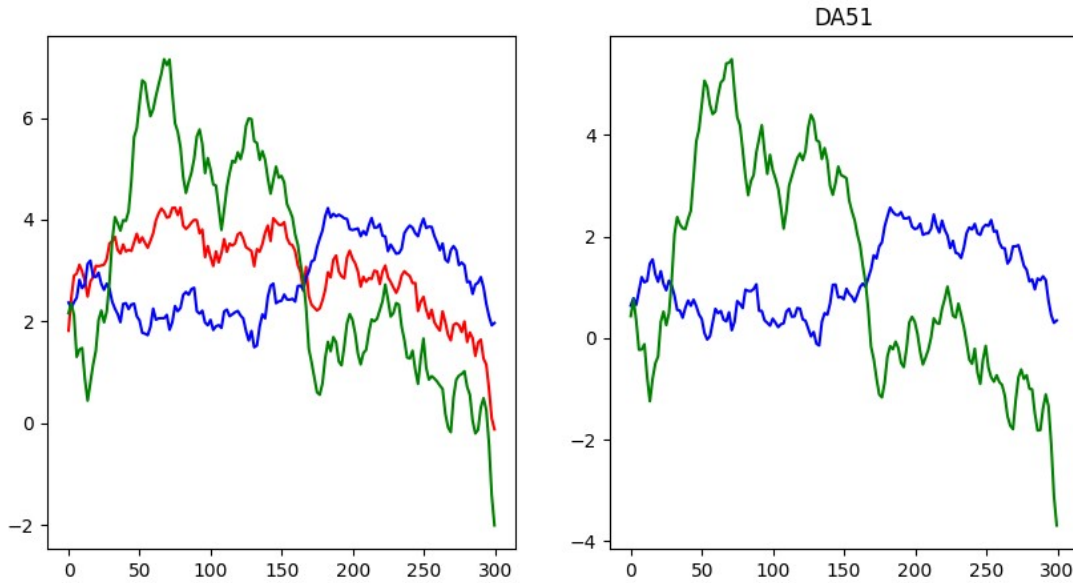


Figure 26: Like the previous figure, but now for antenna DA51. For YY pol. our PWV-based approach failed. Note the anti-correlation between PWV- and WVR-corrected phases for pol. XX (more in the text).

Ideally, the WVR-based phases and their dPWV-derived counterparts should be the same up to a constant offset (dPWV method corrects just the phase variations, not the time-averaged value over a scan). In fact, from the so far data it seems that difference between these two quantities exhibits systematic increase with time – see, e.g. Fig. 23. In order to study this systematically, we have performed similar analysis as we did above for time trends between dPWV and the phase delay, but now we have done mutual correlation between the phases corrected by two different methods: by our dPWV proxy and `applycal()` of the `wvrgcal()` table. Again, as we are allowing for the time-trend as an independent variable, we use our routine `paregression()` based on Eqs. (2.2.19) – (2.2.21). This time-detrending cross-correlation is implemented in the routine `dpwvXwvr_corrs()` in the `sol_wvrgcal.py` collection.

Illustrative sample results are shown in Figs. 27 and 28, a table of the cross-correlation coefficients and time-trends (export form XLS) as well as the full antenna-set plots are available online in the project repository.

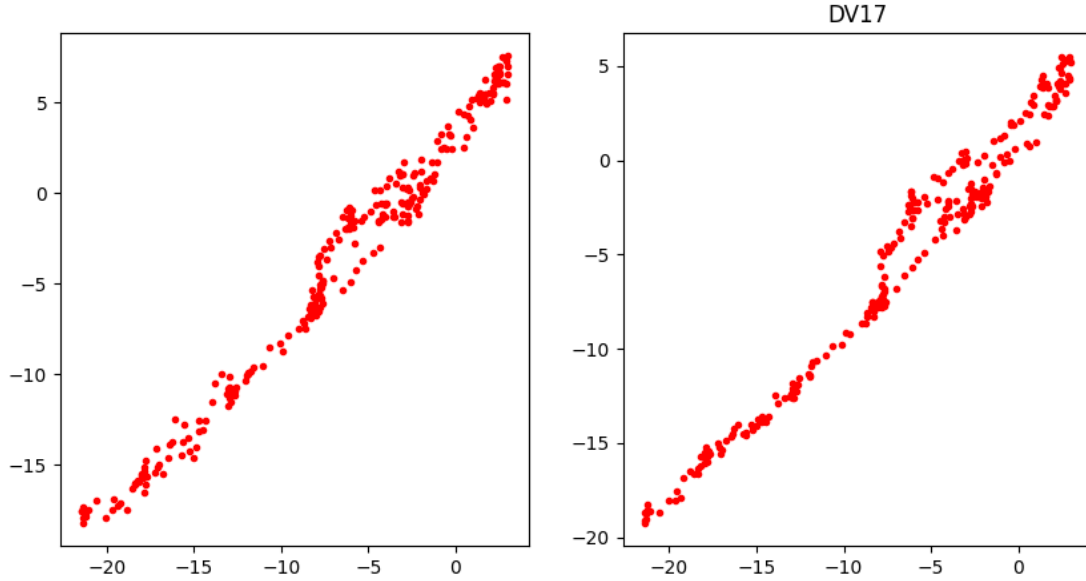


Figure 27: Cross-correlation plots between the dPWV- and WVR-corrected phases – the red and blue curves in Fig. 25 for the well-working example of antenna DV17. PWV has been calculated from channel-integrated SPW 6. Correlation coefficients for both XX (left) and YY (right) pols are very close to 1 (see the DV17 line in the XLS table online), time trend is small positive – meaning that the WVR-corrected phases grow a bit with time. w.r.t. the PWV-corrected ones (as is actually visible also in Fig. 24).

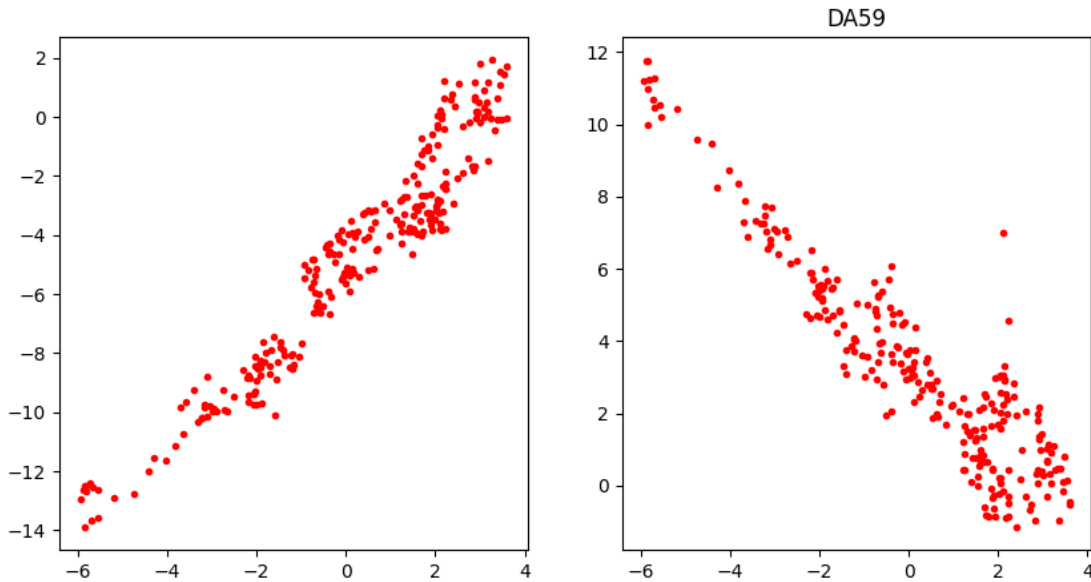


Figure 28: Like Fig. 27 above, now for antenna DA59. Clear anti-correlation is visible for the YY pol (right panel). Moreover, for the anti-correlated phases we have also negative time-trend.

From the results (see the XLS table online) it seems that

- WVR- and PWV-corrected phases are mostly (but not always) well correlated, either positively or negatively (anti-correlation).
- We have systematically small positive time-trend for phases corrected by WVR w.r.t. those corrected by dPWV (or, if we take the WVR as reference, then we can speak about systematic decrease with time for the dPWV-corrected phases) for the positive cross-correlation coefficient K (direct correlation), while negative time trend for negative K (anti-correlation).

We interpret this findings the following way: The factors, which are used for re-calculation of the water-vapor contents to the phase delays, i.e. our sensitivity factor $SF = -25\text{rad/mm}$ for dPWV proxy and the dL/dT coefficients determined in `WVRgcal()` calibration, may be under- or over-estimated, which leads to under-/over-correction in the dPWV and WVR methods. This effect may lead to anti-correlation as already suggested above. For future development of our dPWV-based phase corrections we suggest to follow the `WVRgcal` approach, where dL/dT coefficients are also not based on the theoretical assumptions, but rather empirically found for each antenna separately. The same way we shall do it for our PWV-based method. We shall check the stability of the coefficient through the EB by establishing it not only for the band-pass scan but also for all phase-calibrator scans in the MS.

2.2.8 Comparison of different source SPWs

As we have mentioned in the beginning of this WV-based phase-fixing chapter, we have three options in selecting the primary data for the dPWV calculation: It can be either (i) auto-correlations in the channel-resolved or (ii) channel-averaged SPWs, or (iii) the SQLDs.

For the results shown in the previous paragraphs we have calculated dPWV from Eqs. (2.2.16) and (2.2.17) using the channel-averaged AC SPW 6. In order to test its robustness, we have repeated it with corresponding SQLD SPW 0, and also with channel-resolved SPW 5, but the results do not differ much.

For the SQLD we use the same equations as for the single-channel ACs, just we have to take into account the non-zero SQLD offset value P_0 . For the channel-resolved SPW 5 we need to use directly the Eq. (2.2.14), which is implemented in method `dPWV_spwPart()` in our collection `sun_wvgcal.py`. For now, we can define the band-pass BW, over which the integration goes, just as a continuous band from `startChan` to `stopChan` in that method.

We applied it again to the BP calibrator of our solar Band 5 dataset. Somehow arbitrarily we have chosen integration in Eq. (2.2.14) over the lower half of the SPW5, i.e. next to the WV line core, where we can expect highest sensitivity of the power variation to fluctuating water-vapor contents. We also skip the very edge channels 0 – 7, as usually. Main motivation for using the channel-resolved SPW lies in the two facts: (i) we can select just the part of the SPW closer to the WV line and increase sensitivity, and, (ii) we can avoid the other weak atmospheric lines like O_3 , present in the SPWs 5 and 6, which ‘contaminate’ the WV signal by the dry component.

The applicable result – comparison between the raw, dPWV-corrected (now dPWV being calculated by the alternative *dPWV_spwPart()* method), and WVR-corrected, data analogical to that shown in Fig. 26 (where we used the single-channel SPW 6), is presented in Fig. 29. As one can see, there is no significant difference to the ‘traditional’ method that uses channel-averaged SPW6. This holds for all antennas, as can be seen in the full set of figures available on-line – there are differences, but very little, at least for the given dataset. In any case, expected improvement due to better sensitivity implying from using just the highly-absorbing part of the SPW, did not come true. Comparison between single-channel ACs and SQLDs is available in the online repository, too. Also in this case the differences are negligible.

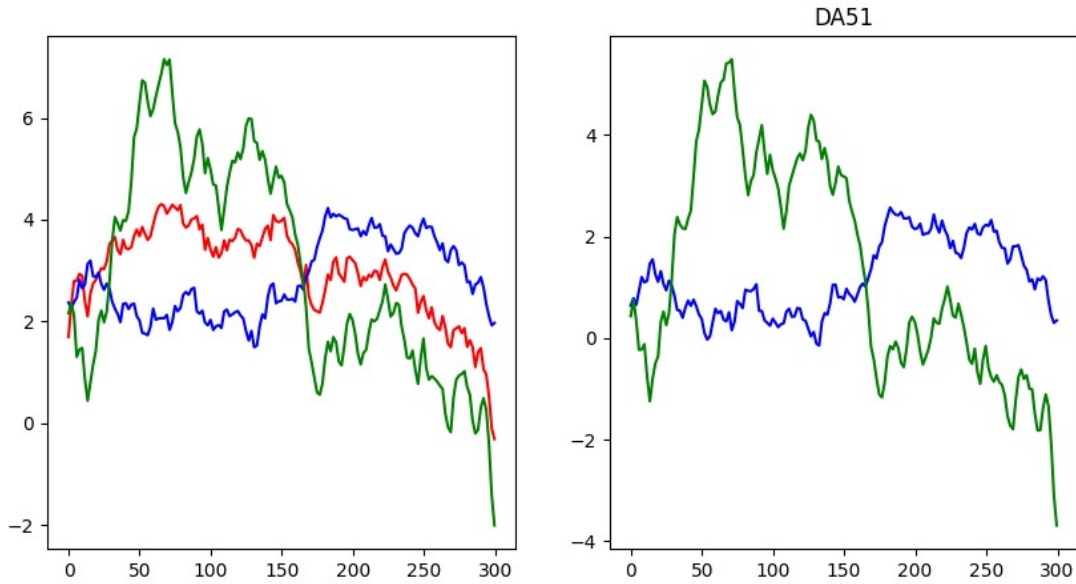


Figure 29: Like in Fig. 26: Green uncorrected phases, blue: corrected by WVR, and red: corrected by our dPWV proxy. Now the dPWV is calculated by Eq. (2.2.14), i.e. via integration over just a part of the spectrally-resolved SPW5. Displayed for the same antenna DA51 as in Fig. 26, for easy comparison.

2.2.9 Testing the approach with different dataset with low median PWV

Band-pass calibrators are present in basically all ALMA interferometry datasets and thus this analysis can be repeated for a MS with similar spectral setup and different, i.e. much lower, median PWV.

By the archive search we have found an observation, which fulfills the following conditions: (i) it has very similar frequency coverage for the SPW closest to the WV line – the lowest SPW being between 190 GHz and 192 GHz, like our solar dataset, (ii) observation is continuum, optimally in TDM regime (or FDM with low frequency resolution, covering the full base-band), and (iii) has low PWV, below

0.8mm. By this criteria we selected project 2021.1.00306.S, uid___A002_Xf8b429_X8ef3.asdm, science target IRAS_F14348-1447, observed on May 10, 2022 by 12m array.

We had to generalize our routines in `sol_wvgcal.py` in order to apply them to the band-pass calibrator for the above MS. Namely, the SPWs, for which the ATM scan has data for ACD calibration (hot, ambient, and sky sub-scans) are not the same as the science SPWs; we had to use the T_{sys} *spwmap* for their correct identification.

Before analysis, the *scriptForCalibration.py* has been generated via standard script generator

```
execfile('QA2/science/analysis_scripts/almaqa2csg.py')
```

```
generateReducScript(asdm_name)
```

just with *asis=CalDevice*, *CalAtmosphere* parameters set on top of the ‘standard’ settings (without that the ACD procedure fails on not having the ambient and hot-load temperatures). The calibration has been run up to its *Step 6* (antpos, T_{sys} and WVRgcal calibrations), skipping the AC-flagging step for obvious reason.

Then, we have run our *phiFixCompare()*, like in the step described above for the solar MS – as a result we got, again, the raw BP-calibrator phases (green curve in the following figs), and their reduced versions corrected by (i) WVRgcal (the blue curve), and (ii) our dPWV-proxy method (red curve), respectively. The straightforward application of this procedure led to a strange result. It seems that for the channel-integrated AC SPW 26, from which we have calculated the dPWV proxy, there is complication with the ACD calibration: The procedure reports T_{sky} temperature calculated by Eq. (11) from gain G and receiver temperature T_{rx} derived from our ACD calibration scan to be 3000K (!). This is far beyond our expectation – the T_{sky} for the ATM scan (i.e., OFF_SOURCE) shows temperature around 130K and the calibrator itself has its flux density only 1.6 Jy. When taking into account the typical Jy-per-Kelvin for the 12m antenna being ~ 42 , the source should cause increase of the temperature by a small fraction of Kelvin ($\sim 40\text{mK}$). Since the corresponding channel-resolved SPW 25 has 960 channels and different integration time than the SPW 17, which was identified as being adjunct to it by T_{sys} *spwmap* (has only 128 channels), one can understand that the gain should be perhaps re-scaled by the factor corresponding to ratio of integration times and channel widths.

In order to include this gain-scaling uncertainty, we tried the same procedure *phiFixCompare()* but now with the gain G arbitrarily down-scaled by a factor of 50, which gives us realistic T_{sky} for the BP-cal on-source scan. The result follows in Fig. 30.

The situation after gain-rescaling significantly improved, at least for some antennas – for the case of DA56 we can really speak about “correction” – the red curve (dPWV-corrected phases) has less variations than the uncorrected phases in green.

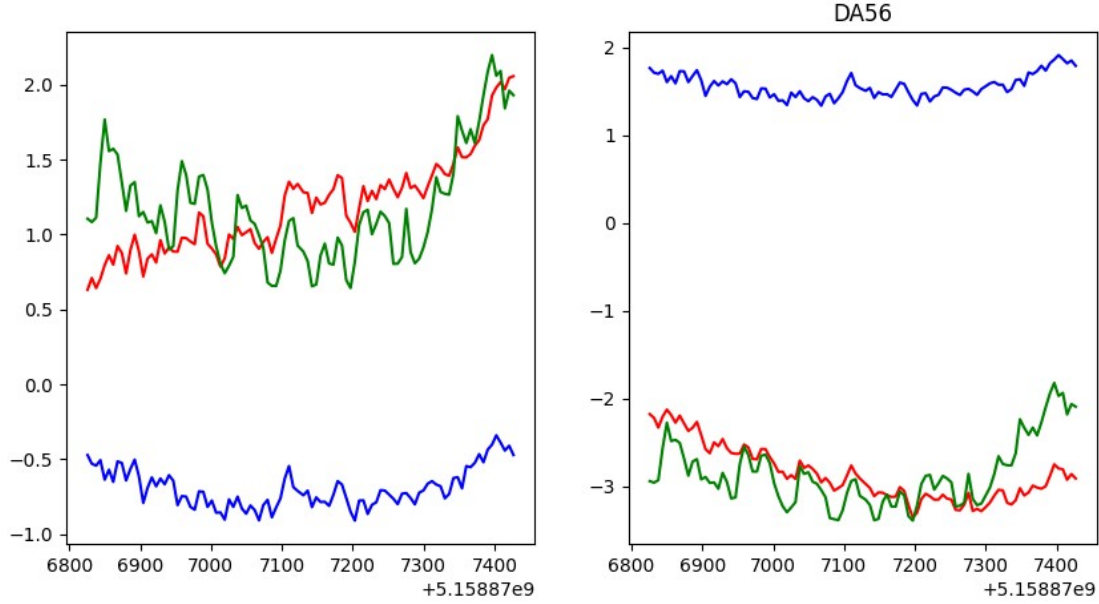


Figure 30: Like Fig. 25, now for the low-PWV dataset (IRAS). The gain has been arbitrarily re-scaled by a factor 50, in order to fit the sky temperature for the source. Displayed antenna is DA56.

We have tried it yet for the SQLD windows, where the ATM scan and science scan SPWs have the same resolution 128 channels. However, the T_{sky} temperature after the dual-load calibration is 300K, so lower, but still unexpectedly high. The results of application of our procedure, without the gain scaling are similar as for AC SPWs. We have to admit that the transfer of gain and T_{rx} from the ATM scan in T_{sys} SPWs to the corresponding science SPWs (via *spwmap*) for this MS is unclear and needs further investigation.

Nevertheless, in spite the confusing results, still it looks that there is a correlation between dPWV and phase variations, just we need different factor for transforming it into the phase difference. For this dataset, the change of the correction factor may be related not only to the (unknown) gain re-scale between T_{sys} and science SPWs, but also it is likely to attribute at least part of it to influence of the dry component of the phase variations for this MS with ultra-low PWV.

2.2.10 Concluding remarks to this topic

We have developed the legacy idea of Richard Hills (2019) and derived a proxy measure dPWV (in mm) to the water-vapor contents in the atmosphere. This can be re-calculated to the phase delays; the scaling coefficient for the SPW 5 (the closest one to the WV line) in our solar MS is $\sim 25\text{rad/mm}$.

The original Richard's formula derived in the Annex 3 to the MTR can be in our notation written as

$$\frac{\delta T_{\text{sky}}(t)}{\langle T_{\text{sky}} \rangle} = -\tau_1^{\text{mean}} \cdot dw(t)$$

It has been derived for the limit, where T_{RX} and T_{atm} are negligible when compared to the brightness temperature of the Sun. This, however, is not fulfilled in our data as can be seen from Figs. 11 and 12 - the T_{rx} can reach the values around 1000 K while the solar brightness is attenuated (by WV absorption) in the low-frequency part of the SPW 5 down to 3500K. Nevertheless, it can be easily shown that in the limit $T_{inst} \rightarrow 0$, $T_{atm} \rightarrow 0$ our Eq. (2.2.14) when applied to reasonably narrow bandwidth BW exactly matches the original formula above.

It has been found in the clear test case of the BP calibrator that the correction based on the dPWV works actually quite well and qualitatively it is comparable with that based on *wvrgcal()*. There are still some problematic antennas, for which the results are less convincing. Most likely reason is to be found in the errors of the ACD/dual-load calibration: The denominator of formula (14) contains a difference of two comparable and relatively large quantities, which both depend on gain or instrumental (noise) temperature. Even a small relative error in determination of both is thus strongly amplified, the resulting dPWV is quite sensitive to them.

In this respect, further development of the method is needed. We intent to

- Use also the phase calibrator scans and find the scaling factor between dPWV by regression over all scans of the MS, similarly as *wvrgcal()* does for its dT/dL coefficients.
- For the sources, where we have both *wvrgcal()* and our dPWV signal we may consider the parameters G and T_{rx} in our formula (2.2.14) as unknown and try to find them independently of the ACD/dual-load calibration by mutual fitting of the *dPWV* and *wvrgcal()* signals. These modified instrumental-calibration parameters will be then used also for improved T_{sys} calculation. This application might be of interest to non-solar community, too. Note, that for the case of solar data we have been able to find corrections to G and T_{rx} by solving an extended optimization task, which involves both the science and ACD scans data – see details below in Section 2.3.3.12.

2.3 Science SPW-based PWV corrections for the solar data

In the previous Chapter 2.2 we have described the procedure for differential (i.e., w.r.t. its mean value averaged in time over a scan) PWV changes, called *dPWV-proxy*, and successfully demonstrated its application on the band-pass (BP) calibrator. The method is based on measurement of signal variations in one science SPW, usually that closest to the WV line center, as it is most sensitive to the PWV variations. Depending on whether we are using (i) the channel-resolved auto-correlation (AC) SPWs (spws 5,7,...) or, (ii) the single-channel AC (spws 6,8,...) or SQLD SPWs (spws 0,1,...), we use expressions (2.2.14) for channel-resolved SPWs, and Eqs. (2.2.16) + (2.2.17) for channel-integrated SPWs (both AC and SQLDs). It has been shown that the results do not significantly differ – AC resolved/integrated and SQLD spws provide very similar dPWV(t) profiles which, moreover, correlate well (at least for part of antennas) with both raw BP-cal phases and with the *wvrgcal()* caltable.

For the Sun, however, such a simple procedure may be risky, because the signal variation can be caused by (i) varying absorption – i.e. truly PWV related, (ii) intrinsic source (the Sun) brightness variations or, (iii) apparent variations of the solar brightness caused by lensing effects of the atmospheric turbulence. For the BP calibrator we could neglect the last two contributions because the single-dish (i.e., AC or SQLD) sky brightness is far dominated by the terrestrial atmosphere emission. In the case of Sun this, however, need not to be true – the contribution (ii), the intrinsic variation of the solar brightness, would be for sure present in case of solar flares (and one such has been successfully observed in May 2024 by White et al.), and the contribution (iii), caused by focusing/defocusing effects of the bubbles in the terrestrial atmosphere to the incoming solar radiation, is, to some extent, present always – we devote specific section to it in further text below. In other words, for the Sun we cannot neglect the last term in Eq. (2.2.9) and we have to take it into account.

Well, do we have ever a chance to distinguish between those three contributions? Fortunately yes. The keys are following:

1. Frequency dependency: Variations of atmospheric absorption caused by varying PWV increase as we approach the WV line center, while the solar brightness variations – both intrinsic (flaring) and apparent (atmospheric lensing) – are more prominent with more transparent atmosphere, i.e. further from the WV line.
2. Antenna dependency: Intrinsic source variations (flaring at the Sun) are the same at all antennas, while the apparent brightening/darkening (atmospheric lensing) is randomized per antenna.

Based on that we have developed the improved procedure described in detail below.

2.3.1 Theory

We start with Eq. (2.2.9) in its full form, including the source-brightness variations, both intrinsic and apparent as these two are not distinguishable with just one antenna. Moreover, now we do not split the total opacity $\tau(f)$ into dry and wet parts $\tau(f) = \tau_d(f) + \tau_l(f) \cdot w$, where just the wet part has been insofar considered as varying due to temporal dependence of $w(t)$. Instead of that, we consider general split of the opacity into scan-averaged and varying part $\tau(f, t) = \langle \tau(f) \rangle + \delta \tau(f, t)$. The equation (2.2.9) then reads

$$\overline{T_{sky}}(t) - \langle \overline{T_{sky}} \rangle = -\frac{1}{BW} \int_{spw} d\tau(f, t) [\langle T_{sky}(f) \rangle - T_{atm}] df + \frac{1}{BW} \int_{spw} \delta T_{src}(f, t) e^{-\langle \tau(f) \rangle} df \quad (3.1.1)$$

where, as in previous reports, the $\langle \rangle$ -brackets mean time average over a scan and over-bar denotes frequency averaging over the BW band pass (typically an SPW or its part).

Now, in order to move ahead, we have to assume something about the source variation $\delta T_{src}(f, t)$, namely we decompose its temporal and spectral dependence as

$$\delta T_{src}(f, t) = F(f) \cdot \delta T_s(t) \quad (3.1.2)$$

where for spectral profile we assume (for now) a power-law in the form

$$F(f) = \left(\frac{f}{f_o} \right)^s \quad (3.1.3)$$

The spectral index s shall be $s=2$ for the Rayleigh-Jeans approximation of thermal radiation and f_o is the reference (central) frequency.

This is going to be a ‘zero-order’ approximation if the solar flaring activity at THz/sub-THz waves is of the thermal nature. However, in the following text (gradual improvements of the methodology) we have ‘converged’ to more realistic thermal-radiation spectrum, based on the paper by Alissandrakis et al. (2022). Discussions on thermal vs. gyro-synchrotron (GS) mechanism for solar bursts at mm wavelengths are lively and ongoing in the solar community – some authors suggest strong GS component with completely different spectral index. Anyway, we have nothing better so let us the spectrum in the form above for now.

If we apply the standard ACD calibration technique (Eq. 2.2.11), we can express the sky temperatures in (1) in terms of the directly measured powers. Including source-variation assumption expressed here as (2) and (3), and assuming (as we did insofar) the opacity variations to be connected solely with the PWV dynamics, i.e. $d\tau(f,t) = \tau_1(f) \cdot dw(t)$ (this will be yet revisited in below) we can re-express Eq. (3.1.1) here as

$$\begin{aligned} \int_{BW} \frac{1}{G(f)} \cdot (P(t, f) - \langle P(f) \rangle) df = & -dw(t) \cdot \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \tau_1(f) df \\ & + \delta T_s(t) \int_{BW} \left(\frac{f}{f_o} \right)^s \cdot e^{-\langle \tau(f) \rangle} df \end{aligned} \quad (3.1.4)$$

Here, T_{inst} is the total ‘instrumental temperature’ – essentially T_{RX} generalized by the offset P_o for the SQLD SWPs:

$$T_{inst}(f) \equiv \frac{P_o}{G(f)} + T_{RX}(f)$$

Eq. (4) is the central formula for further calculations. We have two independent variables $dw(t)$ and $dT_s(t)$ but since (3.1.4) can be expressed for all SPWs, we can form even over-determined set of linear equations for those two variables in every single time instant. In practice, in order to improve SNR, we use 12 SPWs: 4 channel-resolved ACs, 4 channel-integrated ACs, and 4 SQLDs. Eq. (3.1.4) for those 12 SPWs represents the linear system $A \cdot x = C$, where row i of the matrix A and right-hand side vector C

$$A_{i1} \cdot x_1 + A_{i2} \cdot x_2 = C_i$$

represents the formula (3.1.4) for a given SPW i ; i ranges from 1 to 12. The components of matrix A and the RHS-vector C are thus written as

$$\begin{aligned}
C_i &= \int_{BW} \frac{1}{G(f)} \cdot (P(t, f) - \langle P(f) \rangle) df, \\
A_{i1} &= - \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \tau_1(f) df, \\
A_{i2} &= \int_{BW} \left(\frac{f}{f_o} \right)^s \cdot e^{-\langle \tau(f) \rangle} df
\end{aligned} \tag{3.1.5}$$

and the unknown 2-D vector is $x = (dw, dT_s)^T$; here the superscript ‘ T ’ denotes matrix transposition (making x a column vector). The system (3.1.5) is then solved for every time sample in the on-source sub-scan of the solar scan. As already adduced above: The different frequency dependence of A_{i1} and A_{i2} [which represent the sensitivity factors for PWV variation $dw(t)$ and total (intrinsic+apparent) source brightness dynamics $dT_s(t)$] assures that the columns of the matrix A are independent vectors (for the SPWs occupying different frequency domains they are far from co-linearity), so the system (3.1.5) is well conditioned and can be solved.

Since the system (3.1.5) is over-determined (we have 12 equations for 2 variables), we use the least-square approach (minimizing the residual $(A \cdot x - C)^2$), what leads to the system of so called ‘normal equations’:

$$A^T \cdot A \cdot x = A^T \cdot C \tag{3.1.6}$$

In our case it contains 2x2 matrix $A^T \cdot A$ and 2-D vectors $A^T \cdot C$ and x , so we have a unique solution for x .

To sum up: Solving the system (3.1.6) in each time instant t , we are able to decompose signal variations measured in science SPWs into: (i) variations of the PWV – the $dw(t)$ time profile, and (ii) apparent+intrinsic brightness dynamics of the Sun (actually its part exposed in the antenna PB) – the $dT_s(t)$ time series.

Entire procedure is implemented in a new routine *dWdB()* in the *sol_wvrgcal.py* collection attached as Annex 4.

2.3.2 Early results

We have applied the method to our December 2023 test data – project 2022.1.00031.CSV, SB Sun_10_a_05_INT. First, we have been interested in quality assessment of the procedure – namely the residual error in solution of the system (3.1.5). Once we have the solution vector $x = (dw, dT_s)^T$, we can calculate the difference $R_i = A_{i1} \cdot x_1 + A_{i2} \cdot x_2 - C_i$ representing the residuum for SPW with index i . An example for antenna DV12 is shown in Fig. 31. Left and right sides represent results for XX and YY, respectively. The upper panels show the residua R_i for SPWs 5,7,9,11 (red, green, blue, black solid line), 6,8,10,12 (red, green, blue, black dashed), and 0,1,2,3 (red, green, blue, black dotted). As it can be seen, the residua are comparable for all SPWs in this case (in just a few antennas some SPWs are problematic, far most of antennas are good) and reach low units of Kelvins (except of a few peaks they lay within -3 K to +3 K limit). The residual error thus represents just a small fraction of the total sky brightness variations that are displayed in the middle row of panels and which oscillate between -100 K

and 100 K: We plot the right-hand-side vector C_i from Eq. (3.1.5) normalized to the band-width BW, over which we integrate, in order to get the SPW-averaged T_{sky} ; the color-coding for SPWs is the same as in the residuals displayed in the top row.

The bottom row of panels shows, how much to the total sky-brightness variations is contributed by the apparent source brightness variations, i.e., the time series $dT_s(t)$ found as component x_2 , modified by the sensitivity factor A_{i2} (which has a meaning of average transmissivity weighted slightly by the Rayleigh-Taylor frequency scaling) – we plot $A_{i2}.x_2$ for each SPW with index i , using the same color-coding as above. As it can be seen, the brightness dynamics of the source (apparent + intrinsic) represents smaller but already non-negligible fraction of the total signal variations, namely it contributes to its ultra-short

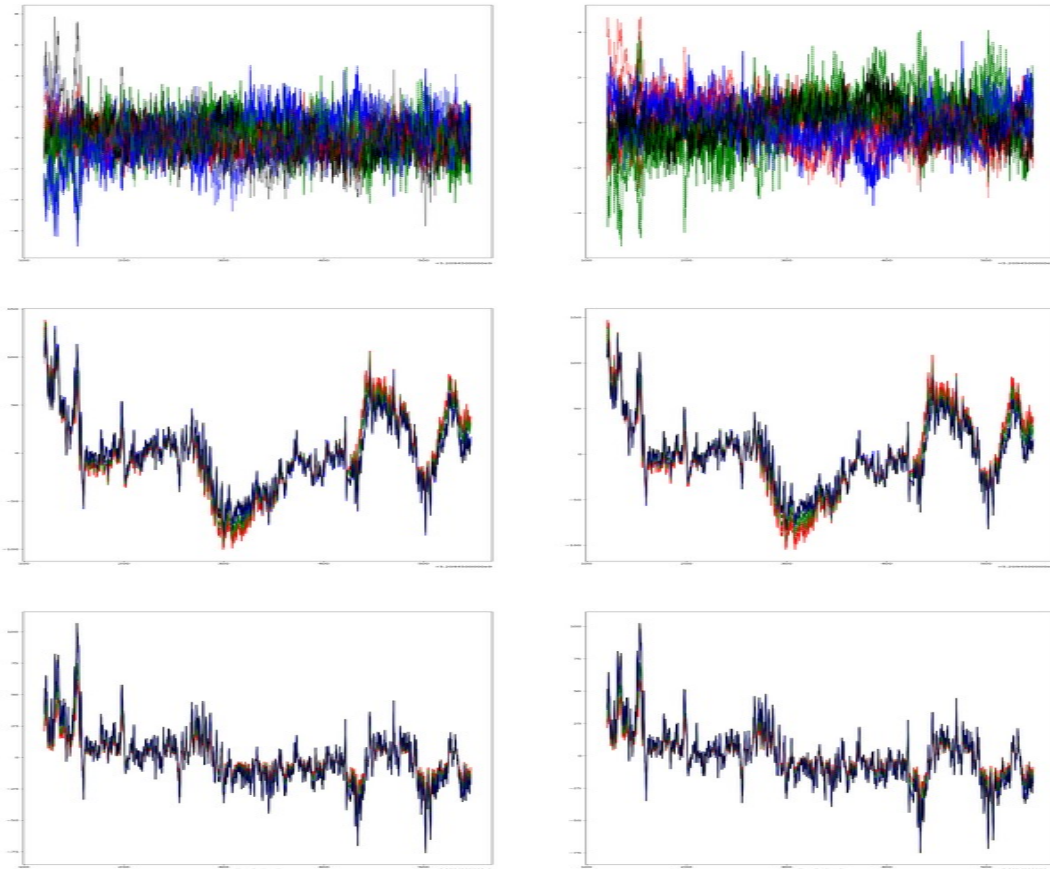


Figure 31 Residuals (top row), total sky brightness variations (middle) and source brightness contribution to the total sky brightness variations as extracted by solving the system in Eq. (5). Left and right columns were obtained from XX and YY pols in science SPWs.

and intense peaks, like that at around time 150 ($+5.20945 \times 10^9$) MJD secs. Signal in all SPWs is ‘synchronous’. Figures for all antennas are available on-line as *Annex 1*.

As the procedure seems to be fine – residuals are much smaller than any measured or derived quantity and all the SPWs show very similar results (the profiles are synchronous), we now check the solutions itself. Fig. 32 shows decomposition of the signal to the source-brightness variations $dT_s(t)$ in left panel and the PWV variations $dw(t)$ in right panel. Blue and red colors correspond to XX and YY polarizations.

When the new PWV variation profile is compared with the same quantity but derived using the old, simple single-SPW approach according to Eq. (2.2.14), they are actually very similar – only significant difference is in the ultra-short high peaks (spikes) that are present in the single-SPW $dPWV(t)$ curve, while not in the new one in Fig. 32 (right). Clearly, these spikes have moved into the left panel – they represent the source (apparent) brightness spikes, not a sudden, significant and at the same moment short-living (which is unlikely) PWV enhancements.

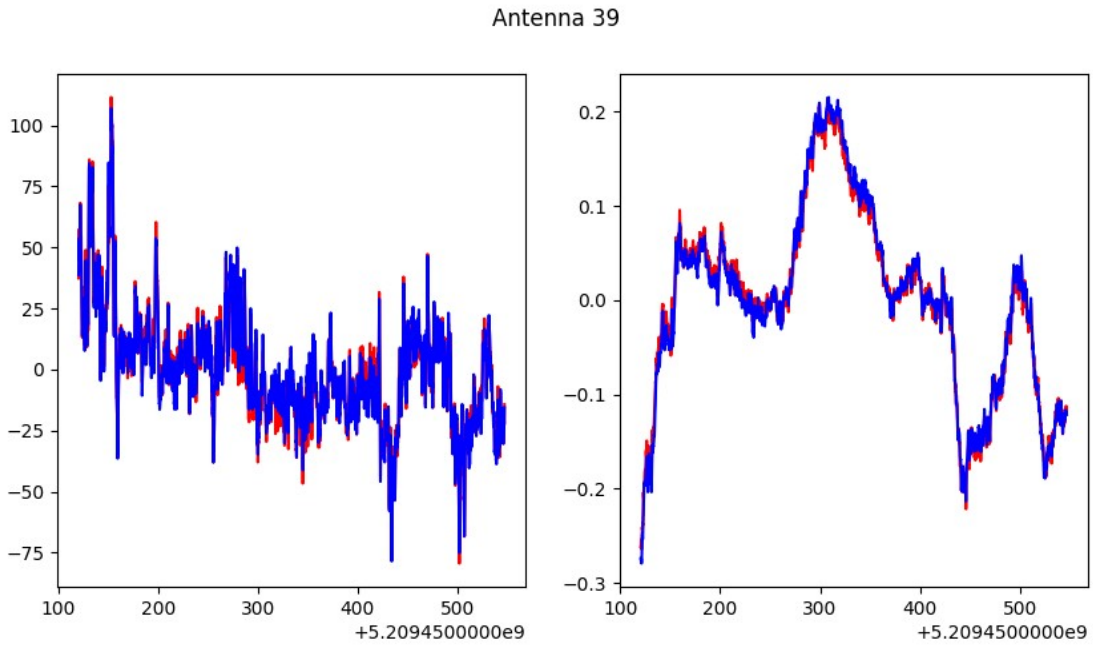


Figure 32: Source brightness dynamics (left, in Kelvins) and PWV variations (right, in mm) for antenna DV12. Blue and red curves are for XX and YY pols. Time at the x-axis is in MJDsecs.

Inclusion of the source-brightness fluctuations explains yet another observation that was a bit mysterious to us insofar: If we use the single-SPW calculations by Eq. (2.2.14), it is clear that the signal (T_{sky}) variations are linearly proportional to the PWV variations, in any considered SPW. The proportionality coefficient (which we call ‘sensitivity factor’ from obvious reason) clearly differs from SPW to SPW, the highest being in the SPW closest to the WV line. Hence, if we make a scatter plot between measured T_{sky} variations in, say SPW 5 and SPW 11, we would expect a straight line with the slope that corresponds to the ratio of those sensitivity factors. Such a scatter plot is displayed in Fig. 33 – x-axis shows T_{sky} variations (in K) in SPW 5 (closest to the WV line) while the y-axis T_{sky} variations in

SPW11 (farthest from the WV line). We have used continuous line (the red curve) instead of single dots in the scatter plot in order to trace the dynamics – the line connects the points that are consecutive in time.

What is eye-striking in the red curve that it looks like there are two effects working at two different timescales: The fast variations with the steeper slope, and the slower dynamics with the overall milder slope. Our first interpretation was in terms of the PWV dynamics – that it may have (i) the fast part related to fast compression/dilution of the atmospheric bubbles (as the total air pressure changes, it changes also its partial pressure of the water vapor), and, (ii) the ‘slow mode’ related to slow changes of water-vapor relative abundance (humidity). However, searching for decomposition into pressure and humidity variations failed (we returned to this topic yet in the discussion further). Nevertheless, inclusion of the $dT_s(t)$ dynamics changes the game – the fast variations may be interpreted as the apparent brightness dynamics of the source. And indeed: We have over-plotted the red scatter curve with two straight lines: The green one corresponds to ratio of matrix coefficients $A_{11,1}/A_{5,1}$ i.e. the sensitivity factors for the $dW(t)$ variations, while the blue one corresponds to ratio of matrix coefficients $A_{11,2}/A_{5,2}$ i.e. the sensitivity factors for the $dT_s(t)$ variations. Clearly, the blue line follows the fast dynamics of the red curve while the green one corresponds to the slow ‘global drift’.

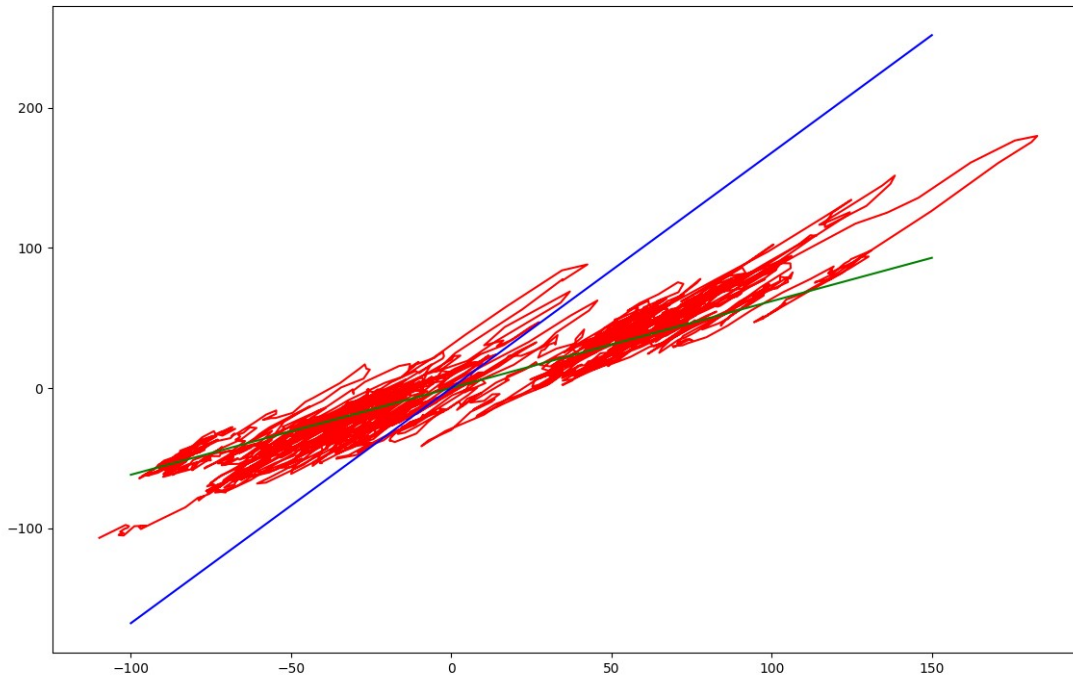


Figure 33: Red curve: Scatter plot (connected points with consecutive times) of T_{sky} variations in SPW 5 vs SPW 11. Green and blue lines: Ratio of matrix coefficients (sensitivity factors) for dW and dT_s , respectively (see details in the text).

Conclusion: The signal measured as T_{sky} variations in science SPWs for the Sun has two components: Slower one, which corresponds to PWV dynamics over a given antenna, and fast one related to the apparent brightness variations of the observed part of the Sun.

In order to increase our confidence in this interpretation, we can do a similar scatter plot for the BP calibrator. There is no source (neither intrinsic nor apparent due to atmospheric lensing) variation expected because the emission in ACs and SQLDs is far dominated by the terrestrial atmosphere. If, on the other hand, the slow/fast component splitting was related to humidity/pressure variations (i.e. purely atmospheric effect), we should see it in the BP cal, too.

As it can be seen, there is, indeed, no fast component present, except of the random noise, and the only linear structure in the figure is the ‘global drift’ that follows the green line corresponding to the ratio of sensitivity factors for the PWV variations.

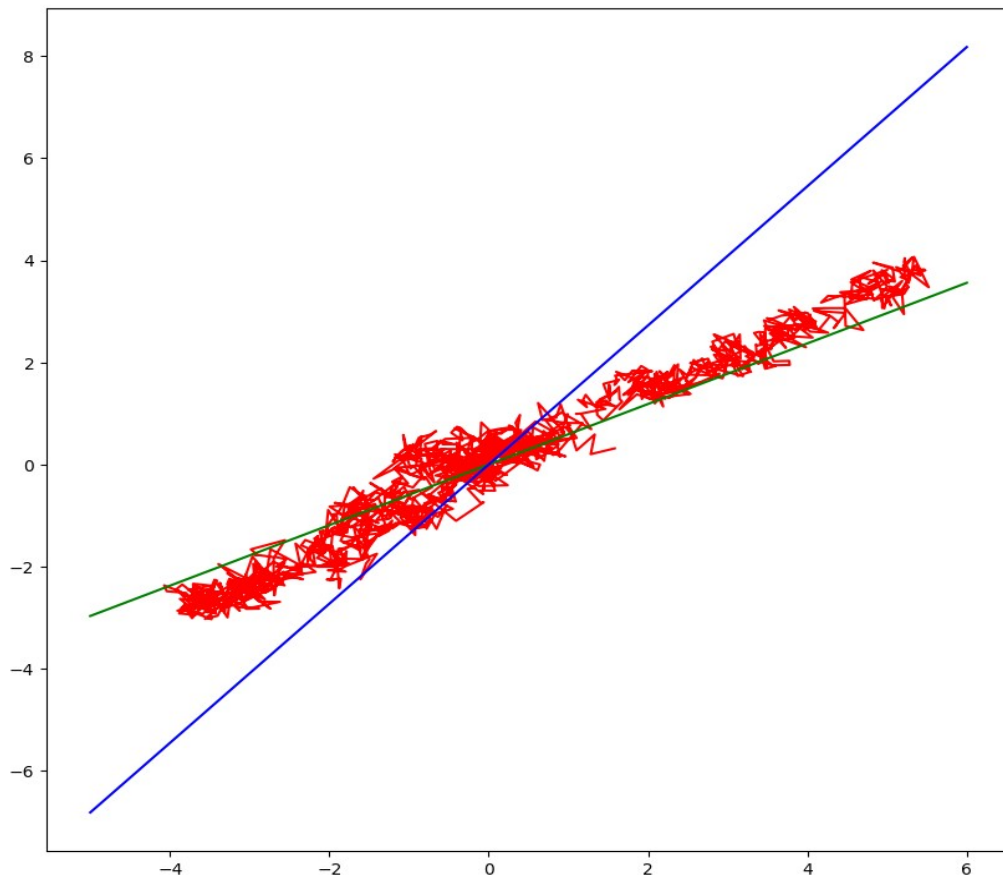


Figure 34: Like in Fig. 33, now for the BP calibrator (note the much smaller ranges of T_{sky} variations on both axes), variations in SPW 8 vs. SPW12

As already described above, the decomposition of the two components – PWV variations and the T_{src} dynamics – are possible only thanks to different frequency dependence of the sensitivity factors for $dw(t)$ and $dTs(t)$. This different frequency functions lead to independence of columns of the matrix A in the system (5). Graphical illustration of that fact is visible in the Figs. 33 and 34 as the angle between the green and blue straight lines: The bigger is the angle, the more exact we have the decomposition (better conditioned matrix).

2.3.2.1 Nature of the source brightness variations

We have done the decomposition into $dw(t)$ and $dTs(t)$ for all antennas – see *Annex 1* online. Already at the first look it is clear that the $dTs(t)$ is significantly different for different antennas. Hence, its main component hardly can be connected with the intrinsic brightness variation of (the observed part of) the Sun. Most likely interpretation lies in atmospheric lensing – the turbulence eddies have focusing and de-focusing effects on the transmitted solar radiation. If a focused-radiation ‘hot spot’ falls into an antenna, we record short and relatively intense brightening. Such an effect is well known in the long-wavelength radio observations, where the larger-scale traveling ionospheric disturbances (TIDs, basically ‘bubbles’ of enhanced electron density) may even cause ‘fake’ radio burst in ground-based receivers. These ‘pseudo-bursts’ draw characteristic features in their dynamic radio spectra and are known as *spectral caustics* (e.g., Koval et al. 2019, <https://iopscience.iop.org/article/10.3847/1538-4357/ab1b52> and references therein). We do not have time for deep investigation into this topic but as an indication we present Figs. 35 and 36, which show correlation between the $dTs(t)$ and the time derivative of the $dw(t)$. The idea (not really developed in depth) is the following: If $dw(t)$ changes rapidly, it means that the phase delay changes rapidly, too. Rapidly changing phase may indicate focusing irregularity in the atmosphere.

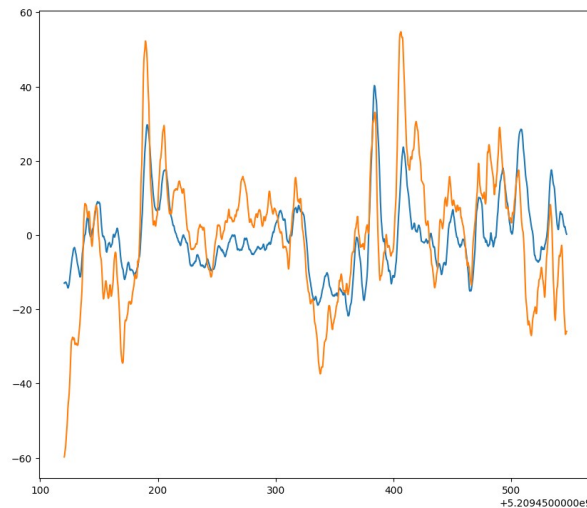


Figure 35: $dw(t)/dt$ - smoothed and linearly scaled to the same range of values (orange) and $dTs(t)$ (in blue) both as a function of time. There are apparent similarities in both curves.

Albeit, as written above, the idea is not really developed (and will not be, as it is off-topic for this study), the correlation between $dw(t)/dt$ and $dTs(t)$ indicates dominantly atmospheric origin of the T_{src} variations: If it was extra-terrestrial, there would be no reason why the two quantities should be related, because solution of the system (3.1.5). is performed in each time instant independently, so it ‘knows’ nothing about dynamics hidden in the $dw(t)/dt$ derivative.

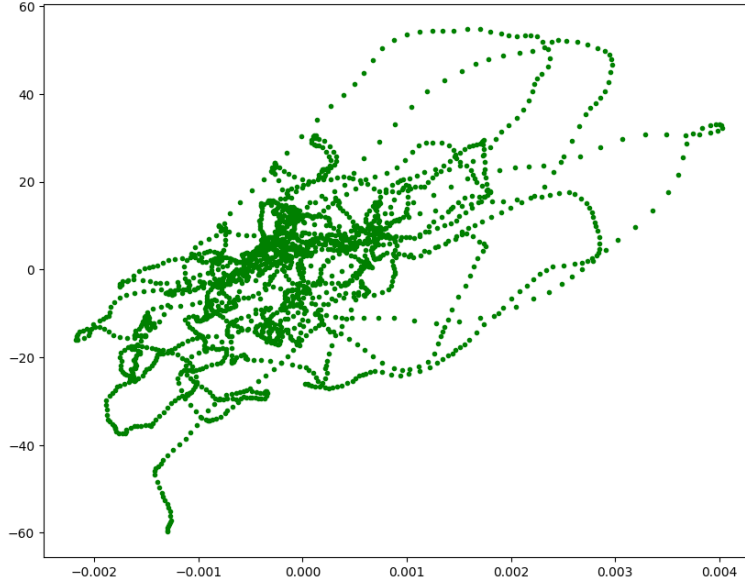


Figure 36: Cross correlation (scatter plot) between $dw(t)/dt$ and $dTs(t)$

Anyway, we are interested in the question, whether there is also some intrinsic source brightness variation. The key, as it has been adduced above, is in the difference between $dTs(t)$ time series for different antennas. If they contain just the random atmospheric component, the sum (or, better, the mean) of all $dTs(t)$ profiles over entire array would tend to zero as the random variations tends to cancel each other. On the other hand, intrinsic solar brightness fluctuation should come out in the sum/mean $dTs(t)$ curve.

We have tried it and the result is displayed in Fig. 37. Clearly, there is non-zero variation in both polarizations in units of Kelvins. Saying it is the intrinsic source T_B variation is too strong, the lensing component may have some amount of mutual correlation as the ‘hot’ and ‘cold’ spots on the Earth surface created by focusing/de-focusing effects are transported over the array by the mean wind. On the other hand, it is quite smooth and variations are not big, so its interpretation as the intrinsic source variability is plausible.

Anyway, whatever is the nature of the apparent source brightness variations, for this project it is important just one matter: We are able to separate them from the measured signal and thus produce more trustworthy the remaining cause of signal – the PWV variations.

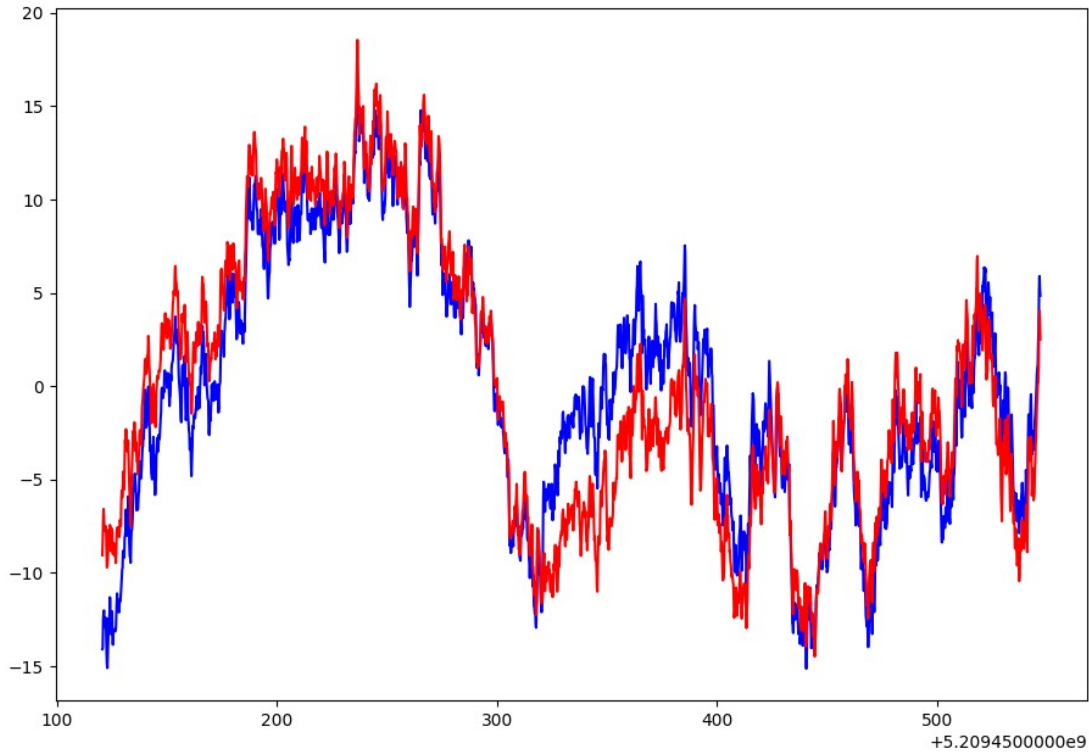


Figure 37: Sum of $dTs(t)$ curves over all antennas (with exception of the two flagged): Red XX, blue YY, time in x-axis in MJDsec.

2.3.2.2 Discussion to separation of pressure/humidity or dry/wet components in dPWV variations

Before finding that the two components (slow and fast) with two different slopes manifested in the scatter plot in Fig. 33 are actually related to PWV variations (slow component) and dynamics of the (apparent/scintillating) source brightness (fast component), we have been thinking about interpretation in terms of varying relative humidity (slow) and total pressure/density of the air (fast). Alternatively, we may think about making a decomposition into *dry* and *wet* component variations. Here we show that both decompositions (humidity/pressure and wet/dry) are actually equivalent. Nevertheless, as we will see, the results of an attempt to disentangle additional components in the signal are remaining negative.

Basically, the total PWV content $w(t)$ should be proportional to integrated air mass (integral of air density) $R(t)=\int \rho(l,t) dl$ and the (ray-path-averaged) relative abundance of the water vapor $a(t)$

$$w(t)=C \cdot R(t) a(t)$$

where C is appropriate constant coefficient (basically conversion between units). If we express each quantity as its time-averaged mean value and a small variation, e.g.

$$w(t)=\langle w \rangle + dw(t)$$

and likewise for $R(t)$ and $a(t)$, we arrive (after some simple algebra) to expression

$$w(t)=\langle w \rangle \cdot \left(1 + \frac{dR(t)}{\langle R \rangle} + \frac{da(t)}{\langle a \rangle} \right) = \langle w \rangle + \langle w \rangle \cdot \left(\frac{dR(t)}{\langle R \rangle} + \frac{da(t)}{\langle a \rangle} \right),$$

where we also neglected the second-order term $dR(t) \cdot da(t)$. Here, the latter term clearly represents the PWV variation $dw(t)$.

Now, we can express the wet opacity as

$$\tau_w(t)=\tau_1 \cdot w(t)=\tau_1 \langle w \rangle \cdot \left(1 + \frac{dR(t)}{\langle R \rangle} + \frac{da(t)}{\langle a \rangle} \right)$$

Similarly, the dry opacity will reads

$$\tau_d(t)=\langle \tau_d \rangle \cdot \left(1 + \frac{dR(t)}{\langle R \rangle} \right)$$

as it is related just to the total integrated air density variations and not to WV abundance. So, if we express the total opacity

$$\tau(t)=\tau_w(t)+\tau_d(t)=\tau_1 \cdot \langle w \rangle \cdot \left(1 + \frac{dR(t)}{\langle R \rangle} + \frac{da(t)}{\langle a \rangle} \right) + \langle \tau_d \rangle \cdot \left(1 + \frac{dR(t)}{\langle R \rangle} \right) \quad (3.2.1)$$

we can write it, again, as its mean (time-averaged) value and variation

$$\tau(f,t)=\langle \tau(f) \rangle + d\tau(f,t)$$

where the obvious frequency dependence is now explicitly indicated, and, the variation itself may be expressed in two alternative ways, depending on how we arrange and combine the terms in Eq. (3.2.1):

$$\begin{aligned} d\tau(f,t) &= \langle \tau(f) \rangle \cdot \frac{dR(t)}{\langle R \rangle} + \langle \tau_w(f) \rangle \cdot \frac{da(t)}{\langle a \rangle} \\ &= \tau_1(f) \cdot dw(t) + \langle \tau_d(f) \rangle \cdot \frac{dR(t)}{\langle R \rangle} \end{aligned} \quad (3.2.2)$$

Now, this expression (3.2.2) may be inserted in the general Eq. (3.1.1) to get two alternative generalizations of the central formula (3.1.4):

$$\begin{aligned}
\int_{BW} \frac{1}{G(f)} \cdot (P(t, f) - \langle P(f) \rangle) df = & -dw(t) \cdot \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \tau_1(f) df \\
& - \frac{dR(t)}{\langle R \rangle} \cdot \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \langle \tau_d(f) \rangle df \\
& + \delta T_s(t) \int_{BW} \left(\frac{f}{f_o} \right)^s \cdot e^{-\langle \tau(f) \rangle} df
\end{aligned} \quad (3.2.3a)$$

and

$$\begin{aligned}
\int_{BW} \frac{1}{G(f)} \cdot (P(t, f) - \langle P(f) \rangle) df = & -\frac{da(t)}{\langle a \rangle} \cdot \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \langle \tau_w(f) \rangle df \\
& - \frac{dR(t)}{\langle R \rangle} \cdot \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \langle \tau(f) \rangle df \\
& + \delta T_s(t) \int_{BW} \left(\frac{f}{f_o} \right)^s \cdot e^{-\langle \tau(f) \rangle} df
\end{aligned} \quad (3.2.3b)$$

As a result, we get a linear system of 12 equations similar to Eq. (3.1.5) (if we use all AC and SQLD SPWs), now for three independent variables: In the approach (3.2.3a) the 3-vector of unknown variables is composed of the PWV variation, relative air-mass (integrated density) variations, and the (apparent) source-brightness dynamics; in case of (3.2.3b) the independent variables are relative ‘humidity’ variation, relative air density variation, and again, the source-brightness dynamics. The approaches 3.2.3a and 3.2.3b are equivalent, just the 3-vector of the unknown variations is expressed in another basis.

We first tried the approach (3.2.3b) but the results are absolutely disappointing – the relative variations in ‘humidity’ $a(t)$ and air (averaged) density $R(t)$ are both oscillating in interval much broader than -1 to +1 (which is non-sense – they represent the *relative* change of the respective quantities, so expected variations were at the level less than 1%), essentially in anti-phase, compensating each other in order to get the finite and correct right-hand-side vector of the system (the sky brightness variations). The reason is clear – their matrix coefficients differ only to that extent as the time-averaged wet opacity $\langle \tau_w(f) \rangle$ differs from the total opacity $\langle \tau(f) \rangle$. Since the difference is negligible, the system is very ill-conditioned (inverse value of the matrix determinant is of the order of 10^5). In this sense, the result is not surprising.

With the approach (3.2.3a) the result is somewhat better, but also not answering the question: Eq. (3.2.3a) essentially differs from Eq. (3.1.4) only by presence of the 2nd term at the RHS. Now we have, indeed, different frequency dependence of the matrix elements – the dry opacity $\langle \tau_d(f) \rangle$, controlling the 2nd term at RHS, is very different function of frequency than $\tau_1(f)$, so the rows of the linear system formed by different SPWs are independent vectors. As a result, we reproduce the $dw(t)$ variations with a curve that is indistinguishable from that produced by Eq. (3.1.4) (i.e., without dry/wet decomposition). On the other hand, the relative variations of the integrated air density are, again, very high. The reason is, that the matrix coefficient (sensitivity factor) for the air-density variations is very

low – it is proportional to the dry opacity – and, thus, its inverse value, used for the density variation calculation, is of the order of 10^4 . So, in fact, we just effectively multiply the noise by this high factor.

Summary: We did not succeed with decomposition of the opacity into dry/wet (or humidity/density) sub-components: In a first case we reproduce the $dw(t)$ curve correctly, but the resulting calculated density variations are essentially just an amplified noise, in the latter case, the humidity and total-density variations are both wrong as the system is ill-conditioned.

2.3.3 From dPWV fluctuations to full-PWV time series: Rigorous approach

In so far, when speaking about “*dPWV proxy*” calculated from science SPW signal, we have been always working with the PWV *fluctuations* – differentials to the mean PWV value, time-averaged over a scan. We have presented many various approaches to dPWV calculation, the most sophisticated one in the last Section 2.3.1. It decomposes the science-SPW signal fluctuations into (i) varying absorption (due to changing PWV), and, (ii) to the apparent brightness changes of the target, where ‘apparent’ stands here for (de)focusing effect of atmospheric ‘bubbles’ (convective turbulence vortices), i.e. the scintillation. Nevertheless, even this advanced method, implemented in *dWdB()* routine in our *sol_wvrgcal.py* collection, works also just with differences to the average PWVs and brightness temperatures.

In order to achieve the *full phase corrections*, not just removing (or significantly reducing) the fluctuating part, we, however, need the *complete* PWV time series for each antenna. To that end we have developed a new, rigorous methodology for calculation full PWV time series from the science data. This text represents a full-detail report on this topic, including all incremental steps that improved the procedure. Impatient reader may skip directly to Section 2.3.3.13, where the final result implementing all accumulated gradual improvements is presented, and return back to the mathematical derivations, if interested.

2.3.3.1 Starting point

In the previous sections 2.3.1 and 2.3.2 we have presented a method, which decomposes fluctuations of signal recorded in the science SPWs around its mean (scan-averaged value) into two parts: (i) Variations due to changes of atmospheric transmissivity, which changes because of varying PWV, whose fluctuations we describe by a quantity $dw(t)$, and, (ii) variations of intrinsic or apparent (due to scintillation) brightness of the solar target, described as $dT_s(t)$. The decomposition is possible thanks to different frequency dependencies of transmissivity (determined by absorption profile of WV line @183GHz) and solar brightness (given by the solar thermal spectrum), and the fact that we observe over a relatively broad frequency domain of Band 5 in the range of 190 GHz – 206 GHz. We have eventually arrived to a linear system (3.1.4) for an unknown 2D vector $x \equiv (x_1, x_2) = (dw, dT_s)^T$ in the form of

$$A_{i1} \cdot x_1 + A_{i2} \cdot x_2 = C_i, \quad (3.3.1)$$

where matrix coefficients A and the RHS-vector components C (recorded temperature fluctuations) read (see Eq. 3.1.5, we repeat it here for readers convenience):

$$\begin{aligned}
C_i &= \int_{BW} \frac{1}{G(f)} \cdot (P(t, f) - \langle P(f) \rangle) df \equiv \delta T_i^{rec}, \\
A_{i1} &= - \int_{BW} \left(\frac{\langle P(f) \rangle}{G(f)} - T_{inst}(f) - T_{atm} \right) \tau_1(f) df, \\
A_{i2} &= \int_{BW} \left(\frac{f}{f_o} \right)^s \cdot e^{-\langle \tau(f) \rangle} df
\end{aligned} \tag{3.3.2}$$

Index i goes over our available SPWs (i.e., AC-resolved 5,7,9,11, AC-integrated 6,8,10,12, and SQLD 0,1,2,3) from 1 to 12. We solve this system independently for each polarization (XX/YY) and each time instant. The system is over-determined, because we have – for each time instant and each polarization – 12 equations and only 2 variables, so we solve it in the least square sense, i.e. the $(A.x - C)^2 \rightarrow \min$. The minimizer of this quadratic form leads to the system of so called normal equations $A^T.A.x = A^T.C$. The matrix coefficients A_{i1} and A_{i2} represent the *sensitivity factors* for dw and dT_s , respectively – the bigger they are, the larger are responses to dw and dT_s variations seen in data in form of the fluctuating received power. For each SPW id $S=i$ we write $SF_w^S = A_{s1}$ and $SF_B^S = A_{s2}$, where we have chosen index ‘B’ for brightness-temperature sensitivity in order to not mixing it with ‘t’ for time.

Now we present the incremental steps in the development of our procedures towards mathematically rigorous handling of the problem of full PWV(t) time series.

2.3.3.2 Polarization coupling in PWV

Easiest incremental improvement is connected with improved handling of the differential approach governed by Eq. (3.3.1) and concerns ‘double’ solution for $dW(t)$. Insofar we have been handling the system (3.3.1) independently for both XX and YY pols. This way, we have got two time series for both $dw(t)$ and $dT_s(t)$ – see, e.g., Fig. 32, which presents the time-series as red and blue curves per polarization. While the brightness variations $dT_s(t)$ can, in principle, change independently in each polarization, there is no logical reason for $dw(t)$ should do so, too.

In addition of independent per-polarization solutions, we have also viewed Equations (3.3.1) as being independent for each time instant, i.e., we have been, in fact, handling $nPol \times nTimeSamples$ independent systems to be solved for each time (integration) and each polarization separately. It is, however, useful to conceptually change the view and look at it as to one large linear system like in Fig. 38: The large matrix of this system is composed from blocks of the matrix A of the basic system (3.3.1), and the new, large RHS vector is a formed as a stack of the basic RHS vectors C .

The vector of unknowns dw and dT_s is also formed as a stack of elementary (per time and polarization) solutions of system (3.3.1), the polarization and time (technically, index of correlator-data integration) now represent (multi-)index in this large vector of unknowns. Mathematically, both views are equivalent, because the blocks in the large matrix apply only on the relevant part of the composed unknown-variable vector and assure that the product will be equaled just with the appropriate (for a given time and polarization) segment of the RHS (data) vector. But this large-system view allows us to understand changes applied further in the following text.

Figure 38: Linear system (3.3.1) when viewed not like $nTimeSamples \times nPols$ independent subsystems but as one equivalent large linear system of equations. The spw and polarization indices are written directly as super-scripts, the time index is traditionally written in parentheses (albeit it is discrete, too) and indexed from 1 to N , where N is a shortcut for $nTimeSamples$. Only green areas in the large matrix are occupied by non-zero numbers (the sensitivity factors for dW and dT), the white areas are all zeros.)

Our changed view on the system (3.3.1) gives a hint, how we can move forward:

- We re-arrange the unknown-variables vector (the order of its component) such a way, that variables of the same kind (dw , dT) are now next each other i.e., from the scheme in Fig. 38

$\{dW^{XX}(0), dT^{XX}(0), dW^{YY}(0), dT^{YY}(0), dW^{XX}(1), dT^{XX}(1), dW^{YY}(1), dT^{YY}(1), \dots, dW^{YY}(N), dT^{YY}(N)\}$
we come to $\{dW^{XX}(0), dW^{YY}(0), dT^{XX}(0), dT^{YY}(0), dW^{XX}(1), dW^{YY}(1), dT^{XX}(1), \dots, dT^{XX}(N), dT^{YY}(N)\}$

The figure shows a large matrix equation: $\text{Matrix} \times \text{Vector} = \text{Vector}$.

The matrix is a large square with a block-diagonal structure. The diagonal blocks are green and contain sub-matrices for each time step t . The off-diagonal blocks are white and represent zero matrices. The matrix is multiplied by a column vector of unknowns, resulting in a column vector of constants.

The green blocks are of shape 24×3 . The rest white areas of the large-system matrix are zeroes.

The column vector of unknowns is:

$$\begin{bmatrix} dW(0) \\ dT(0) \\ dW(1) \\ dT(1) \\ dW(2) \\ dT(2) \\ \vdots \\ dW(N) \\ dT(N) \end{bmatrix}$$

The column vector of constants is:

$$\begin{bmatrix} 5XX \\ C(0) \\ 5YY \\ C(0) \\ 7XX \\ C(0) \\ 7YY \\ C(0) \\ \vdots \\ 3YY \\ C(0) \\ 5XX \\ C(1) \\ 5YY \\ C(1) \\ 7XX \\ C(1) \\ 7YY \\ C(1) \\ \vdots \\ 3YY \\ C(1) \\ \vdots \\ 3YY \\ C(N) \end{bmatrix}$$

Figure 39: Re-arranged linear system from Fig. 38 with reduced set of unknowns: We have now single solution for $dw(t)$ variations. The green blocks are of shape 24×3 , rest white areas of the large-system matrix are zeroes.

- We further re-arrange the order of equations: So far, in Fig. 38, they were ordered by time, polarization, and SPW, so that the SPW-index changes fastest, time-index slowest. We make the

polarization to be the fastest-changing index, i.e. the RHS column components will take the form $\{C^{5XX}(0), C^{5YY}(0), C^{7XX}(0), C^{7YY}(0), \dots, C^{3YY}(N)\}$.

- Now we can merge the variables $dw^{S,XX}(t)$ and $dw^{S,YY}(t)$ for SPW S and pols XX/YY together into a single one $dw^S(t)$ (note, still it is a vector, the SPW index S and the time index t remained, just the pol dimension contracted).

The above operations form a reduced system shown in Fig. 39, whose forming blocks are now matrices of the 24(rows) x 3(columns), where 3 comes from 3 sub-variables $\{dw^S(t), dT^{SXX}(t), T^{SYY}(t)\}$, still a vector with S and t indices, and 24 is a product of $nPol$ s x $nSPW$ s.

The structure of the system keeps its block-diagonal form and the same hold for the corresponding normal equations $A^T.A.x = A^T.C$. The normal matrix $A^T.A$ is formed from 3x3 blocks at its diagonal with all other elements being zero. This has a practical consequence – we can still solve the problem for each time instant independently, having $nTimeSamples$ small systems of size 3x3.

Results: See Fig. 40. The brightness variations have now more pronounced (scintillation) peaks, some 40 – 60 % higher, the $dw(t)$ curve did not change much from its ‘double’, per-pol solution (it looks a bit smoother), because already when it was decomposed into two polarizations, they were almost identical – c.f. Fig. 32.

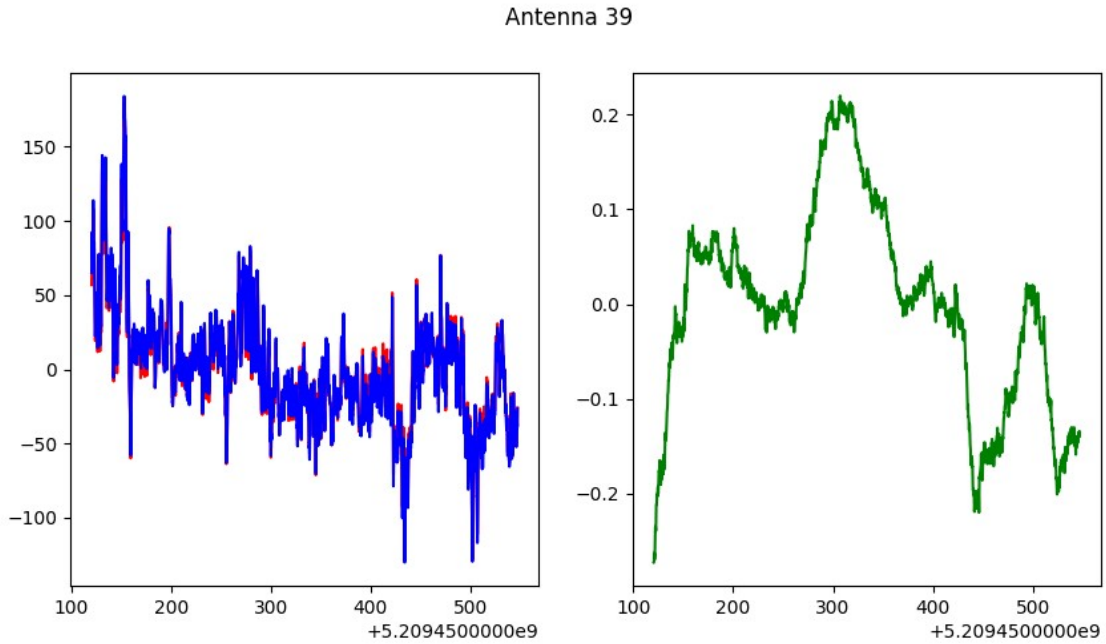


Figure 40: Left: Fluctuations of apparent (incl. scintillations) brightness temperature in XX (red) and YY (blue) pols as a function of time (MJDsec). Right: Fluctuations of PWV (mm) as a function of time. Both for antenna id 39 (DV11). We now have just a single $dPWV(t)$ time series, not two per-polarization solutions.

2.3.3.3 Making use of the scan-averaged signal

Now we come to the main point – upgrade of our insofar differential (fluctuations-only) approach used for science SPW data in order to derive the dPWV proxy, towards the full-signal handling. The main change lies in our handling of atmospheric properties – its transmissivity and intrinsic emission. Insofar we have been using just differences/derivatives, now we go using their full ‘expressions’ as a functions of PWV. This change makes the set of our governing equations non-linear because the brightness temperature of the sky is non-linear function of PWV.

Mathematical background

We start with separating instrumental (internal) and external sources of the recorded signal

$$T_{rec}^{SP}(t) \stackrel{\text{def}}{=} \frac{P_{rec}^{SP}(t)}{G^{SP}} = T_{inst}^{SP} + T_{ext}(t) \quad (3.3.3)$$

where the ‘external’ contribution to intensity of radiation, expressed by its equivalent temperature, is by the Sun, diminished by atmospheric transmissivity $tr()$, and intrinsic contribution of the terrestrial atmosphere (namely WV-line wing) and cosmic background (CMB), handled together as T_{sky} :

$$T_{ext}(t) = tr[w(t)] \cdot T_{Sun}^B(t) + T_{sky}[w(t)] \quad (3.3.4)$$

The ‘internal’ T_{inst} contribution has been already defined in Section 2.2.2 as $T_{inst} = T_{RX} + P_o/G$, in the AC SPWs it equals the receiver temperature T_{RX} , in SQLDs it accounts also for instrumental offset P_o .

We assume all quantities in Eq. (3.3.4) are integrated (averaged) over frequency channels of the SPW in a given polarization. The external brightness temperature can be thus written as an integral over the bandwidth BW of the SPW S in polarization P (indicated by sub- and super-scripts):

$$T_{ext}^{SP}(t) = T_{Sun}^B(t) \cdot \int_S tr(w(t), f) \cdot S(f) \cdot g^{SP}(f) \frac{df}{BW} + \int_S T_{sky}^S(w(t), f) \cdot g^{SP}(f) \frac{df}{BW} \quad (3.3.5)$$

Here we define the spectral gains $g^{SP}(f)$ normalized to unity as

$$G^{SP} \stackrel{\text{def}}{=} \int G^{SP}(f) \frac{df}{BW}, \quad g^{SP}(f) \stackrel{\text{def}}{=} \frac{G^{SP}(f)}{G^{SP}}$$

For the channel-resolved SPWs we calculate $g^{SP}(f)$ from the ACD calibrations, for the integrated AC SPWs and SQLDs they are set to unity. $S(f)$ is the already earlier introduced solar spectrum normalized to 1.0 at the reference frequency f_o , where we evaluate the T_{Sun}^B , i.e. $T_{Sun}^B(f, t) = T_{Sun}^B(t) \cdot S(f)$. The source spectrum $S(f)$ is expressed (for now – see improvements below) as a power-law $S(f) = (f/f_o)^s$, and we assume spectral index $s=2$, which corresponds to Rayleigh-Jeans law for black-body radiation.

In fact, we have no analytical formula expressing the transmissivity and brightness temperature of the sky (atmosphere + CMB) as a function of frequency and PWV, but we can obtain their numeric values from the ATM model (Pardo et al., 2001; implemented as *atmosphere/at* tool in CASA) for any frequency channel and any user-set PWV value (plus further parameters that are preset by

`at.setAtmosphere()` in order to match the atmosphere-model to actually measured meteo conditions). Specifically, we evaluate T_{sky} in SPW S as

$$T_{sky}^S[w(t),f]=at.getTebbSkySpec(spw=S, userH2O=w(t)) \quad (3.3.6)$$

Inserting (5) into (3) we get

$$T_P^B(t) \cdot \int_S tr(w(t),f) \cdot S(f) \cdot g^{SP}(f) \frac{df}{BW} + \int_S T_{sky}^S(w(t),f) \cdot g^{SP}(f) \frac{df}{BW} = T_{rec}^{SP}(t) - T_{inst}^{SP} \quad (3.3.7)$$

where we have dropped the subscript ‘Sun’ for the brightness temperature as it cannot make confusion and we need the space for polarization index P : As can be seen from (3.3.7), we allow the (apparent) solar brightness temperature to be dependent on polarization P , while solar spectrum and atmospheric transmissivity are dependent only on frequency, hence they need to be calculated for each SPW separately (therefore subscript S at the integral, but no polarization dependence). As in the case of differential (fluctuation-only) approach, we can write equation (3.3.7) for all spw:pol combinations, which represents in total $numSPW \times numPOL = 12 \times 2 = 24$ equations for each time instant. At the same moment, it must be valid for all time instants (integrations), for which we have our measured data $T_{rec}^{SP}(t)$. So, in total, the Equation (3.3.7), in fact, represents a system of $numSPW \times numPOL \times nTimeSamples$ equations. Basically, we are in similar situation as in the differential (fluctuation) approach (Fig. 39), but now we face a large set of *non-linear* equations. We have, again, three sub-vectors of unknowns: $w(t)$, $T_{XX}^B(t)$, and $T_{YY}^B(t)$, each of them having $nTimeSamples$ components, so in total it represents a long vector of $3 \times nTimeSamples$ unknown variables. Clearly, the system is over-determined as in case of fluctuation-only (linear) approach, because number of equations exceeds the number of unknown variables. It can therefore hardly be fulfilled exactly and we, again, search for a solution in the least-square sense. Our system of equations (3.3.7) takes the general form

$$f_k(x_j) - b_k = 0 \quad (3.3.8)$$

where $x_j = \{w(t), T_{XX}^B(t), T_{YY}^B(t)\}$ is our unknown vector, the vector function $f_k(x_j)$ represents the LHS of Eq. (3.3.7), and b_k stands for the vector of the RHS values ($T_{rec}(t) - T_{inst}$). Index k goes from 1 to $numSPW \times numPOL \times nTimeSamples$. In the least-square approach we search for such a vector x_j so that

$$\sum_k (f_k(x_j) - b_k)^2 \rightarrow min. \quad (3.3.9)$$

For the minimum of Eq. (3.3.9), gradient in the x_j -space should be zero, which leads to equation

$$J_{ik}^T(x_j)[f_k(x_j) - b_k] = 0 \quad (3.3.10)$$

where J_{ik}^T is the transposed Jacobian matrix of the vector function $f_k(x_j)$, i.e.

$$J_{ik}^T = \left(\frac{\partial f_k(x_i)}{\partial x_i} \right)^T$$

Calculation of the Jacobian J_{ik} is performed by differentiation of the LHS of Eq. (3.3.7). For $T_{sky}^S[w(t),f]$, whose values are obtained numerically via interface-functions to the ATM model (CASA *at* tool), we calculate numerical derivative w.r.t. the unknown $w(t)$ also numerically, by evaluation the function in two close adjacent points for current solution of PWV +/- 0.05mm. Actually, the Jacobian

matrix J_{ik} is identical with our block-composed matrix in Fig. 39 – we can identify that the Jacobian elements represent exactly the expressions for sensitivity factors SF_W and $SF_B^{XX/YY}$ for the dW and dT^B variations, used in our older approaches derived without mathematical rigorosity, which backwards justifies their correctness.

Eq. (3.3.10) represents, in fact, a system of $3 \times nTimeSamples$ non-linear (because J is evaluated at the exact solution x_j , as well as $f_k(x_j)$ is) equations, so we have now the same number of equations as variables. Still it is difficult to be solved, so we do one more step forward and perform linearization of the system around some operational point x_j^0 . Using Taylor expansion to the first order we eventually get

$$J_{ik}^T(x_j^0) \cdot J_{kj}(x_j^0) \cdot (x_j - x_j^0) = J_{ik}^T(x_j^0) \cdot [b_k - f_k(x_j^0)] \quad (3.3.11)$$

This is the “master equation” for our approach. It is a linear approximation of Eq. (3.3.10) for displacements of the solution from the operation point, but it can be used iteratively such a way that the displacement

$$\Delta x_j^K \stackrel{\text{def}}{=} x_j^K - x_j^{0,K}$$

found as a solution of Eq. (11) in the iteration K , can be added to the original operation point x_j^0

$$x_j^{0,K+1} = x_j^{0,K} + \Delta x_j^K$$

for getting the new operation point in iteration $K+1$. This way the operation point converges to the exact solution of the non-linear system (3.3.10). When we are there, the RHS of Eq. (3.3.11) is exactly zero (because (3.3.10) is fulfilled), and so are then the displacements from the operation point ($x_j - x_j^0$) on the LHS (because the LHS matrix $J^T \cdot J$ is regular), so we stop moving in the x_j -space for further iterations (convergence completed). Essentially, this iterative approach is an implementation of general Newton-Raphson method for solution of non-linear systems specific for our problem.

In practice, Eq. (3.3.11) is a good approximation already if the operation point x_j^0 is close to the exact solution. We conveniently chose x_j^0 such a way that $\{w^0(t), T_{xx}^0(t), T_{yy}^0(t)\} = \{\langle w(t) \rangle, \langle T_{xx}^B(t) \rangle, \langle T_{yy}^B(t) \rangle\}$, i.e. replacing the actual values of the PWV and brightness temperatures for XX and YY pols in each specific time instant t by their scan-averaged means. Practically it means that we use iteration scheme

$$x_j^{0,K+1} = x_j^{0,K} + \langle \Delta x_j^K \rangle. \quad (3.3.12)$$

The displacements ($x_j - x_j^0$) at the LHS of Eq. (3.3.11) then represent fluctuations around means of our unknown quantities, exactly as we have had in our older approaches. Eq. (3.3.12) means that we iteratively correct the operation point by the averages of fluctuation, ensuring that the operation point at the end of iterations is really the time average, so that the sum of fluctuations around it is zero. When we work essentially with fluctuations again, what is then the practical difference to our older approach? Now we can always add the fluctuations (displacements found by Eq. 3.3.11) on top of the average (our operation point, which is found iteratively by Eq. 3.3.12) and get the full values of PWV and brightness temperature at each time instant!

Practically, this ‘uniform’ (time-independent) selection of the operation point assures that the matrix $J^T J$ of the linearized system (3.3.11) is composed of $nTimeSamples$ identical 3x3 blocks at the matrix diagonal, with all other LHS matrix elements being zero – it can be easily seen, when one makes right-sided matrix multiplication of the matrix in Fig. 39 with its own transposition (i.e. $J^T J$). Such a simple structure of the linearized system facilitates the solution a lot. Because of the block-diagonal nature of the LHS matrix, it can be, in fact, solved for each time instant independently, resembling the system in Eq. (3.1.5) (the differential/fluctuation approach), but now using also information contained in time-averaged (over a scan) values of received powers (not just their fluctuations), which, in turn, allows for the full PWV and full T^B time-series solutions.

The above described procedure has been implemented in the new routine `dWdB_Tbs()`. Now we shall present its sample results for antenna 43 (DV17), the full dataset is available online.

2.3.3.4 Early results

Fig. 41 shows the time series of the PWV at antenna DV17 – we almost identical fluctuations as in our differential approach, varying around mean PWV~2.7mm, which is too low: Median PWV for the observation as reported by `aU.getMedianPWV()` (it reads it from the SYSCAL table) is 4.86mm. So, there is clearly some issue.

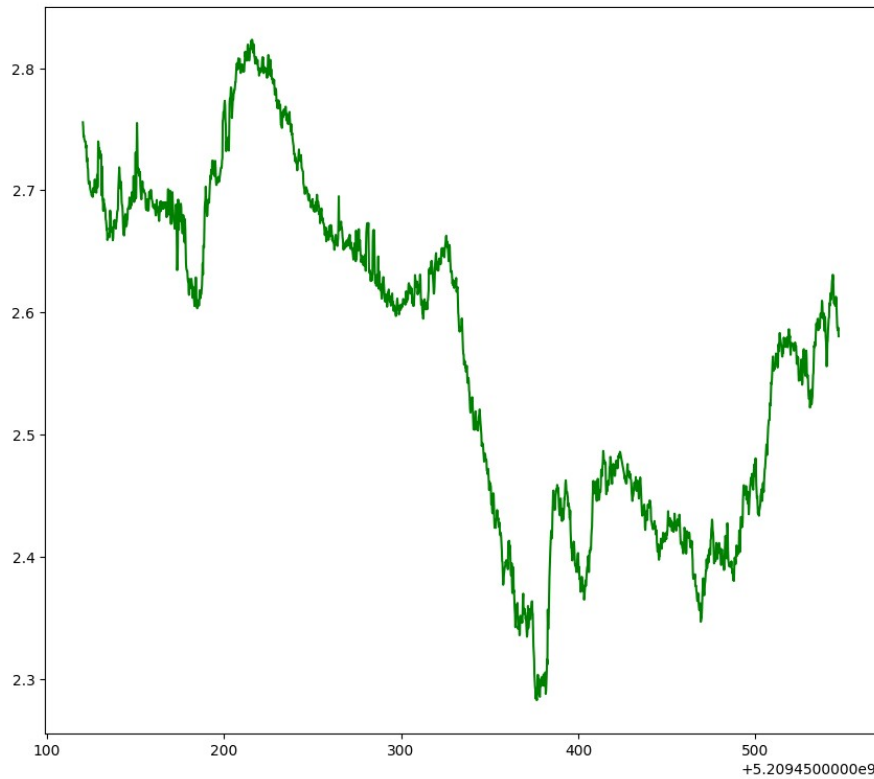


Figure 41: PWV in mm as a function of time (MJDsec) for Scan 9 at antenna id43 (DV17).

Complementary brightness temperatures for XX and YY pols are shown in Fig. 42. Again, the variations of the signal is almost identical to that found by our original differential approach, but the mean value ~ 4900 K lags far behind expectations: The nominal/reference brightness temperature for the solar disc center in Band 5 used in the TP imaging (White et al. 2017) is 6300 K. If the limb brightening (our target is very close to limb) is taken into account – see the next subsection “Improving the solar spectrum” – expected value is even ~ 7000 K. Hence, even in this respect the early result is not satisfactory.

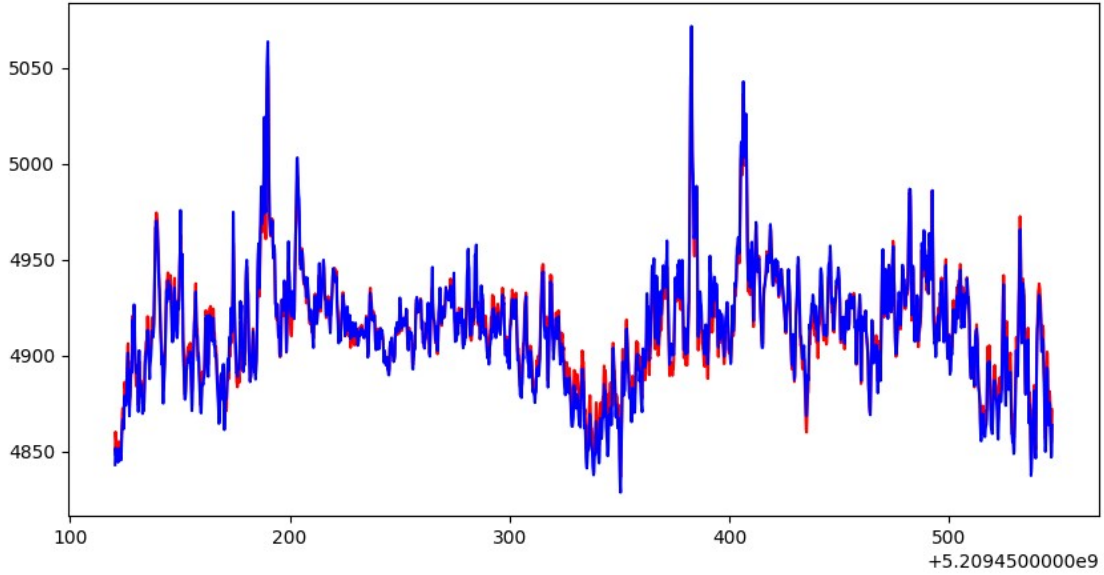


Figure 42: Solar-target brightness temperatures (in Kelvins) in XX (red) and YY (blue) polarizations as a function of time (as MJDsecs) in Scan no. 9.

In the previous Fig. 31 we have shown also analysis of the quality of the solution. We present similar analysis here, for our new approach. The results for antenna 43 (DV17) are displayed in Fig. 43. Again, like in Fig. 31 we can calculate the residual value for each equation in the system (3.3.8) by inserting its solution found by the iterative usage of linearized Eq. (3.3.11). These residuals are shown in the top row of panels in Fig. 43, color- and line-style coded for specific SPW: The colors code the base-bands (BB1 – BB4), line-style within the same color shows the specific SPW covering the BB (AC resolved = solid, AC integrated = dashed, SQLD = dotted, see also description in Fig. 31), left column shows XX and the right the YY polarizations. The difference between the model of the sky+src brightness temperature (LHS of Eq. 3.3.7) and the scan-averaged received temperature corrected by instrumental offset (time-average of the RHS of Eq. 3.3.7) are presented in the middle row. Finally, the bottom row shows the ‘net’ source contribution to the recorded-signal variations (the fluctuation of T^B multiplied by its sensitivity factor). As it can be seen at the first look (comparing the top rows in Fig. 31 with Fig. 43 here), we have now much bigger residuals then in the purely differential approach. The main difference

is in the mean values of the residuals for *spw:pol* combination (all color- and line-style coded curves are far from zero), while the fluctuations are basically of the same magnitude as in Fig. 31. It is clear that we did not manage to fulfill the set of Eq. (3.3.7) simultaneously for all *spw:pol* data channels, because it is hard to fit the mean (scan-averaged) sky+src brightness temperatures for all *spw:pol*s at once. Moreover, the line of the same color – i.e. the same BB – groups together, we see a clear trend of changing mean residual with frequency.

Summary: We have now, for the first time, the full PWV(t) time series and also the time series of apparent (including atmospheric scintillation effects) solar brightness temperature. On the other hand, the early results are quite disappointing:

- The PWV variations are basically the same as in the differential approach – this result is, indeed, robust. On the other hand, the mean value of $\sim 2.7\text{mm}$ is far low from expected – the SYSCAL reports PWV median to be 4.8mm for this MS.
- The same holds for time-averaged solar brightness temperature – variations are OK, but the expected mean value in Band 5 for the target at the solar limb is $\sim 7000\text{ K}$, while we have got $\sim 4900\text{ K}$.
- The residual errors (specifically their time-averages) are now much higher than for our older treatment via signal-fluctuations only.
- Moreover, the residuals have been found to have a clear trend with frequency – see how the lines in Fig. 43. are ordered by colors: red, black, blue, green – the colors code the BBs. We address this finding immediately in the next subsection “Improving the solar spectrum”.

We spend a lot of effort in identification of the source of this discrepancy between model and data and our way towards improving the model/data fit is described in the following subsections.

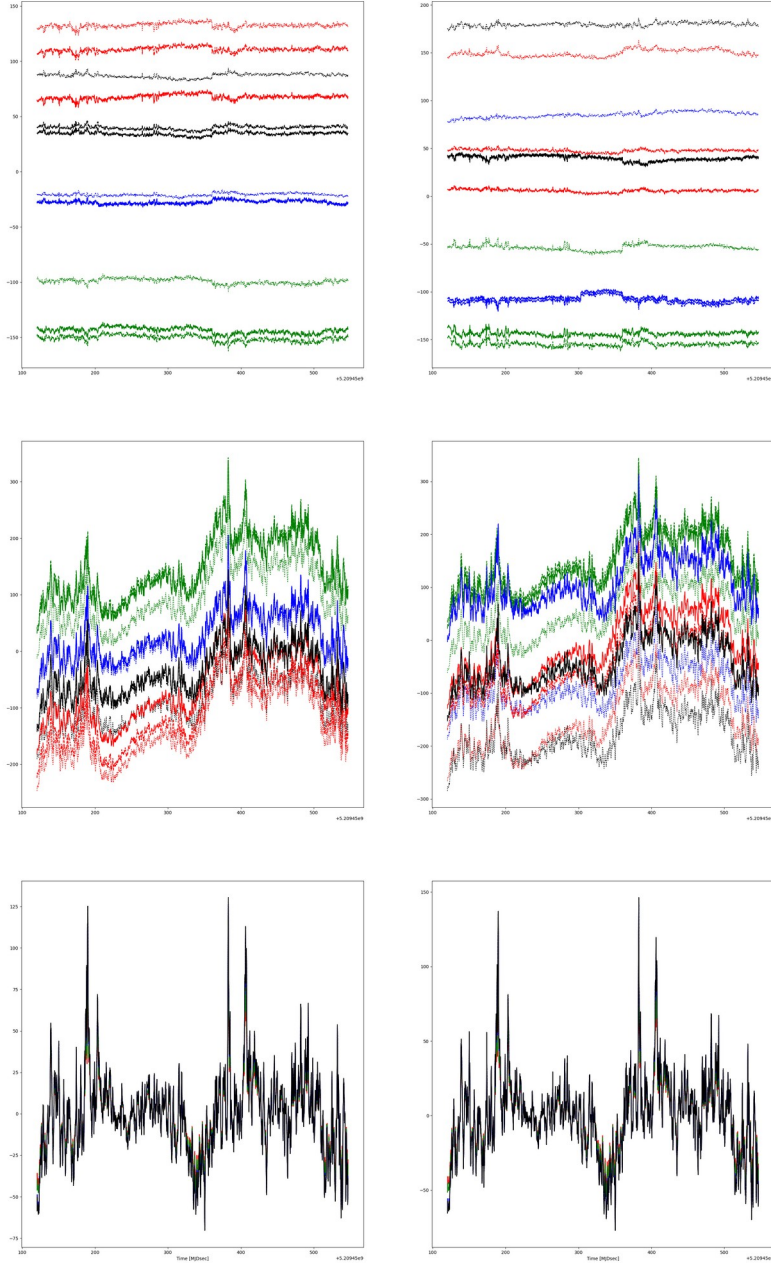


Figure 43: Residuals of Eq. (3.3.7) as temperatures in K (top row), sky+src brightness model minus time-averaged observations (middle, in Kelvins) and source brightness contribution (K) to the total sky brightness variations as extracted by solving the system in Eq. (3.3.7). Left and right columns were obtained from XX and YY pols in science SPWs. Can be directly compared with Fig. 31

2.3.3.5 Improving the solar spectrum

The solar spectrum (scaled by the atmospheric transmissivity) forms the sensitivity factor for brightness changes – see the 3rd equation in (3.3.2). Insofar we have adopted a naive ‘zero-order’ approach of the Sun as a black-body and thus the intensity increases with frequency-squared as prescribed by Rayleigh-Jeans law (see Eq. 3.1.3). However, the results of our fitting procedure show that it is most likely wrong, as we can see systematic trend in residual errors for different frequency domains – while the low-frequency SPW 5 (and its integrated and SQLD companions 6 and 0) seem to have ‘under-rated’ temperatures, the opposite side of Band 5 frequencies (SPWs 11,12, and 3) are over-rated. We start playing with the spectral index s in the last Eq. of (3.3.2), which is nominally set to 2 for the R-J law, and succeeded to improve the results with negative s around -0.5 . However, such an *ad hoc* change of the spectral law is highly unsatisfactory, so we started research in literature. We have realized that situation is, in fact, much more complicated than the black-body assumption and have eventually found semi-empiric formula for quiet-Sun brightness temperature, which fits observations, in Alissandrakis et al. (2020). The brightness temperature, in this approach, is a function of frequency and μ , which is the direction cosine of the position angle – the angle between the normal to the solar surface in the location of our target and the line of sight. Hence the object in the solar disc center has this angle zero and $\mu=1.0$, while close to the solar limb angle grows to 90 degs and its cosine approaches zero. The exact expression for $T^B(f, \mu)$ adapted from Alissandrakis et al. (2020) reads:

$$T^B(f, \mu) = T^B(f_o, 1) + a_2 \ln(\mu) + 2a_2 \ln\left(\frac{f}{f_o}\right) \quad (3.3.13)$$

where the semi-empirical constant a_2 has a value -580 K and the reference brightness temperature $T^B(f_o, 1)$ for $f_o=198$ GHz at the solar disc center ($\mu=1$) is 6300 K (White et al. 2017). Dependence on the position angle means that we have to read ephemeris file in order to locate the target w.r.t. the solar disc center.

Results: Clear improvement in all respects

- Mean PWV for antenna DV17 is now 4.78 mm, i.e. much closer to the expected value, the mean solar brightness temperature increased to 5900 K, still being lower than expectations: Eq. (3.3.13) gives for out close-to-limb target with low μ estimated value 6990 K.
- The residual-means are lower – they diminished from amplitude of 150 K to 60 K, and rather random – the frequency trend disappeared – see Fig. 44.

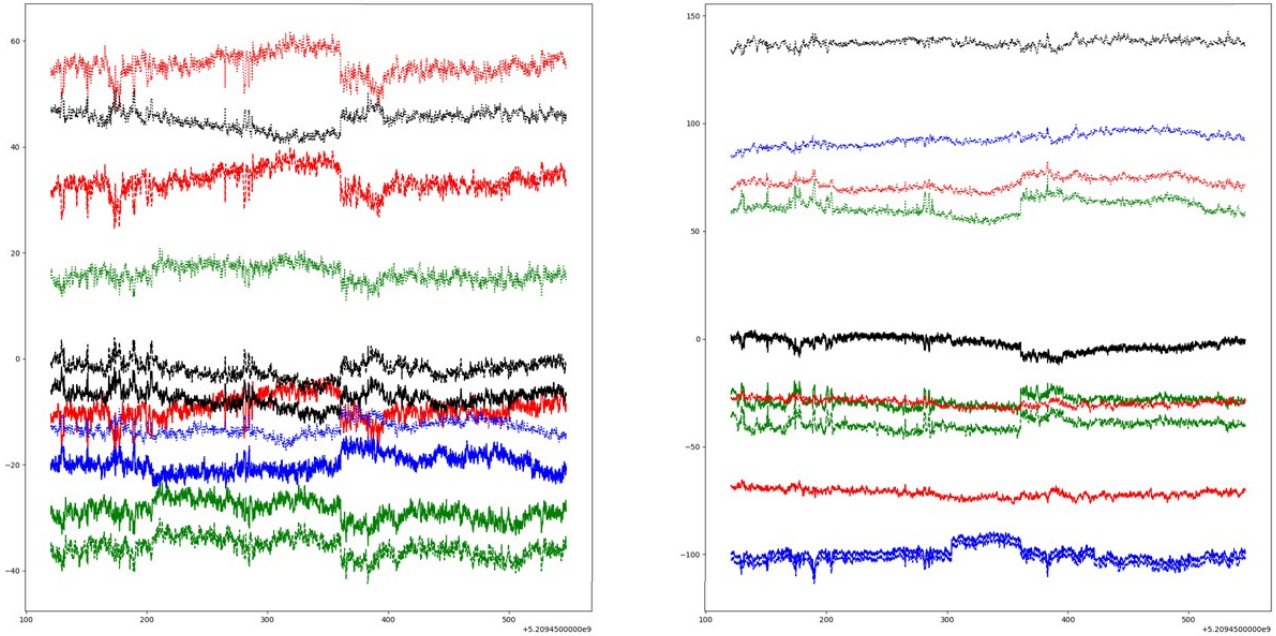


Figure 44: Residuals as in the top-row of Fig. 43. Clearly they diminished and do not show trend with the frequency.

2.3.3.6 Validating the linearization approach

In our routine $dWdB_Tbs()$ we move the operation point, around which the linearization is performed, in each iteration not exactly to the improved solution but, for practical reasons, to its time averages (see Eq. 3.3.12). This allows us to keep the structure of linearized equations simple and solution cost-effective. Question arises, what price in terms of lost accuracy we pay for that. Can this simplification cause the increased residuals?

In order to answer the question, we have written another routine $dWdB_Tbs_NL()$, which uses ‘brutal force’ for solving the nonlinear (NL) system (3.3.10) directly and we compared the results with the linearized iterative version. The non-linear solution is CPU demanding so we can afford performing it only for few antennas, just as a verification. The difference between the linear (Eq. 3.3.11) and full NL (Eq. 3.3.7) solution are shown in Figs. 44 and 45 for PWV and brightness temperature, respectively.

As it can be seen, the error in PWV is of the order of micro-meters, so absolutely negligible, while in the case of brightness temperature it reaches low units of Kelvins, so, again, less than 0.1% of the nominal value. The magnitude of error correlates with the absolute value of PWV/T^B fluctuations as the exact solution in those points is than more distant from our operation point (in the linearized system) selected conventionally as the time-average of the exact solutions over a scan.

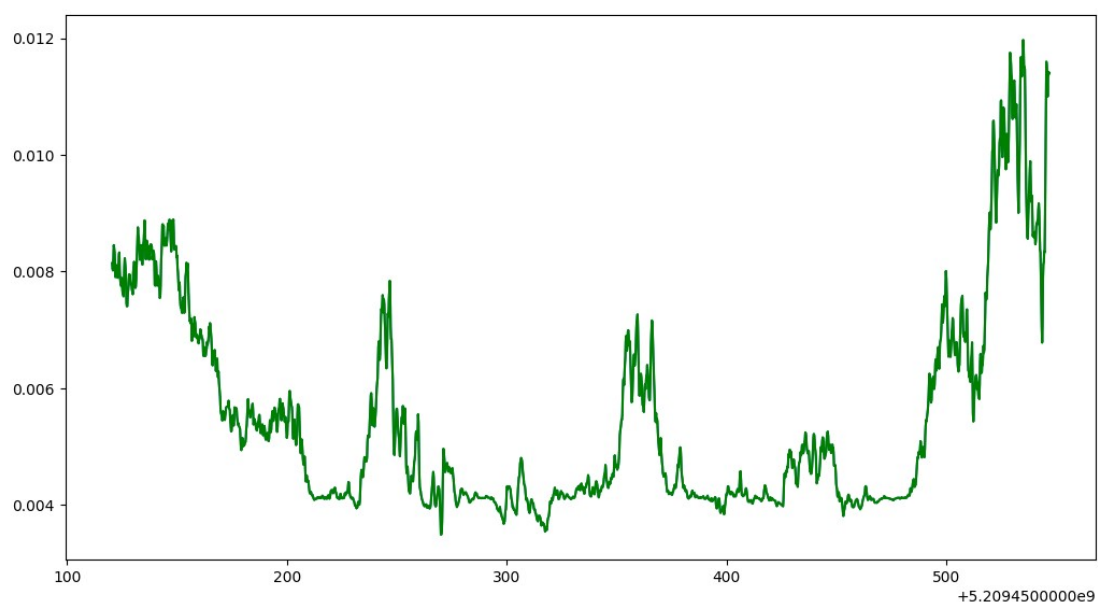


Figure 45: Difference of solutions for PWV (mm) as a function of time (MJDSec) between linearized equations (3.3.11) and full non-linear solution (3.3.10) at antenna id23.

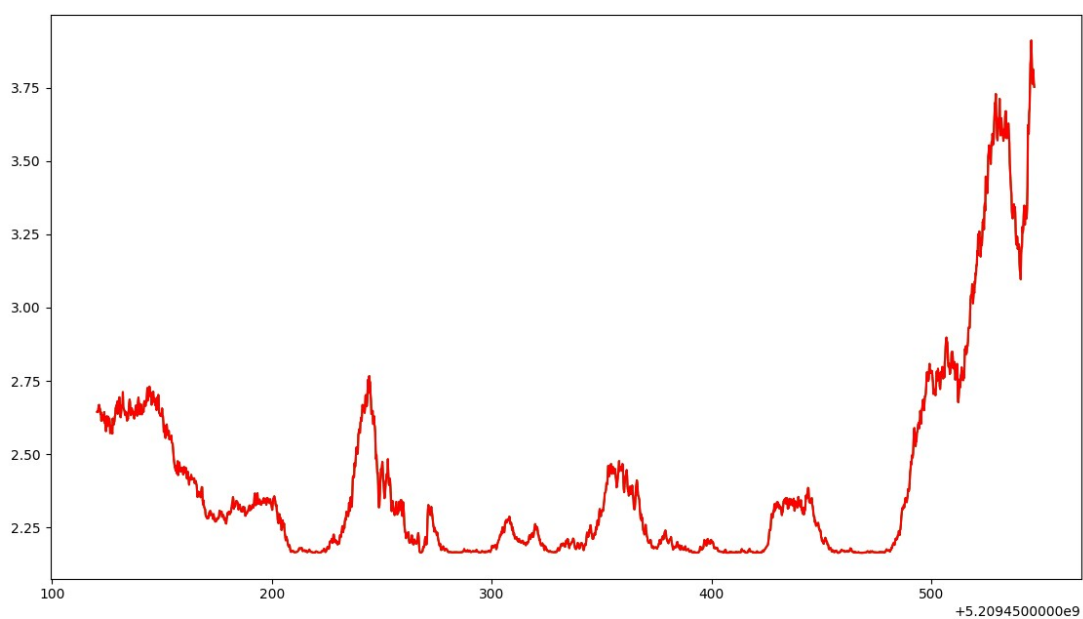


Figure 46: Like in Fig. 45 above, now for the brightness temperature in Kelvins.

2.3.3.7 Introducing the antenna efficiencies

The so far implemented model-upgrades improved the situation what concerns all respects – diminished residual errors of our over-determined system with no systematic trend (i.e. better fit), average PWV is closer to that reported by SYSCAL PWV median. Still the brightness temperatures seem to be underestimated w.r.t. the semi-empirical (and observation-validated) model in Alissandrakis et al. (2020).

We have realized that we have insofar ignored the antenna efficiencies – rear- and forward scattering and spill-over, traditionally designated as η_l and η_F . Earlier, we have somehow expected that they might be absorbed in the other instrumental parameters, namely into gain. But it is not true in full extent, because, in addition to diminishing the target and atmospheric signals (what lower gains does, too), they also bring the scattered radiation from the ground and skies to the antenna.

Inclusion of efficiencies changes the original expression for external temperature in Eq (3.3.5) to a new form:

$$\begin{aligned} T_{ext} = & \eta_l \cdot \eta_F \cdot T_B^{Sun} \cdot \int tr(w(t), f) \cdot S(f) \cdot g(f) \frac{df}{BW} \\ & + (1 - \eta_F + \eta_l) \cdot \int T_{sky}(w(t), f) \cdot g(f) \frac{df}{BW} \\ & + (1 - \eta_l) \cdot T_{amb} \end{aligned} \quad (3.3.14)$$

T_{amb} is the ambient temperature, here considered also as the ground temperature for the rear spillover contribution. The three lines in Eq. (3.3.14) mean, respectively, the solar contribution diminished by imperfect transmissivity through the atmosphere and by non-ideal antenna efficiencies, contribution from the skies, both arriving directly with the ray from the source and that from the forward spillover, and, eventually, the rear spillover contribution from the ground.

Eq. (3.3.14) can be inserted instead of (3.3.5) into the general relation (3.3.3) and we eventually get modified set (3.3.7). The ‘normal’ equations for iterative linearization (3.3.11) keep their form, just the Jacobian elements – the sensitivity factors, get modified by presence of the efficiencies. The solution of the system otherwise stays methodologically unchanged, the results are, of course different.

Results:

- Definitely improved resulting brightness temperature ~6900 K, which is now much closer to the expected value of 6990 K.
- PWV = 4.87 is now fully in line with the SYSCAL estimation – see Fig. 47
- What still makes worries, are the residuals – they are significantly higher than those for the differential (fluctuation-only) approach. Namely, the mean value (over time) is high, at the level of 50 – 70 K, while the fluctuations are small (up to 10K) and comparable with the previous differential approach.

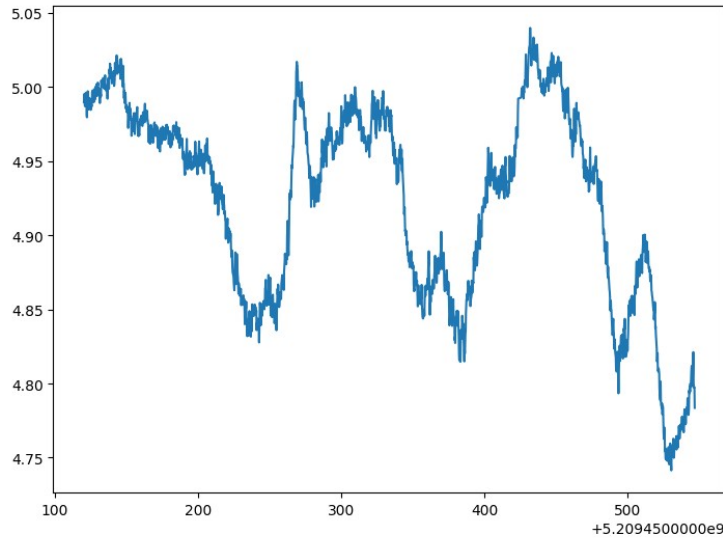


Figure 47: PWV in mm as a function of time (MJDsecs) for antenna id 21 (DA56). The model-upgrades including improved solar spectrum and antenna efficiencies have been applied, which has led to much better result than the ‘naive’ approach displayed in Fig. 41.

2.3.3.8 Improving antenna gains

Large residual mean-values seen in the top row of Fig. 43 and, somewhat improved but still quite high in Fig. 44, indicate that we have bad fit of our model on the LHS side to the measured temperatures at the RHS. Since we have focused on improving the model values in the previous paragraphs (and many improvements have been actually achieved) and we, perhaps, exhausted the possibilities for further upgrade of the model, we now focus on the measurement side. Moreover, the residuals now (after improving the model solar spectrum and including the antenna efficiencies) seem to be quite random with no clear trend related to the SPW frequency domain (the base-band BB), their values non-systematically differ from antenna to antenna – this also indicates problem rather in measurement than in the model.

Actually, the RHS temperatures still do not represent the pure measurement data because in Eq. (3.3.7) we are comparing the model and recorded temperatures, where the latter are only derived from the actual primary data (voltages/powers) via instrumental characteristics like gains G , receiver temperatures T_{RX} , and – in case of SQLD data – also the power offsets P_o . These instrumental parameters are pre-determined by the ACD (dual-load) calibration. The ACD calibration equations are solved by our routine *acdCaltabs()*.

In case of solar observations we have two levels of gain: One for the ‘standard non-solar’ (i.e. low) attenuation, which is derived from the ACD calibration solution, and second, with high attenuation, used during the solar scan. The scaling/attenuation factor AF between those two is derived by two

atmosphere scans: One with low (the ‘la’ index) and one with high (i.e., standard for the Sun) attenuation (see Shimojo et al. 2017)

$$AF \stackrel{\text{def}}{=} \frac{G}{G_{la}} = \frac{P_{sky} - P_o}{P_{off} - P_o}, \quad (3.3.15)$$

where P_{off} data are taken in the solar off-source sub-scan with solar (increased attenuation). Technically, in general, this relation should be used for calculating the gain G solely in the SQLD spws, because the P_{off} signal is typically too low to be detected sufficiently accurately in the AC SPWs. In Band 5, however, the solar attenuation level is not so high and we are able to measure the P_{off} signals in AC SPWs, too.

Nevertheless, uncertainty around this method (using P_{off} scans in AC SPWs) has led us to searching for further improvement of determination of the scaling factor (3.3.15) for antenna gains between the low and high attenuation levels. The RHS values are typically $\sim 3000\text{K} - 4500\text{K}$ (indeed – the absorption in low-frequency SPW 5 at $PWV \sim 5\text{mm}$ – is more than 50%!). So, just 5% error in gain determination easily produces miss-fit of $150\text{K} - 200\text{K}$, therefore it is important to estimate the gain as much accurately as possible.

The idea is following:

- We calculate the gain-to-gain scaling by Eq. (3.3.15) solely in the SQLD spws, in line with the approved procedure in solar ObsMode (see Shimojo et al. 2017).
- We shall transfer the scaling from the SQLD to the AC spws, that occupy the same frequency domain / base-band (e.g., from SPW 0 to SPWs 5 and 6).
- For the channel-resolved SPWs we assume that the scaling factor to gain is applied homogeneously to all frequency channel, i.e. that the spectral gain normalized to 1.0 does not change, when higher attenuation is switched on/off.

The transfer between SQLD and co-located AC spws is based on the idea that for the SPWs occupying the same frequency domain we have the same contribution in terms of radiation intensity (i.e. temperature) both from the Sun and from the atmosphere. Hence, the difference in received signal (powers) is just due to different instrumental characteristics (gain and generalized instrument-related power offset) in different SPWs. Hence, if we insert Eq. (3.3.14) for the external temperature together with definition of the instrumental temperature $T_{inst} = T_{RX} + P_o/G$ into the general expression (3.3.3), after frequency averaging we get the following relation for power recorded in the SPW with index S

$$\overline{P_S} = \overline{G_S} \left[\overline{T_{RX}^S(f)} \cdot \overline{g^S(f)} + \eta_l \eta_F \overline{tr^S} \cdot \overline{T_B^S} \cdot \overline{g^S(f)} \overline{tn(f)} \overline{S(f)} + (1 - \eta_F + \eta_L) \cdot \overline{g^S(f)} \overline{T_{sky}(f)} + (1 - \eta_l) \overline{T_{amb}} \right] + P_o^S \quad (3.3.15a)$$

Here the over-bar means frequency averaging over the SPW and $g^S(f)$ is the spectral gain normalized to unity, defined already in Eq. (3.3.5), i.e. $g^S(f) = G^S(f)/\overline{G_S}$ and, similarly, $S^S(f)$ is the normalized solar spectrum in the SPW S , defined above (Eq. 3.3.5), too. $\overline{T_B^S}$ is the frequency-averaged solar brightness temperature in the SPW S and $tr^S(f)$ is atmospheric transmissivity in the same window. We define also

its normalized (to unity) counterpart as $tn(f)=tr(f)/\bar{tr}$, analogically to spectral normalized gain and normalized solar spectrum. In case of SPWs that are not channel-resolved, all these normalized quantities are 1.0 by definition, and the frequency-averaged quantities in the relation (3.3.15a) above (e.g., gain and receiver temperature) are, obviously, represented by their single-channel values as directly obtained from the MS data. The atmospheric + CMB brightness $T_{sky}(f)$ is defined by Eq. (3.3.6).

From Eq. (3.3.15a), applied onto two SPWs – the AC and SQLD in the same base-band (what assures the same source and atmospheric contributions) – one can express relation between the received powers in those two SPWs. The relation should be linear, in the form

$$P_{SQLD} = K \cdot P_{AC} + Q$$

where we have skipped the over-bars, as it is clear that we work with frequency-averaged (per-SPW) quantities, and the parameters K and Q can be expressed from Eq. (3.3.15a) as:

$$K = \frac{G_{SQLD}}{G_{AC}} \cdot SWF$$

$$Q = G_{SQLD} \cdot \left[T_{RX}^{SQLD} - SWF \cdot \overline{T_{RX}^{AC}(f) g^{AC}(f)} + (1 - \eta_F + \eta_l) \cdot [\overline{T_{sky}} - SWF \cdot \overline{g^{AC} T_{sky}}] + (1 - \eta_l) T_{amb} (1 - SWF) \right] + P_o^{SQLD}$$

where the spectral-weight factor SWF is the abbreviation for

$$SWF = \frac{1}{\overline{g^{AC}(f) tn^{AC}(f) S^{AC}(f)}}$$

The parameters K and Q of the linear relation between P_{AC} and P_{SQLD} can be determined “experimentally”, by means of linear regression (correlation) between actual recorded values of P_{AC} and P_{SQLD} in the target scan – see Fig. 48. This way we may eventually express the gain in the AC SPW in relation to the gain (with solar high-attenuation) in the corresponding SQLD spw as

$$G_{AC} = \frac{G_{SQLD}}{K} \cdot SWF \quad (3.3.15b)$$

where the parameter K has been obtained as a regression coefficient as described above. For the AC channel-resolved SPWs we assume, as adduced above, that the low/high attenuation switch affects just the gain scaling, not its (normalized) spectral profile $g^{AC}(f)$. Hence, the per-channel high-attenuation gain in the AC-resolved SPWs, transferred from the same-baseband SQLD SPW reads:

$$G_{AC}(f) = \frac{G_{SQLD}}{K} \cdot SWF \cdot g^{AC}(f) \quad (3.3.15c)$$

To sum up the methodology:

1. We calculate the high-attenuation (solar-scan) gains according to Eq. (3.3.15) solely for SQLD spws.
2. Using linear correlation between the powers P_{AC} and P_{SQLD} , recorded during the solar scan in the AC and SQLD spws in the same BB, we find regression coefficient K .

3. Using Eqs. (3.3.15b) or (3.3.15c) for AC-integrated or AC-resolved SPWs, respectively, we transfer the high-attenuation gains from SQLD to the respective AC spws.

The entire procedure is implemented in the improved ACD calibration routine *acdCaltabs2()*.

Results: The situation got further improved – PWV slightly raised to 4.83mm, solar brightness temperature remained 6900 K/6800 K for XX/YY polarizations, the residuals diminished by 30 – 70% depending on antenna. Still they remain high – their means are much higher than their fluctuation part, which signalizes imperfect fitting of the model and observed data – see Fig. 49.

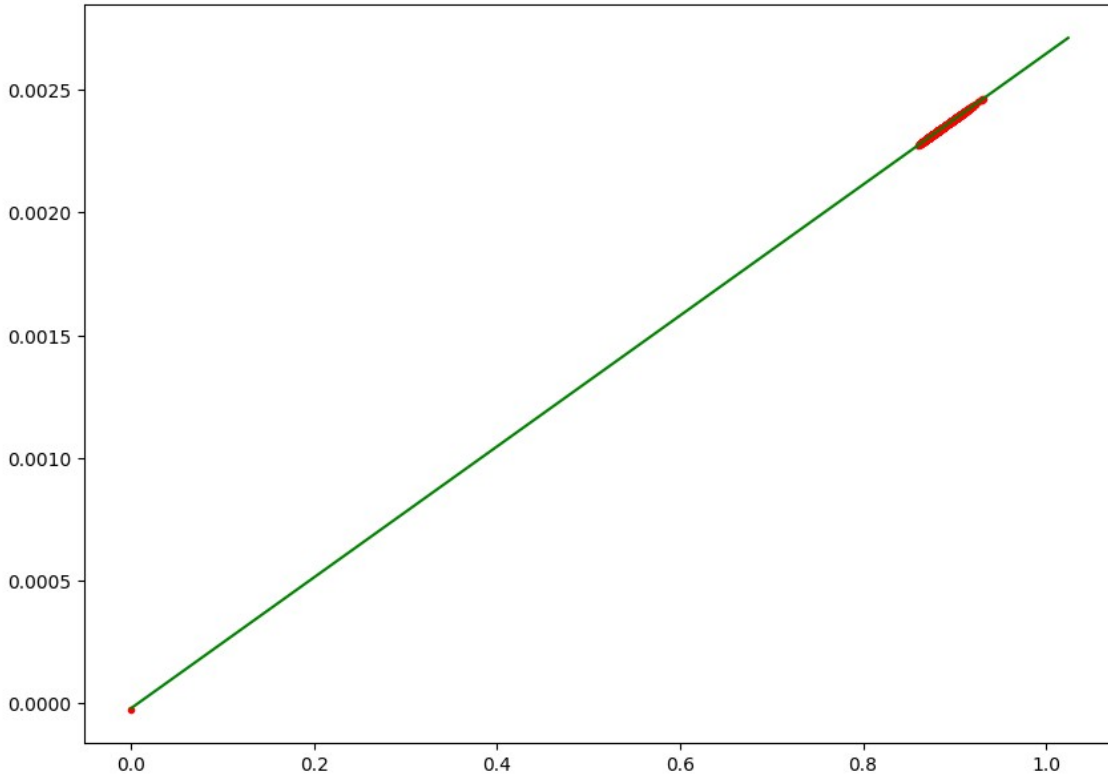


Figure 48: Cross-correlation between recorded powers in the AC SPW 5 (channel-averaged) and SQLD SPW 0 in the same BB. The red point close to zero corresponds to the point $[0;Q]$, where Q is the regression offset, the higher values (around 0.9 for P^{AC}) represent the scatter-plot between P^{AC} and P^{SQLD} for the solar on-source sub-scans.

2.3.3.9 High- vs. low-attenuation gain scaling: Issue with time dependence

In the previous paragraph we have developed a method on how to scale the gain and the instrumental power-offset from SQLD spws to the AC spws, in line with the approved *Solar ObsMode* strategy.

Note, that the insofar used ‘official’ solar calibration implemented as `sol_ampcal2()` routine in `sun_reductionUtils.py` in fact works with antenna temperature T_{ant} (see Shimojo et al. 2017; White et al. 2017) and not directly with gains. Nevertheless, the principle and formulae used for calculations are the same, albeit in our method `acdCaltabs2()` we, moreover, in addition to the ACD ambient and hot loads (used in `sol_ampcal2()`), make also use of correlations for the time-varying solar signal between AC and SQLD spws, what improves statistics.

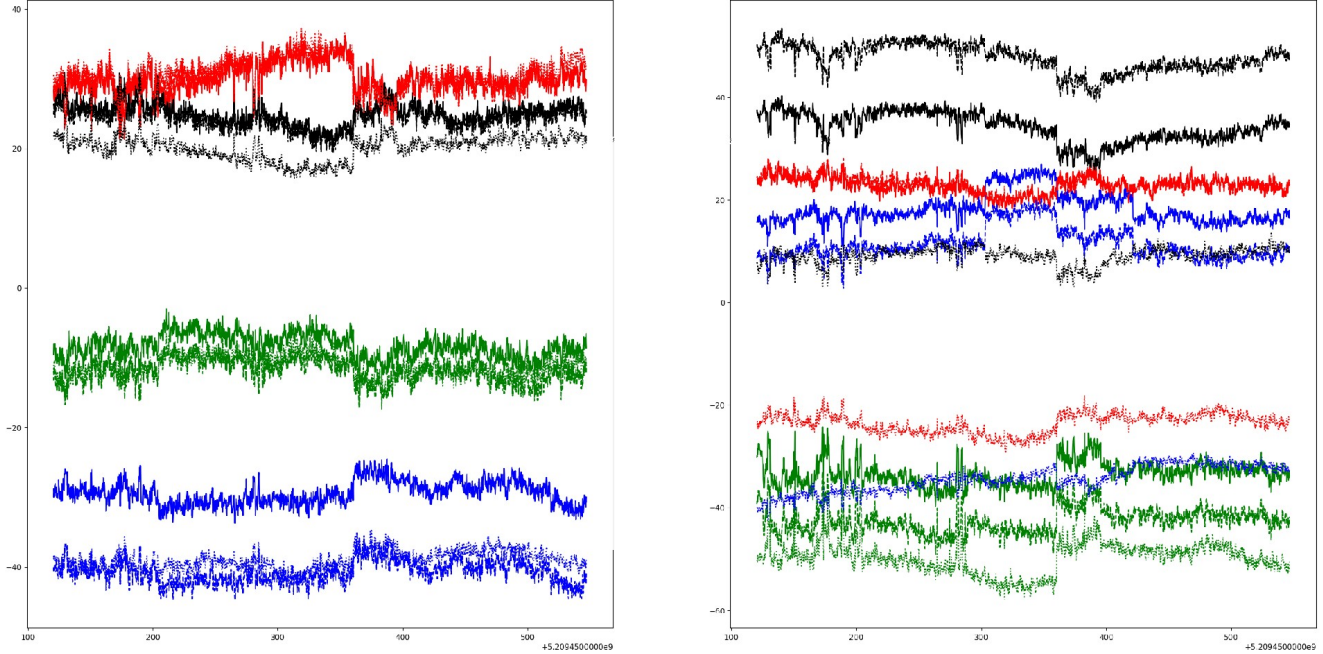


Figure 49: Residual errors like in Fig. 44 after the “solar” (i.e. high-attenuation state) gain has been transferred to the AC SPW from their respective SQLD counterparts in the same BB via Eqs. (3.3.15b) or (3.3.15c). The mean values of fluctuation residuals are further diminished. The same colors, i.e. the same Bbs, are now grouped together, but in this case the residual in each BB does not systematically increase with frequency, the order is random. Clearly, the error in the BB is determined by accuracy of the solar gain in the leading SQLD SPW.

Now, the question is, how good is the scaling in line with Eq. (3.3.15) for the SQLD spw itself, which is used as a gauge. The issue is related to time-dependence of the P_{off} values used in the relation (3.3.15). We have it determined from two (off-source) sub-scans of the target scan – one very first on the beginning of the target scan and another one at the very end. The value of P_{off} clearly drops in our MS between the leading and trailing sub-scan. There are basically three ‘attitudes’ to this fact:

- One interpretation may say that it is because the antenna-gain itself is time dependent. In such a case, it has sense to make linear time interpolation for the scaling factor in Eq. (3.3.15) between the leading and trailing sub-scans and apply the time-dependent high-attenuation (solar) gain to the solar data. This was our original (until April 2026) implementation in `acdCaltabs()`.

- Second approach, silently applied in the standard solar calibration in *sol_ampcal2()* assumes that the differences in P_{off} between the leading and trailing sub-scans are more-or-less random, so it takes time average from all integrations in both sub-scans together, creating a single value of P_{off} . So, in fact, we are taking the average of P_{off} values from leading and trailing sub-scans.
- Eventually, and we are convinced that it is the right way, we may interpret the decrease of P_{off} as a consequence of decreasing PWV (the decrease is clearly indicated by the PWV curve from SYSCAL table), and consequently decreasing emissivity of the atmosphere. With this interpretation, it makes sense to collect the P_{off} data from the first sub-scan only (ignoring the trailing one), because the leading sub-scan follows immediately after the ATM scan, so the PWV, hopefully, did not change too much.

We compared the approaches no. 1 (time-interpolation between leading/trailing sub-scan) and 3 (leading sub-scan only) and combined them with two methods of determining the scaling factor (3.3.15): (i) each SPW, including the ACs, collects P_{off} values directly from its own data (using the original *acdCaltabs()*) and, (ii) we transfer the scaling factor (3.3.15) to ACs from the SQLDs using Eqs. (3.3.15b) or (3.3.15c) (*acdCaltabs2()*). Results are shown in Fig. 50. We present the residual errors as we did in the top row of the Fig. 43 and in Figs. 44 and 49. Here, the top row shows the residuals for P_{off} collected in each SPW and time interpolation between leading/trailing sub-scans, the middle row displays approach with SQLD-only based (and transferred to AC spws) gains, still with the time-interpolation, and the bottom row displays the gains calculated from SQLD-only approach, with P_{off} collected in the leading sub-scan only. Namely the top row with high residuals shows sort of ‘divergence’ of the residuals, indicating that the time interpolation for the gain actually worsens the situation – so the P_{off} drop in time is not a consequence of the gain change. To some extent this effect is visible also in the middle row, which has, however, largely diminished residuals thanks to transfer of the high/low-attenuation gain-scaling (3.3.15) from SQLDs to ACs. Eventually, the bottom row has no time trend and as a whole the lowest residuals, indicating that the first sub-scan is the best choice for the P_{off} measurement and subsequent use in the gain scaling according to (3.3.15).

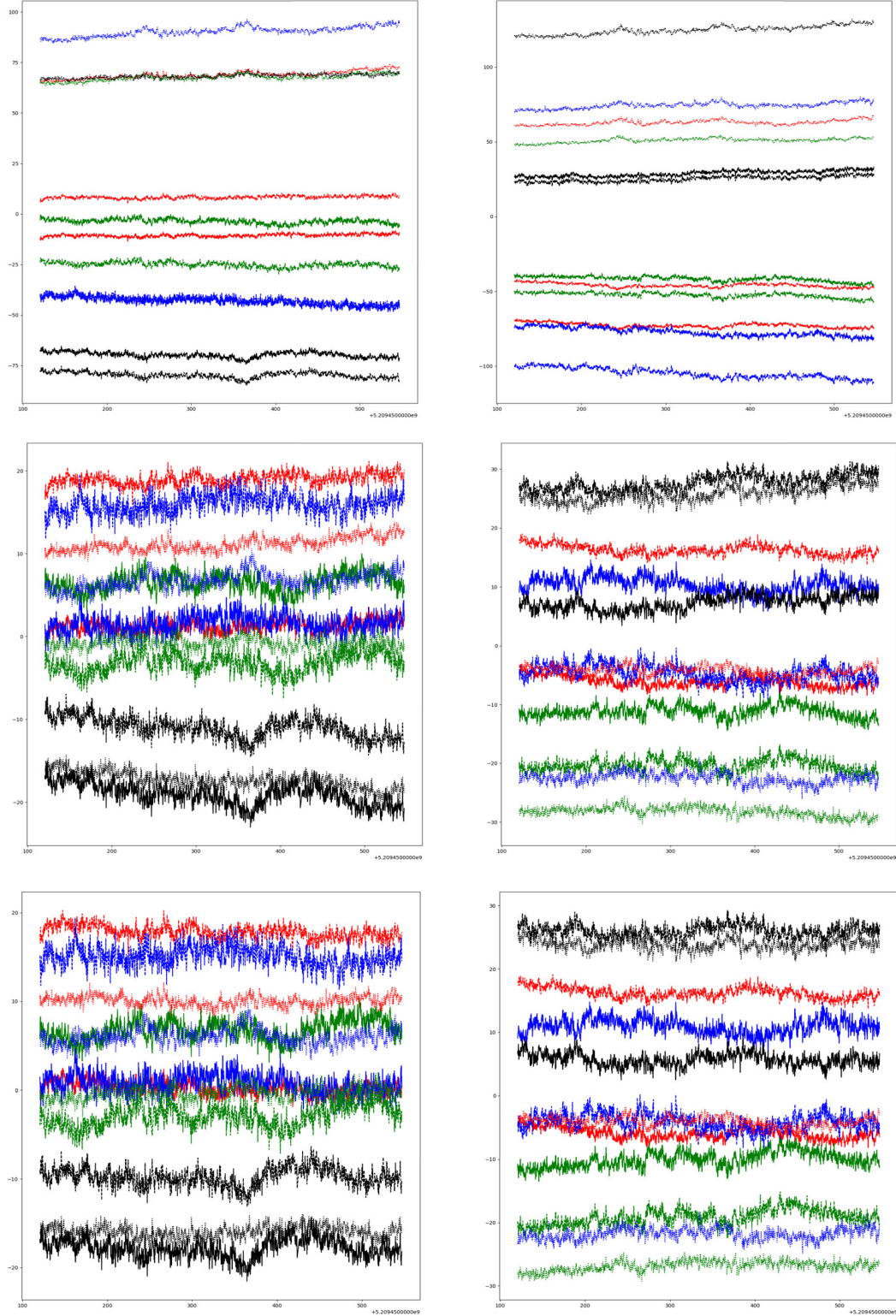


Figure 50: Comparison of residual errors (see Fig. 43 for description) for various approaches to solar (high-attenuation) gain. Top: P_{off} measured in all SPWs (incl. the ACs), leading and trailing off-subscans linearly interpolated. Middle: P_{off} only in SQLDs, transfer to ACs using Eq. GGG, linear interpolation of leading and trailing subscans. Bottom: With transfer to ACs, using leading sub-scan only for P_{off} .

Results: We see time variation (a drop) of the P_{off} between the leading and trailing off-source sub-scans of the target scan in our MS. This fact rises a question, which sub-scan should be used for P_{off} determination and subsequent calculation of the gain-scaling between the low- and high-attenuation regimes according to Eq. (3.3.15). We can use either the average of both (which the standard calibration procedure *sol_ampcal2()* does), or linear time interpolation between both sub-scans (what we have done until April 2026), or, take just the first sub-scan that is immediately following the ATM scan, from which the P_{sky} value in Eq. (3.3.15) is derived. Having the best results with the third option, we are using solely this setup in the following. Anyway, it looks like more general problem, at least for all the Band 5 solar data (in other ALMA Bands the emissivity is not so strongly dependent o the PWV). Also in view of the next paragraph, where we allow for limited corrections of gains, it would worth considering to measure the gain-scaling factor between the low (standard-sky) and high (solar) attenuation levels not between the two off-source sky observations (as the sky brightness may change due to changing PWV), but rather during the ACD calibration: Having the hot and ambient sub-scans once with low and once with increased attenuation levels. We discuss this further in the final Chapter 4, where recommendations for improvements of the *ALMA Solar ObsMode* are summarized.

2.3.3.10 Instrumental parameters as a part of solution

From the above results it is clear that we are making progress towards lower residuals (i.e. better fit) when trying to increase accuracy in gain estimation. Insofar we have considered the instrumental parameters: gain G^{SP} and generalized instrumental power-offset $P_{inst}^{SP} \equiv G^{SP} \cdot T_{RX}^{SP} + P_o^{SP}$ for each *SPW:POL* (as indicated by the superscript SP) data channel as a set of constants, pre-determined apriori by the ACD calibration (implemented in *acdCaltabs()* routine or its improved version *acdCaltabs2()* in *sol_wvrgcal.py* collection). However, in fact, we may think about parameters G^{SP} and P_{inst}^{SP} to be the variables of the system as well and further improve the results. As argued above, we have $nTimeSamples \times nPols \times nSPWs$ equations in the set (3.3.8), while, adding the instrumental powers and gains to the set of variables (i.e., appending them to the already calculated ‘natural’ variables PWV(t), $T_{xx}^b(t)$, and $T_{yy}^b(t)$), we got $(3 \times nTimeSamples + 2 \times nPols \times nSPWs)$ total variables, which is still much less than the number of equations (having the quite high number ~ 2300 of time samples/integrations). Hence, the system of equations is still (over)determined.

Formally, we extend our unknown vector of variables by the instrumental parameters, so it can be written as $x_j = \{w(t), T_{xx}^B(t), T_{yy}^B(t), P_{inst}^{SP}, G^{SP}\}$, where the indices t , S , and P take ranges $t=1..nSamples$, $S=\{5,7,9,11,6,8,10,12,0,1,2,3\}$, and $P=\{XX,YY\}$. Inclusion of G and P_{inst} into the vector of unknown variables requires that we have to return back to measured powers, not transformed to temperatures by the apriori preset gain scaling. This makes situation cleaner – the RHS now contains just directly measured quantities, while instrumental parameters G and P_{inst} become part of the model.

The master equation used in our further calculations for power $P_{rec}^{SP}(t)$ directly recorded in SPW S and polarization P in time instant t reads as follows:

$$P_{rec}^{SP}(t) = P_{inst}^{SP} + G^{SP} \cdot T_{ext}(t) \quad (3.3.16)$$

The instrumental power offset has been already defined above as $P_{inst}=G.T_{RX}+P_o$, where P_o is non-zero only for SQLD spws. T_{ext} characterizes radiation intensity from all external contributions outside of the receiver, i.e. the observed target (diminished by absorption and antenna inefficiencies), intrinsic radiation of the terrestrial atmosphere, and spillovers from ground (rear-spillover + antenna resistivity at ambient temperature) and skies (at lower T_{sky} temperature).

If we insert (3.3.14) into (3.3.16), we get, again, a very large set of equations, because the combined system must hold for every *spw:pol* data channel and for every time instant (integration):

$$\begin{aligned}
P_{inst}^{SP} + G^{SP} \eta_l \cdot \eta_F \cdot T_B^P(t) \cdot \int tr^S(w(t), f) \cdot S^S(f) \cdot g^{SP}(f) \frac{df}{BW} \\
+ G^{SP} (1 - \eta_F + \eta_l) \cdot \int T_{sky}^S(w(t), f) \cdot g^{SP}(f) \frac{df}{BW} \\
+ G^{SP} (1 - \eta_l) \cdot T_{amb} = P_{rec}^{SP}(t)
\end{aligned} \tag{3.3.17}$$

Eq. (3.3.17) again takes the general form of Eq. (3.3.8) and it is solved by finding minimum of the respective quadratic form (the least-square of residuals in Eq. 3.3.9), which leads to the non-linear system (3.3.10). As a consequence of widening of the unknown-variables vector by the instrumental part, we got also a bit wider Jacobian matrix of the system (by $2 \times nPol \times nSPWs = 48$ new columns). The system (3.3.11) in its ‘narrowed’ form $J_{kj} \cdot (x_j - x_j^o) = b_k - f_k(x_j^o)$, extended by the instrumental variables, is schematically shown in Fig. 51. The RHS vector $C_k \equiv b_k - f_k(x_j^o)$ represents the difference between measured quantities and the model value in its operation point. The next Fig. 52 shows the ‘normal’ equations (3.3.11), i.e. after multiplication by J^T , for our extended system. In order not to repeat all the matrix elements, we work with *N-blocks* and *G-blocks*, which are the sub-Jacobians for the ‘natural’ (i.e. $w(t)$ and $T_B^{XX/YY}(t)$) and the ‘gain/instrumental’ (i.e., G^{SP} , P_{inst}^{SP}) subset of unknown variables. If we define our operation point x_j^o , around which the linearization is performed, again by the scan-averaged ‘natural’ quantities complemented by the current-iteration ‘gain/instrumental’ parameters, i.e. $x^o = \{ \langle w(t) \rangle, \langle T_{XX}^B(t) \rangle, \langle T_{YY}^B(t) \rangle, P_{inst}^{SP}, G^{SP} \}$, the *N*- and *G*-blocks are all identical (the *G*-blocks do not depend on time, so in their case it is, actually, automatic), what simplifies situation a lot. The matrix $(J^T \cdot J)$ for extended linearized system displayed in Fig. 52 has a bit more complicated structure: Its main (far biggest) part is composed of *nTimeSamples* identical 3x3 diagonal blocks of the form $(N^T \cdot N)$, otherwise zeros – which is the same as we have had with the constant gains and instrumental power offsets – and, as a consequence of new (‘gain/instrumental’) columns in the Jacobian we have two ‘belts’ – one, the lower belt, as 48 appended rows and one, which is just the transposition of the former, is appended as 48 last columns at the right side of the matrix. The low and side belts are formed from blocks of mixed matrix products: $(N^T \cdot G)$ and $(G \cdot N^T)$, respectively. The entire square matrix $(J^T \cdot J)$ is eventually completed by 48 x 48 block in the very lower-right corner, which takes form of $nTimeSamples \cdot (G^T \cdot G)$.

The figure illustrates a large system matrix equation, represented as a block matrix multiplication. On the left, a large matrix is partitioned into three main horizontal sections. The top and bottom sections each contain a green block (N-block) on the left and a blue block (G-block) on the right. The middle section is empty. These blocks are separated by dashed green lines. To the right of this large matrix is a multiplication sign 'X'. Further right is a tall, narrow vector of blocks. This vector consists of a series of small green blocks (N-blocks) and blue blocks (G-blocks) arranged vertically. The final part of this vector is a blue block. To the right of this vector is an equals sign '=', followed by another tall, narrow vector of blocks. This vector consists of a series of small green blocks (N-blocks) and blue blocks (G-blocks) arranged vertically. The final part of this vector is a blue block.

Figure 51: System for 'narrowed' Eq. (3.3.11) with a vector function $f_k(x_i)$ prescribed by Eq. (3.3.17), extended by the instrumental variables (blue-framed block at the end of the unknown-variables vector), and corresponding identical blocks (the blue squares at the right) in the Jacobian matrix. The empty spaces in the matrix blocks mean zero elements. In further figures we will not explicitly show the elements and will refer to the green blocks as N-block ('natural' variables) and the blue one as G-block (for 'gains').

Linearized system (3.3.11) with the extended Jacobian (Fig. 52) cannot be directly separated to $nTimeSamples$ independent systems for each time extra. The instrumental part, which is common for all time instants, introduced sort of coupling among the single-time solutions: This is manifested by the matrix structure, whose additional side- and lower-belts destroy the simple block-diagonal form (which we have with constant instrumental parameters) and thus prevent the decomposition. In principle, it could be solved by *np.linalg* package, but it takes really long time as the matrix of the system is very large (~ 8000 squared). Nevertheless, the main part of the system matrix, composed of $nTimeSamples$ identical ($N^T.N$) blocks (which are of 3×3 size each) is easily invertible (it is a block-diagonal matrix) and there is an expression related to the Woodbury formula (see Fig. 53), which allows for inversion of such a square matrix, when it is extended by a row, a column, and the lower-right corner element. The formula have been found with help of Gemini AI in Google with no reference to literature reported. But we proved that it is correct: While finding the inversion is difficult, its correctness can be shown easily by direct multiplication of inverted and original matrix. Moreover, we have tested it by a simple routine

`extMatInv()` in our collection `sol_wvgcal.py`. The formula works even if the added row/column are, in fact, belts, so it can be used for our purpose, too. Inverted large-system matrix (after additional inclusion of ACD calibration equations – see next paragraph below) is schematically shown in Fig. 56.

$$\begin{bmatrix}
 N^T \times N & & & N^T \times G \\
 & N^T \times N & & N^T \times G \\
 & & N^T \times N & \vdots \\
 & & & \ddots \\
 & & & N^T \times N & N^T \times G \\
 G^T \times N & G^T \times N & \dots & G^T \times N & n\text{Samples} \times G^T \times G
 \end{bmatrix}
 \times
 \begin{bmatrix}
 dW(0) \\
 dT(0) \\
 dW(1) \\
 dT(1) \\
 dW(2) \\
 dT(2) \\
 \vdots \\
 dW(N) \\
 dT(N) \\
 dP_{\text{inst}}^{5XX} \\
 dG^{5XX} \\
 dP_{\text{inst}}^{5YY} \\
 dG^{5YY} \\
 \vdots \\
 dP_{\text{inst}}^{3YY} \\
 dG^{3YY}
 \end{bmatrix}
 =
 \begin{bmatrix}
 N^T & \\
 N^T & \\
 N^T & \\
 \vdots & \\
 N^T & \\
 G^T & \sum_{t=1}^{n\text{Samples}}
 \end{bmatrix}
 \times
 \begin{bmatrix}
 5XX \\
 C(0) \\
 5YY \\
 C(0) \\
 7XX \\
 C(0) \\
 7YY \\
 C(0) \\
 \vdots \\
 3YY \\
 C(0) \\
 5XX \\
 C(1) \\
 5YY \\
 C(1) \\
 7XX \\
 C(1) \\
 7YY \\
 C(1) \\
 \vdots \\
 3YY \\
 C(1) \\
 \vdots \\
 3YY \\
 C(N) \\
 5XX \\
 C(t) \\
 5YY \\
 C(t) \\
 7XX \\
 C(t) \\
 7YY \\
 C(t) \\
 \vdots \\
 3YY \\
 C(t)
 \end{bmatrix}$$

Figure 52: Normal equations (3.3.11) for the system extended by instrumental variables – i.e. the system in Fig. 51 multiplied by J^T . The $J^T \times J$ matrix at the LHS is composed from identical blocks, formed by products of the blocks (N and G) of the Jacobian J . The pre-extension original matrix is formed just by the green $N^T \times N$ blocks. The extension by instrumental part has led to a newly added left and bottom belts, composed of mixed instrumental (G) and natural (N) blocks (and their transpositions), and the 48×48 square block in the lower-right corner, composed solely from the G -blocks. The RHS vector represents a multiplication of the original RHS vector C from Fig. 51 by the transposed Jacobian J^T .

◆ AI Overview

If you have an invertible matrix A (size $n \times n$) and you know its inverse A^{-1} , the inverse of the matrix M extended by a row and a column can be computed efficiently. 

If the extended matrix M is defined as:

$$M = \begin{pmatrix} A & b \\ c^T & d \end{pmatrix}$$

where b is a column vector, c^T is a row vector, and d is a scalar, and if the Schur complement $S = d - c^T A^{-1} b$ is non-zero, then the inverse M^{-1} is:

$$M^{-1} = \begin{pmatrix} A^{-1} + A^{-1} b S^{-1} c^T A^{-1} & -A^{-1} b S^{-1} \\ -S^{-1} c^T A^{-1} & S^{-1} \end{pmatrix}$$

Detailed Formula Components

1. Inverse of the Schur Complement (S^{-1}):

Calculate $S = d - c^T A^{-1} b$. The scalar S is the Schur complement of A in M .

$$S^{-1} = \frac{1}{d - c^T A^{-1} b}$$

2. Extended Top-Left Block:

$$A^{-1} + (A^{-1} b) S^{-1} (c^T A^{-1})$$

3. Extended Top-Right Block:

$$-A^{-1} b S^{-1}$$

4. Extended Bottom-Left Block:

$$-S^{-1} c^T A^{-1}$$

5. Extended Bottom-Right Block:

$$S^{-1}$$

Figure 53: Efficient method for calculation of inversion of the matrix that takes the form of easily invertible square matrix, complemented by lower and right belt and the lower-right square element as found by the Gemini AI. Matrix multiplication of the results M^{-1} with the original matrix M easily proves the correctness. Our matrices in Figs. 52 and 55 have exactly this form. We have implemented this method in solution of Eq. (3.3.11) in the routine `dWdBdG()`.

Having the $J^T J$ matrix inverted, we can directly write solution of the extended system (3.3.11). Then, using the iterations (3.3.12) for the operational point we converge to the approximate solution of the

full non-linear system (3.3.10), as usually. Entire this methodology has been implemented in the early version (later updated – see next paragraph) of *dWdBdG()* routine in our *sol_wvgcal.py* collection.

Results: Serious issue has been found with the super-high condition number of our extended ($J^T J$) matrix – is almost singular. The reason is clear – we allowed the instrumental parameters (gains and P_{inst}) to take *any* value, absolutely independent of the pre-calculated ACD calibration solutions. As a result, the system converges to a solution with the least residuals, but it is apparently wrong (solar Tb ~ 2000 K, PWV ~ 7mm, gains shifted by ~100% of their nominal ACD calibration solution). Clearly, we need some anchor for the instrumental parameters.

2.3.3.11 Adding the ACD calibration equations into the system

The naive approach to instrumental parameters as new variables of the system failed as they are completely unbound in the new extended system of equations. We need some anchor for them and this can be achieved by adding also ACD calibration equations into our system. Actually, the scientific and calibration equations take the same form – we repeat here Eq. (3.3.16), which represent a framework for the science data-to model fit, for convenience:

$$P_{rec}^{SP}(t) = P_{inst}^{SP} + G^{SP} \cdot T_{ext}(t)$$

The ACD calibration equations are formally identical, just the external temperatures are replaced by ambient and hot loads:

$$\begin{aligned} P_{amb}^{SP} &= P_{inst,la}^{SP} + G_{la}^{SP} \cdot T_{amb}^{SP} = AF^{SP} \cdot (P_{inst}^{SP} + G^{SP} \cdot T_{amb}^{SP}) \\ P_{hot}^{SP} &= P_{inst,la}^{SP} + G_{la}^{SP} \cdot T_{hot}^{SP} = AF^{SP} \cdot (P_{inst}^{SP} + G^{SP} \cdot T_{hot}^{SP}) \end{aligned} \quad (3.3.18)$$

Here, the ‘*la*’ subscript in G_{la}^{SP} and $P_{inst,la}^{SP}$ stands for low-attenuation gains in spw:pol data channel SP , while high-attenuation gain used in the solar scan has no distinguishing index. The scaling/attenuation factor AF^{SP} between the low and high attenuation gains for spectral window S and polarization P , is calculated as a ratio of signals between the atmospheric scan (low attenuation) and the solar off-source sub-scan (high attenuation) in the SQLD spws – see Eq. (3.3.15). We scale the instrumental power P_{inst} by the same factor, because it is essentially $G \cdot T_{RX}$, (for P_o see Eq. 3.3.18a) and we assume (in line with solar calibration methodology) that T_{RX} remains constant during switching the attenuation from low to high.

The calibration equations (3.3.18) may be actually added with weight different from 1.0 – if we increase the weight to a high number, we force the instrumental parameters to obey the calibration equations more exactly. Indeed, in the limit of the weight going to infinity we arrive to the same solution as we have found with the fixed/constant parameters G^{SP} and P_{inst}^{SP} and shown in Figs. 47 and 49. On the other hand, with the low weight we allow for lower residual errors of the combined system on account of large shifts of the gains from their nominal ACD solutions. So the weight factor has to be optimized. We leave this task to the next paragraph and finish here with construction of matrices J_{ik} and ($J^T J$) for the system extended by Eq. (3.3.18). We implement the increased weight by repeating the calibration Eq. (3.3.18) multiple times – let *calWeight* factor is the number of repetitions. In the regular

system of linear equations such a repetition does not add any new information into the system. But here, in the least-square solution of the over-determined system (3.3.8), we enforce that the calibration equations shall be better fitted. With low *calWeight* ~ 1 the dominance of the science-data equations (we have ~ 2300 time samples in sci-data vs. single calibration measurement) would tend to diminish the sum of residual-squares by fitting better the science data. If we repeat the calibration subset *calWeight*-times, we can balance the dominance of the science-data equations in the system.

After addition of the ACD calibration equations (3.3.18) into system with *calWeight* repetitions, the Jacobian matrix J_{ik} for $f_k(x_j)$ in Eq. (3.3.11) gets extended by *calWeight* $\times 2 \times nPols \times nSPWs$ new rows, where the factor 2 comes from ambient and hot load measurements – see Fig. 54. We refer to those newly added block matrices (48x48) as to *A-blocks* and, specifically, their upper and lower halves for ambient and hot loads will be referred as *AA* and *AH* sub-blocks (24 rows $\times$ 48 cols each). At the right-hand side of equation (3.3.11) we now have appended expressions $b_k - f_k(x_j^o)$ for the ambient- and hot-load powers, i.e. similarly to science-SPW powers we have to make a difference between actually measured P_{amb} and P_{hot} , and the corresponding model-value at the operation point x_j^o . Direct calculations give for those ‘differential’ ambient and hot powers at the RHS of the system shown in Fig. 54 expressions

$$\begin{aligned} dP_{amb}^{SP} &= P_{amb}^{SP} - AF^{SP} \cdot (P_{inst}^{SP} - G^{SP} \cdot T_{amb}) + P_o^{SP} \cdot (AF^{SP} - 1) \\ dP_{hot}^{SP} &= P_{hot}^{SP} - AF^{SP} \cdot (P_{inst}^{SP} - G^{SP} \cdot T_{hot}) + P_o^{SP} \cdot (AF^{SP} - 1) \end{aligned} \quad (3.3.18a)$$

where G^{SP} and P_{inst}^{SP} represent our current-iteration solution for the instrumental variables.

Nevertheless, structure of the LHS matrix ($J^T \cdot J$) of the normal linearized system (3.3.11) is not changed – still we have $(3 \times nTimeSamples) \times (3 \times nTimeSamples)$ main (far biggest) part in upper-left corner, which itself is composed from 3x3 small identical blocks at its diagonal (all off-diagonal matrix elements being zero), the $(2 \times nPols \times nSPWs (= 48)) \times (3 \times nTimeSamples)$ lower belt, which itself is composed of *nTimeSamples* identical blocks of sizes 48x3 each, further its transposition appended to the main part as the right belt, and, finally, 48x48 lower-right corner. The structure of the ($J^T \cdot J$) matrix and its inversion is shown in Figs. 55 and 56. One can see that addition of the ACD calibration equations (3.3.18) appended a set of new identical *A-blocks* into Jacobian *J* (Fig. 54), so the ($J^T \cdot J$) matrix (Fig. 55) is, of course, changed, but its structure remains the same as before the ACD equations have been included. Hence, the efficient-inversion formula from Fig. 53 can be used for the system as well. Namely, inclusion of the ACD equations proscribes into Schur complement *S* (in Fig. 53), which reads:

$$S = nS \cdot G^T \cdot G + cW \cdot A^T \cdot A - nS \cdot G^T \cdot N \cdot J_N^{-1} \cdot N^T \cdot G, \quad (3.3.19)$$

where *nS* and *cW* are abbreviations for *nTimeSamples* and *calWeight*, respectively, and $J_N^{-1} = (N^T \cdot N)^{-1}$.

It is already straightforward to multiply the RHS vector in Fig. 55 by the inverse matrix in Fig. 56 to find the unknown-variables vector.

$$\begin{bmatrix}
 \text{Green Block 1} & \text{Blue Block 1} & \vdots & \text{Green Block 2} & \text{Blue Block 2} & \vdots & \text{Red Block 1} & \text{Red Block 2} \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \text{Red Block 1} & \text{Red Block 2}
 \end{bmatrix}
 \begin{bmatrix}
 dW(0) \\
 dT(0) \\
 \vdots \\
 dW(N) \\
 dT(N) \\
 \vdots \\
 dP_{amb}^{XXX} \\
 dG_{amb}^{XXX} \\
 \vdots \\
 dP_{amb}^{YYY} \\
 dG_{amb}^{YYY} \\
 \vdots \\
 dP_{hot}^{XXX} \\
 dG_{hot}^{XXX} \\
 \vdots \\
 dP_{hot}^{YYY} \\
 dG_{hot}^{YYY}
 \end{bmatrix}
 =
 \begin{bmatrix}
 C(0) \\
 C(1) \\
 \vdots \\
 C(N) \\
 \vdots \\
 dP_{amb}^{XXX} \\
 dP_{amb}^{YYY} \\
 dP_{amb}^{7XX} \\
 \vdots \\
 dP_{amb}^{YYY} \\
 dP_{hot}^{XXX} \\
 dP_{hot}^{YYY} \\
 \vdots \\
 dP_{hot}^{YYY}
 \end{bmatrix}$$

Figure 54: System from Fig. 51 extended by the set of the ACD calibration equations (3.3.18) . The ACD calibration equations appear as two new red sub-blocks at the bottom of the matrix – the red rectangles with Jacobian elements for the ambient and hot loads. In the following they shall be referred as AA and AH sub-blocks, forming a single $(AA \mid AH)^T$ block, referred jointly as the A-block. At the RHS, the added ACD equations form the red-framed vectors of the differential (see Eq. (3.3.18a) in text) ambient- and hot-load powers for each SPW S and each polarization P . The red A-block and correspondent RHS vectors may repeat calWeight-times, this way we increase relative weight of the ACD calibration equations in the system – see explanation in text.

$$\begin{bmatrix}
 N^T \times N & & & & \\
 & N^T \times N & & & \\
 & & N^T \times N & & \\
 & & & \ddots & \\
 & & & & N^T \times N & N^T \times G \\
 & & & & & N^T \times G \\
 & & & & & \vdots \\
 & & & & & N^T \times N & N^T \times G \\
 G^T \times N & G^T \times N & \dots & G^T \times N & nSamples \times G^T \times G + calWeight \times A^T \times A
 \end{bmatrix}
 \times
 \begin{bmatrix}
 dW(0) \\
 dT(0) \\
 dY(0) \\
 dW(1) \\
 dT(1) \\
 dY(1) \\
 dW(2) \\
 dT(2) \\
 dY(2) \\
 \vdots \\
 dW(N) \\
 dT(N) \\
 dY(N) \\
 dP_{mse}^{5XX} \\
 dP_{mse}^{5YY} \\
 dP_{mse}^{7XX} \\
 dP_{mse}^{7YY} \\
 \vdots \\
 dP_{mse}^{3YY}
 \end{bmatrix}
 =
 \begin{bmatrix}
 N^T & & & & \\
 & N^T & & & \\
 & & N^T & & \\
 & & & \ddots & \\
 & & & & N^T & N^T \times G \\
 & & & & & N^T \times G \\
 & & & & & \vdots \\
 & & & & & N^T \times N & N^T \times G \\
 G^T & G^T & \dots & G^T & \sum_{t=1..nSamples} N^T \times G^T
 \end{bmatrix}
 \times
 \begin{bmatrix}
 dW(0) \\
 dT(0) \\
 dY(0) \\
 dW(1) \\
 dT(1) \\
 dY(1) \\
 dW(2) \\
 dT(2) \\
 dY(2) \\
 \vdots \\
 dW(N) \\
 dT(N) \\
 dY(N) \\
 dP_{mse}^{5XX} \\
 dP_{mse}^{5YY} \\
 dP_{mse}^{7XX} \\
 dP_{mse}^{7YY} \\
 \vdots \\
 dP_{mse}^{3YY}
 \end{bmatrix}
 + cW \times AA^T \times \begin{bmatrix} dP_{mse}^{5XX} \\ dP_{mse}^{5YY} \\ dP_{mse}^{7XX} \\ dP_{mse}^{7YY} \\ \vdots \\ dP_{mse}^{3YY} \end{bmatrix}
 + cW \times AH^T \times \begin{bmatrix} dP_{mse}^{5XX} \\ dP_{mse}^{5YY} \\ dP_{mse}^{7XX} \\ dP_{mse}^{7YY} \\ \vdots \\ dP_{mse}^{3YY} \end{bmatrix}$$

Figure 55: Linearized normal system formed from the system in Fig. 54 by multiplication by J^T . Symbol cW at the RHS is abbreviation for $calWeight$, the factor controlling relative importance of the ACD calibration equations in the entire system.

The matrix-vector multiplication can be, again, done block-wise, so the entire complex system with dimension $(3 \times nTimesamples + 2 \times nPolS \times numSPWs \sim 8000)$ squared has been effectively decomposed to $nTimeSamples$ 3×3 systems plus inversion of the Schur complement, which is of size 48 squared. CPU time decreased, in comparison with the ‘brutal-force’ solution of the grand-system matrix (~ 8000 squared), enormously, the routine `dWdBdG()` now spends some 10% of time in the system solution, while its bottle-neck is in reading the MS data.

Just the final technical note: In order to improve numerical accuracy, we calculate the temperatures internally in kiloKelvins – this way the sensitivity factors SF_W^S and SF_B^S for variations of PWV and brightness temperature, which form the Jacobian elements, are numerically comparable. This diminished rounding errors in extensive matrix operations performed in the iterative solution of linearized Eq. (3.3.11).

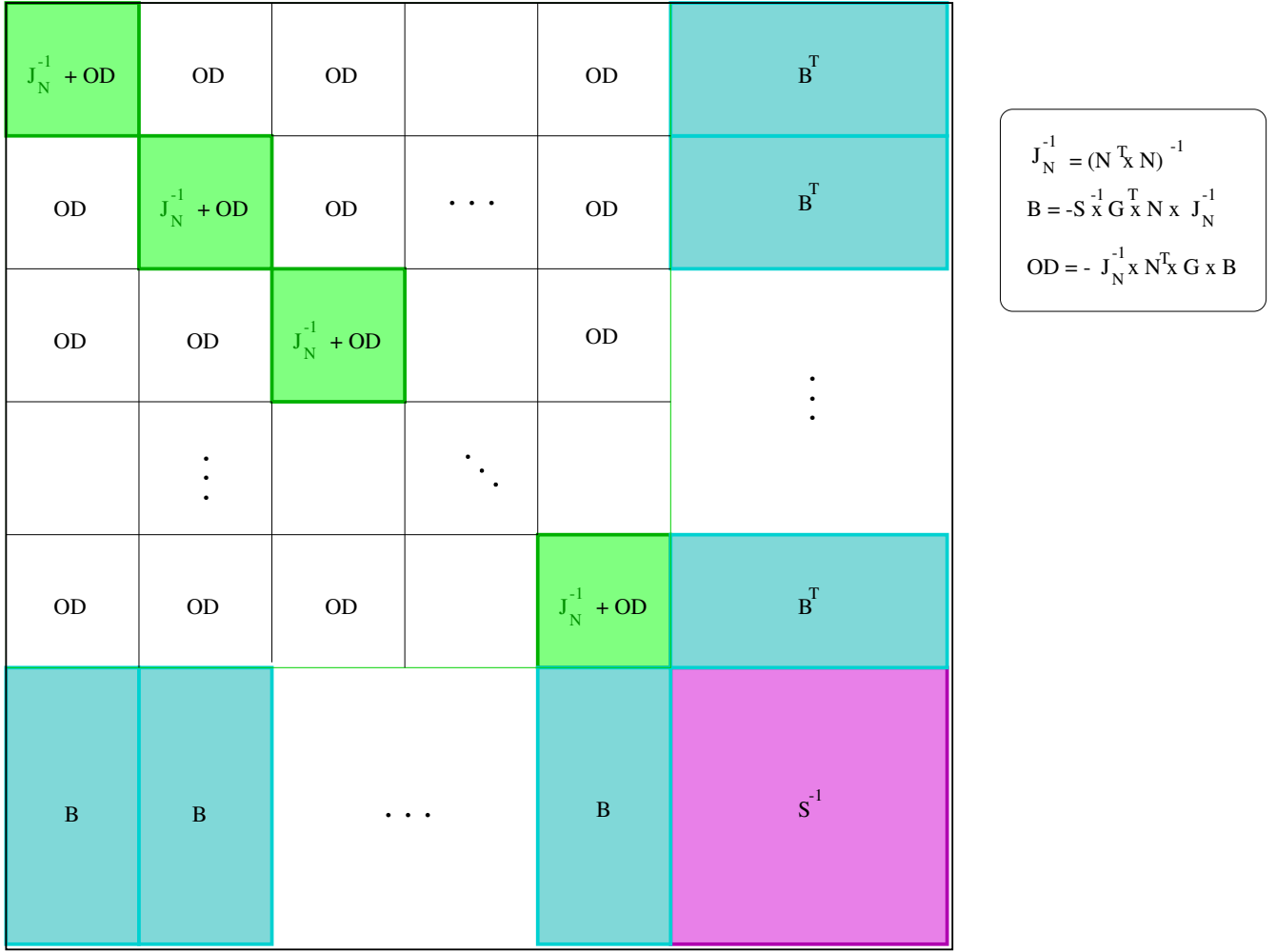


Figure 56: Inversion of $(J^T J)$ grand-system matrix from Fig. 55 using the per-block inversion displayed in Fig. 53. Expressions for the block matrices J_N^{-1} , OD (for Off-Diagonal), and B (Belt) are directly in the figure legend, the Schur complement S, whose inversion S^{-1} is used for calculation of the above mentioned blocks, is – specifically for our system – expressed in Eq. (3.3.19).

2.3.3.12 Finding the optimal calWeight

As it has been explained above, increasing *calWeight* parameter increases importance of the calibration equations (3.3.18) w.r.t. the equations for science data (3.3.16). We can study the effect by splitting the total sum of all squared residuals in Eq. (3.3.9) into two parts: The sum of squared residuals for equations for the science data χ^2_{sci} and the same for the calibration equations χ^2_{cal} . Because the sums grow with the equations count, we normalize them by the that number, i.e. χ^2_{sci} by *nTimeSamples* and χ^2_{cal} by *calWeight*. Now we can vary *calWeight* and study, how these normalized errors behave. The results are shown in Figs. 57 and 58. Fig. 57 presents the normalized sums χ^2_{sci} and χ^2_{cal} as a function of *calWeight* in a broad range of $1 - 10^4$. Because of its extended displayed range, the *calWeight* parameter is expressed in decadic logarithmic scale. χ^2_{sci} and χ^2_{cal} are shown as a red and blue curves, respectively. It can be seen that at *calWeight* $\sim 100 - 300$ an error of the model-to-data fit for science

equations starts increasing rapidly, while we are not reaching further improvement in the fit of instrumental-variables to the dual-load ACD calibration measurements. More detailed view for antenna id 20 is in Fig. 58. We narrowed the range of displayed *calWeights* and show them now in the linear scale. The result is the same – *calWeight* slightly above 200 seems to be optimizing both science and calibration model-to-data fits together. This was demonstrated to be true (most likely) for all antennas – we have selected representatives from CM, DA, and DV groups with the same result. Apparently, the value most likely depends on the *nTimeSamples*, so for different MS it is advisable to re-run the optimization at least for one antenna.

As the last result in this section we present Figs. 59 and 60, which show, how much are our results for scan-averaged PWV and scan-averaged brightness temperatures dependent on the selected *calWeight*. We need not care about the varying parts, these have been shown to be very robust. Fig. 59 displays converged solution of iterations for Eq. (3.3.11) for $\langle PWV(t) \rangle$ as a function of *calWeight* in the range 100 – 1000. It can be seen that within the range 100 – 400, which we may consider as a broader optimum (based on Fig. 58), the resulting PWV varies in the range less than $\pm 0.01\text{mm}$, which is acceptable – in terms of recalculated phase delay it represents less than 10 degs. Fig. 60, shows the same for $\langle T_B^{XX}(t) \rangle$, and $\langle T_B^{YY}(t) \rangle$. The displayed values are in kiloKelvins – they are used for technical reasons as internal unit (see the note at the end of previous paragraph). Again, variation within the displayed range is small, so the obtained results can be considered as quite robust and not dependent on the exact value of the *calWeight* parameter.

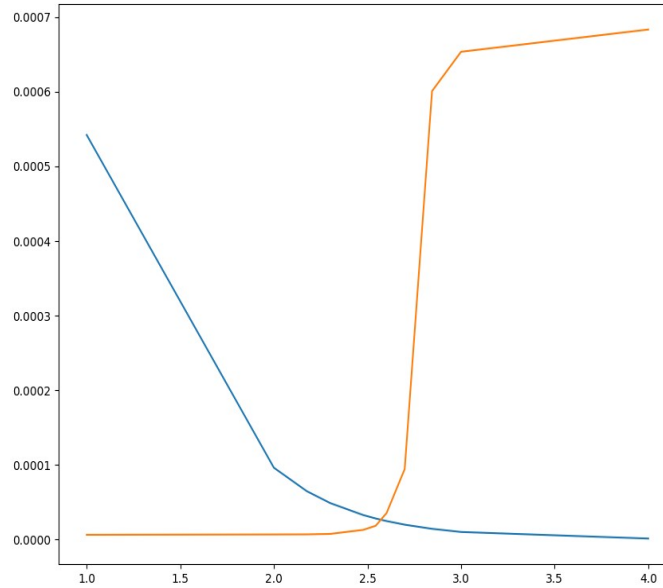


Figure 57: Sum of residuals separately for science-data equations subset scaled down by *nTimeSamples* (red) and ACD calibration equations sub-set scaled down by *calWeight* (blue) vs. decadic log of the *calWeight* parameter.

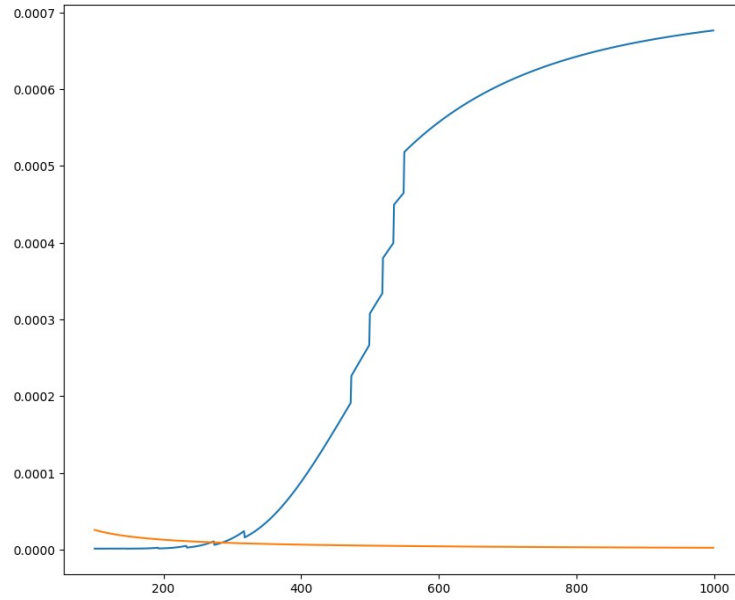


Figure 58: Essentially the same as in Fig. 57, now seen in more detail and in linear-scale for *calWeight* at the *x*-axis.

2.3.3.13 Implementation and results

The entire above described procedure, including all gradual improvements

- combination of XX/YY channels for $dw(t)$ solution,
- non-linear solver for complete PWV(t) (and $T_{\text{Sun}}^{\text{B}}(t)$ as a by-product),
- improved quiet-Sun spectrum,
- inclusion of antenna efficiencies,
- improved transfer of gain-attenuation factors from SQLDs to ACs, and
- making the ACD calibration equations to be a part of extended optimization task

is implemented in the routine *dWdBdG()*. We use the free parameter *calWeight*=200 in this routine, as found by the above described optimization. In order to save CPU time, we call the function *dWdBdG()* in a loop for all antennas in *saveSol_dWdB()* routine, saving the results as Python-native *.NPY* file, which is further re-used for analysis and applications.

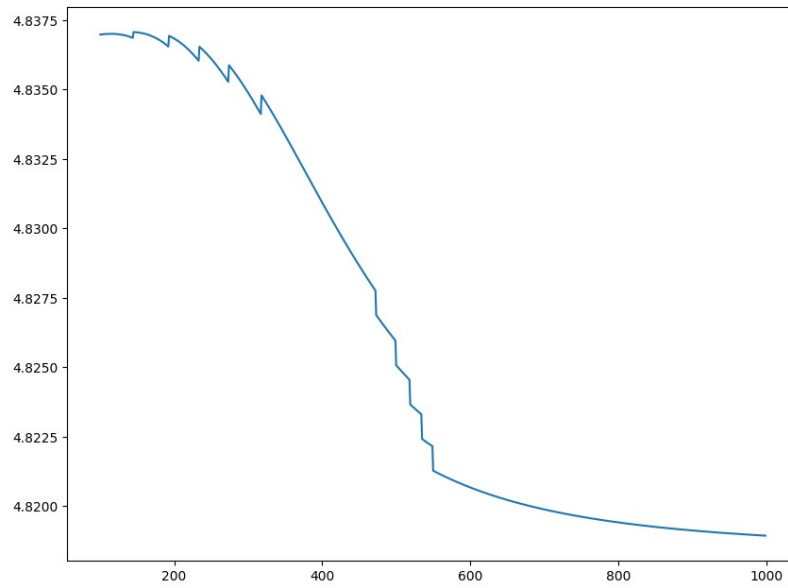


Figure 59: Scan-averaged value of PWV for antenna id20 vs. the calWeight parameter used for balancing weights of the ACD calibration equations with the equations for science data. The teeth visible on the curve are related to the convergence stopper (set to 0.001mm in our code) in the iterative solution of Eq. 3.3.11.

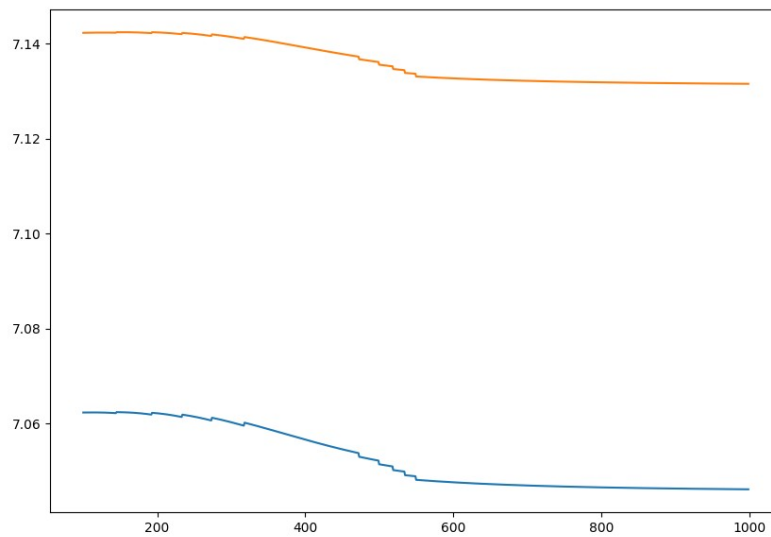


Figure 60: Like Fig. 59 above, now for scan-averaged brightness temperature of the Sun (red XX, blue YY polarizations), in kiloKelvins.

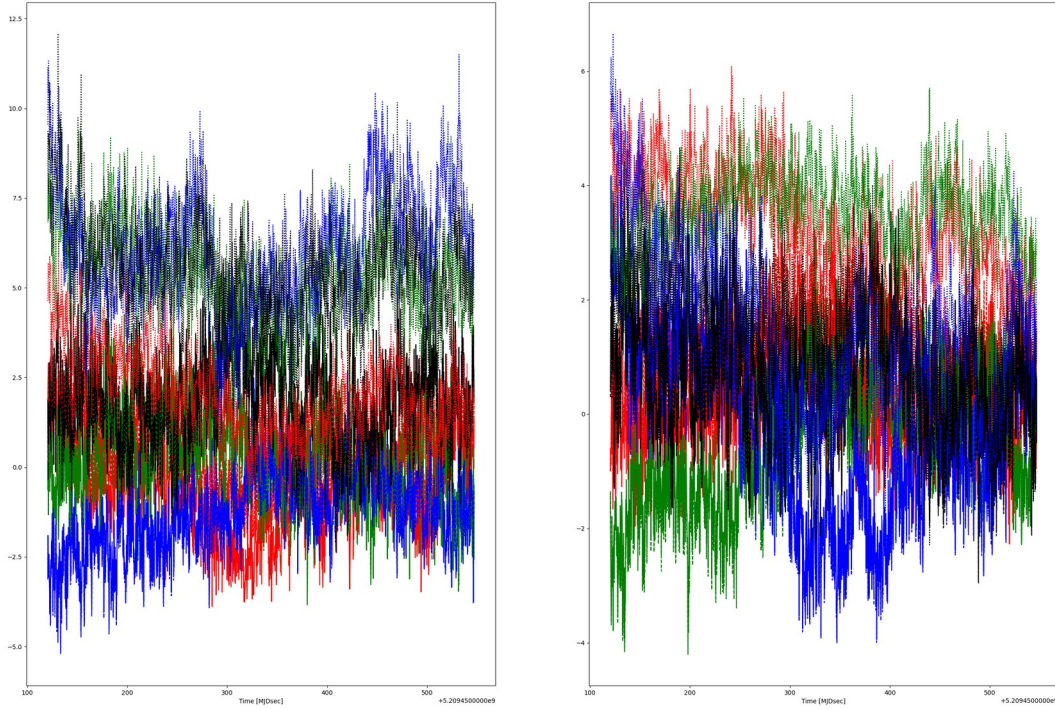


Figure 61: Least-square residuals for antenna id39 in our new procedure $dWdBdG()$, which allows for limited correction of gains and receiver temperatures in frame of the solution of the system. To be directly compared with the top row of panels in Fig. 43.

Results of this improved procedure are shown in Figs. 61 – 65. Fig. 61 shows the residual error in Kelvins for antenna 39 (DA56), so it can be directly compared with the top row in Fig. 43. Clearly, the problem of high (time-averaged) residuals is solved, we now have just fluctuations in order up to 10K. The small fix of gain/ T_{RX} allowed for better fit of the scan-averaged brightness temperatures (sky+src) between model and observation. Figs. 98 – 64 show comparison between results of the older procedure $dWdB_Tbs()$, which uses constant pre-calculated gains and receiver temperatures (blue curves), and our new approach, implemented in $dWdBdG()$, which allows for limited (in order of low units of per-cent) corrections of the instrumental calibration (red curves). Fig. 98 presents comparison for the PWV solution, Figs. 63 and 64 the same for apparent solar brightness temperatures in XX and YY, respectively, all for antenna id38 (DV11). Clearly, the solar brightness temperatures are systematically higher for the gains that are allowed to be corrected as a part of the grand-system of equations (minimizing the least-squares of residuals for the science + ACD calibration measurements together) and they are closer to the expected brightness temperature in Band 5, which is $\sim 6990K$ (Alissandrakis et al., 2020). The constant-gain solution is, on the other hand, $\sim 400 - 800 K$ below it. Again, the PWV variations proved to be really robust characteristics, while its absolute value is subject to some uncertainty. However, having better solution for the solar brightness temperatures provided by the

routine *dWdBdG()*, we trust its full-PWV solution, too. More figures for representatives of CM, DA, and DV antennas are available on-line.

As the last result of this comparative analysis we present Fig. 65. There, the scan- (over times/integrations) and polarization-averaged $((XX+YY)/2)$ values of the solar brightness temperatures are shown for each antenna; the x-axis is discrete and represents the antenna IDs. Again, the red curve represents the correctable-gain solution, the blue one is for constant ACD-based instrumental parameters. Ideally we would expect a constant line around theoretical value of 6990 K. In fact, there are visible the following features:

- Some excesses – namely CM06. It should have been actually flagged (and it was flagged in the standard calibration procedure) but in the PWV analysis we retain all antennas.
- The blue curve is systematically low and – if the excesses in both curves are removed – it has a bit larger RMS.

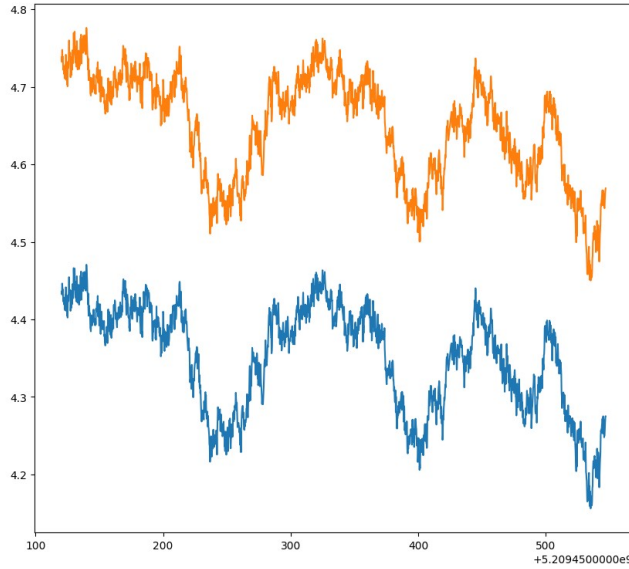


Figure 62: PWV (mm) as a function of time (MJDSec) for antenna DV11. Blue curve: Calculation with constant gains calculated by the ACD calibration equations (*dWdB_Tbs()* routine), red curve: Result of *dWdBdG()* routine, which allows for a limited correction of gains as a part of the grand-system solution.

It seems that the groups of antennas can be identified: CM with IDs 0 – 8, DA (9 – 27) and DV (28 – 51), with the CM antennas exhibiting (in average) lowest T^B , DA a bit higher, and the DV the highest. This is most likely connected with the rear- and forward-spillover efficiencies. We have taken ‘universal’ values of $\eta_l = 0.95$ and $\eta_F = 0.90$ in line with Table 3 in White et al. (2017) for all antennas, but apparently, the DV antennas have a bit higher efficiencies than this values, while the 7m CM antennas a bit lower. We shall come back to this finding in coming Section 2.5.3, where we make use of measurement from an additional test dataset obtained in July 2026. Merely two datasets are not a big

ensemble, but we shall see that, indeed, there are some systematic trends in the scan-averaged solar T_B by antenna type. We have used the result to improved estimation of antenna efficiencies, which are, in turn, used in our improved calculations.

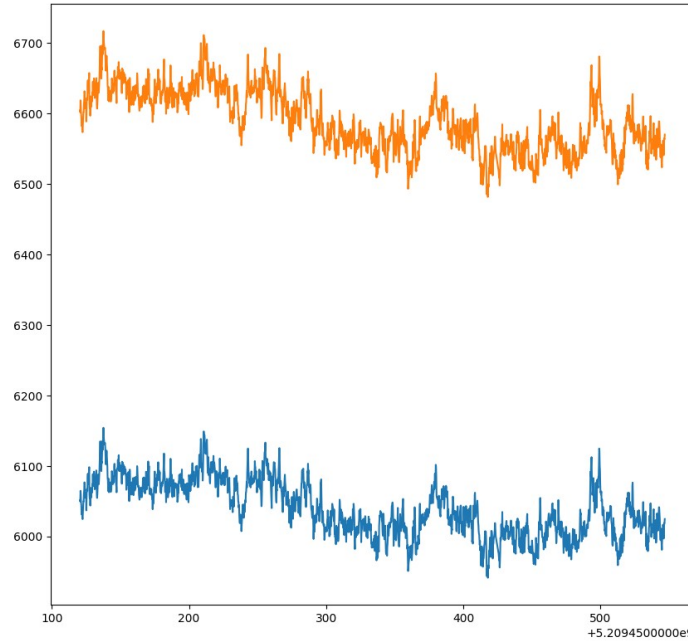


Figure 63: Like in Fig. 62, now we compare solutions for solar (apparent, incl. scintillation) brightness temperature in polarization XX (Kelvins).

Again, the average temperature closer to the expected theoretical value and slightly lower RMS speaks in favor of the method $dWdBdG()$ with the flexible gains being part of the solution, albeit the result is not super-convincing.

Results: We are able – in addition to finding solution for PWV and the T^B time series – to make also small corrections (1 – 2 %) to the instrumental parameters G^{SP} and P_{inst}^{SP} , letting them to become variables of the system (3.3.10). At the same moment we prevent them from large departure from their nominal values found by the ACD calibration solution. This anchor is achieved by inclusion of the ACD calibration equations (3.3.18) into the system with appropriately selected weight. Thanks to clever solution for the system matrix inversion the procedure is, moreover, not so much time-demanding (the most time is spent by reading the received powers and ACD data from the MS, the iterative solution of extended system (3.3.11) takes just about 10% of total time).

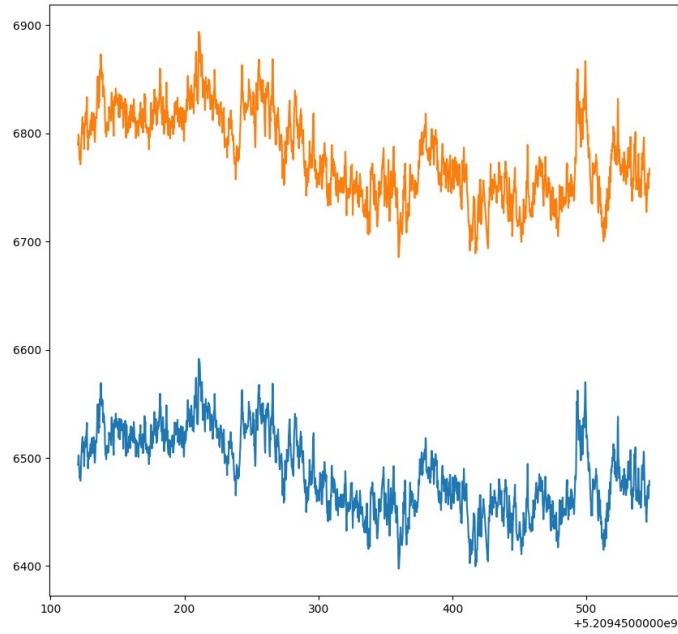


Figure 64: Like Fig. 63, now for YY polarization.

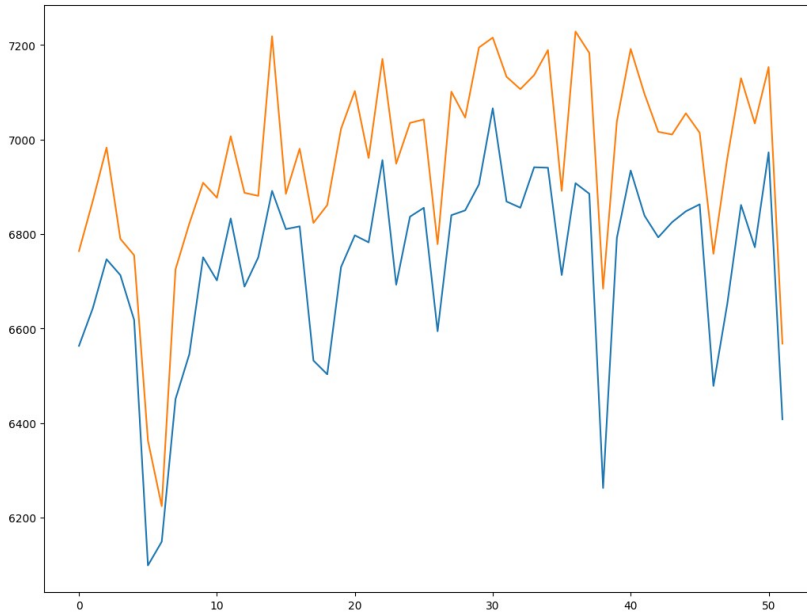


Figure 65: Scan- and polarization-averaged solar brightness temperatures for each antenna (sorted by antenna IDs). Red curve: Flexible-gain solution using $dWdBdG()$, blue: Constant-gain (from the ACD calibration solely) solution.

Summary of this part

By gradual improvements we have developed mathematically rigorous and plausible – as can be indicated by realistic values of scan-averaged solar brightness temperature (close to expected theoretical value) and PWVs (very close to SYSCAL table averages) – method for calculation of PWV(t) time series from the science SPWs of solar scans. Thanks to different spectral dependence of absorption in the terrestrial atmosphere and solar (apparent or actual) brightness changes we are able to decompose both effects and provide ‘the net’ PWV(t) time series. The procedure has been implemented to a collection of Python/CASA routines which, in principle, may be integrated to the general solar ALMA library *sun_reductionUtils.py*. The goal is to use this PWV(t) time profiles as a replacement for direct WVR signal, which is (almost always) unreachable during the solar on-source sub-scans.

2.4 Science SPW-based PWV(t) vs. direct WVR measurements

The question arises, to what extent our PWV(t) time series, calculated by approach developed gradually in previous Section 2.3, indeed correspond to the PWV as measured more directly by means of WVRs.

For the band-pass (or phase, which, however, has quite short scans) calibrator we have been easily able to test the validity of our *Approach 2* based on our differential dPWV(t) proxy to the WV emission/absorption and its subsequent impact to the phase delays. That test case is crystalline clear as we have the ‘ground truth’ – the exact phase solutions for the sources. Moreover, we may (and we did) compare directly the dPWV(t) with WVR data extracted (and transformed to differential phase delays) by means of *wvrgal()*.

In the solar case it is more difficult because the only knowledge about the phase delays of the Sun comes from our *Approach 1*, where we infer them by self-calibration on different time-scales separating the tropospheric turbulence from intrinsic solar dynamics. In fact, the case of BP calibrator is the same, but the BP calibrator has a trivial source model, what is very far from truth with the Sun, which has a complex structure that even changes on the scales only about 10x longer than available cadence of observation. Using solar phase from self-calibration is thus possible, but we actually mix two our approaches (1 and 2) together and it is hard to disentangle their possible errors in the result. Hence, verification of PWV(t) calculations from science SPWs through their impact on phase delays is a bit ambiguous.

The direct comparison with WVR data seemed hard or impossible. In order to make it – at least approximately – we have devised a specific observing test: The 12m array is split into two sub-arrays – most of the antennas watching the Sun, but a few of them (4 - 5), specifically these being part of a close couple with another antenna, look a bit aside off the Sun and measure the (unsaturated) WVR signal synchronously with the solar observation done by the main sub-array. The risky point here is that we observe (i) two a bit different sources – the main array the target at the solar disc, the little sub-array the ‘empty-sky’ source 2 degs apart, and, (ii), the ‘WVR’ antenna, albeit selected very close to its ‘solar’ sibling, is not exactly co-located, so they look through a bit different mass of the atmosphere.

Nevertheless – by an incredibly good luck – we have found yet another possibility of direct comparison of our PWV(t) series with the direct WVR measurements: The 2023 test data has been taken very close to the solar limb. Because the WVR FoV is a bit separated from the Band 5 receiver’s FoV, by chance the WVR were pointing out of the Sun, however very close. The WVR data were thus somehow contaminated by the scattered solar radiation but not saturated. In the following we explore both found possibilities for direct WVR-based vs. sci SPW-based PWV(t) time series. We start (Section 2.4.1) with the contaminated WVR data of 2023 observation, on which we have developed the generalized method on how to calculate the PWV from the WVR channels even under solar contamination. Then, in Section 2.4.2 we present preparation, performance, and results of the specific sub-array test observations designed exactly to the PWV(t) verification.

2.4.1 WVR data in the solar scan of 2022.1.00031.CSV – detailed analysis and comparison with the dPWV proxy.

As it has been already mentioned above, by chance we have found that the WVR data for 2022.1.00031.CSV Band 5 MS are, surprisingly, not saturated, albeit influence of the direct solar illumination is visible. As discussed with Neil Phillips (Development-study team communication), the reason is in luckily pointing to the target very close to the solar limb, while the WVR FoV is separated from the Band 5 receiver’s FoV by ~ 10 arcmin, so the WVR ‘missed’ the solar disc.

The WVR raw data can be extracted directly from the SPW 4 and they represent four time series for four frequency channels of the WVR (see the MTR and references therein). The values extracted from the WVR channels are already calibrated brightness temperatures of the sky in Kelvins. We present here the WVR brightness temperatures as measured in its 4 channels for the solar scan no. 9 as Fig. 66. The jumps at times ~ 120 s (+ MJD offset, up) and ~ 550 s (down) are caused by a nature of the solar scan: It has the very first and very last sub-scan pointing off-source (2deg as verified by our routine *sol_offPos()* in *sol_wvrgcal.py*), while the middle sub-scan(s) points to the target. The reason is in having an additional “atmosphere” (sub)scan but now with the attenuation levels set as for the on-source observation; this is used in calculation of the modified gain (higher-attenuation) gain G^* as explained in Section 2.3.

As one can see, the WVR channel 0 is saturated by the atmospheric WV emission, it is even indifferent to re-pointing of the antenna close to the Sun – there is no jump visible. The remaining channels 1 – 3, however, detect the approaching to the Sun and react by the jumps. Clearly, the signal is contaminated by the solar illumination. Inspired by the success in decomposition of the sky brightness in the science SPWs described in Section 2.3, we attempt here to use similar trick and get rid of the solar-illumination contribution.

The entire process of extraction of the ‘net’ or ‘decontaminated’ (from solar illumination) PWV signal was, again, a sort of gradual procedure, with surprising findings, which demanded change of the approach. We present here the complete evolution, for that sake it is split into two parts, corresponding to history of how we were addressing the problem. Basically, our approach has evolved from linear equations for differential quantities (fluctuations of water vapor dw and brightness of parasitic illumination dL) towards non-linear radiative-transfer equations for complete values of PWV(t) and

illumination L , very similarly to the development documented for PWV-proxy in the previous Section 2.3. And again, the impatient reader may skip to the sub-section 2.4.1.7, where the final procedure, its implementation, and results are presented.

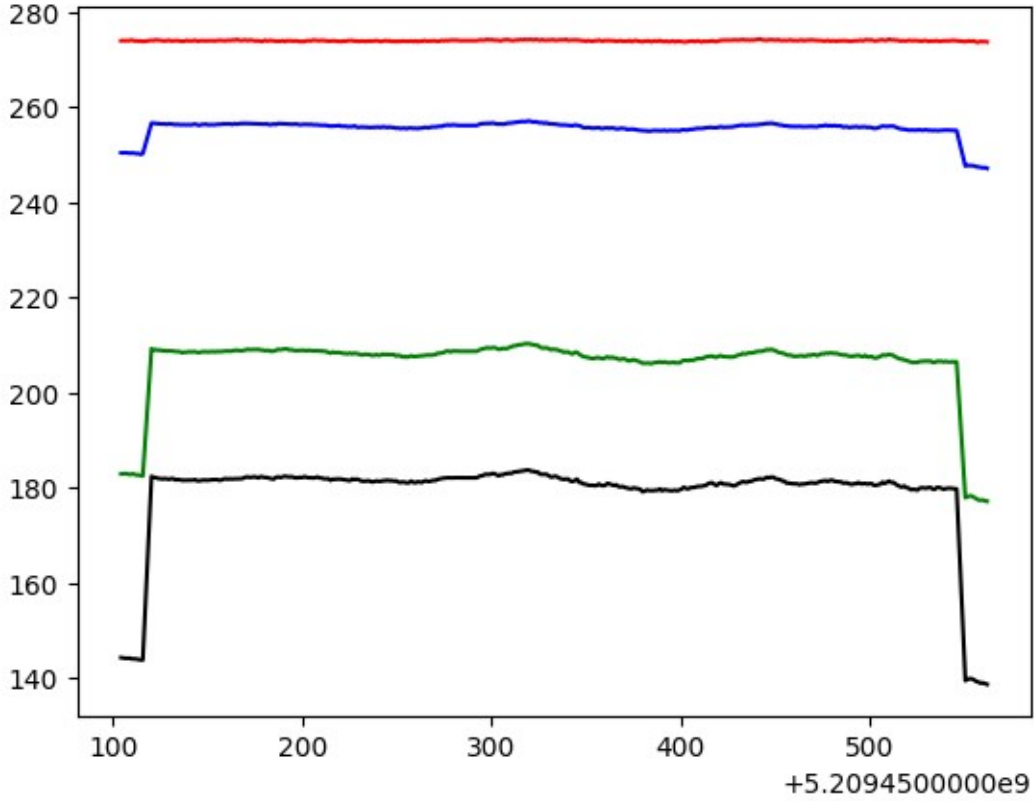


Figure 66: Raw WVR brightness temperatures (in K) as detected in 4 channels in the SPW 4. Channels 0 - 3 are displayed by red, blue, green, and black curves. The jumps at the beginning and end of the solar scan, visible in channels 1 – 3, are caused by moving the antenna towards and outwards of the Sun. Time in the x-axis is in MJDsec.

2.4.1.1 Theory 1: Differential approach

From the jumping WVR intensity, when pointed closer to the Sun, it is clear that the WVR temperatures are contaminated by the solar illumination. Hence, the brightness temperature recorded by the WVR channel contains two components – terrestrial atmosphere irradiation and partial solar illumination, weakened by the absorption

$$T_{\text{WVR}}(f, t) = T_{\text{atm}}(f) \cdot [1 - \text{tr}(f, t)] + L_{\text{sol}} \cdot \text{tr}(f, t) \quad (4.1.1)$$

Here, as experimentally found, we have to allow the effective kinetic temperature of the atmosphere to be dependent on frequency. For the science SPWs treated in Section 2.3 it was found that T_{atm} calculated

by the ATM model varies by $\sim 1\text{K}$ within entire SPW (e.g., SPW 5), but for the WVR's SPW 4, the channel dependence is much stronger. It is obvious – the closer we are to the WV line center, the bigger is the optical depth, so in the line center we are sounding close (i.e., low) atmospheric layers, while in the far wings the radiation comes from the higher altitudes, too. For now, we may think about solar illumination component $L_{sol} = r \cdot T_{sol}$, where r is a fraction of the solar radiation scattered to the WVR FoV and T_{sol} is the solar disc brightness temperature in Band 5 ($\sim 6700\text{ K}$), as being constant, because the solar brightness temperature only weakly depends on frequency (negligible within the relatively narrow bandwidth of the SPW 4) and, one may think that the relative off-pointing of the WVR device from the Sun is kept constant, so there is no reason for the fraction r to be a function of time. However, as we will see in further, this assumption is not true and below, in Part 2, we will have to handle it. But for now we can continue with assuming the illumination L_{sol} being constant.

We can now express the transmissivity from Eq. (4.1.1) as $tr(f,t) = \exp[-\tau(f,t)] = \exp[-\tau_d(f) - \tau_1(f) \cdot w(t)]$ and after decomposing the time-dependent quantities to the time-averaged (over a scan) mean value and the differential variations as we have done in our early dPWV-proxy approach, too, we get an expression for the WVR temperature

$$T_{wvr}(f, t) = T_{atm}(f) + e^{-\langle \tau_d(f) \rangle} \cdot (L_{sol} - T_{atm}(f)) \cdot e^{-\tau_1(f) \cdot (\langle w \rangle + dw(t))}$$

which, in a limit of $dw(t) \ll 1$, becomes

$$T_{wvr}(f, t) = T_{atm}(f) + e^{-\langle \tau_d(f) \rangle} \cdot (L_{sol} - T_{atm}(f)) \cdot e^{-\tau_1(f) \cdot \langle w \rangle} \cdot [1 - \tau_1(f) \cdot dw(t)]$$

For the variation $dw(t)$ set to zero this expression gives the time-averaged value of the WVR temperature $\langle T_{wvr}(f) \rangle$, so we eventually arrive to the expressions

$$T_{wvr}(f, t) - \langle T_{wvr}(f) \rangle = -\tau_1(f) \cdot (\langle T_{wvr}(f) \rangle - T_{atm}(f)) \cdot dw(t) \quad (4.1.2)$$

As it can be seen, we have effectively excluded the explicit solar illumination L_{sol} , however, it is still implicitly contained in formula (4.1.2) by means of the averaged WVR temperature $\langle T_{wvr}(f) \rangle$.

The LHS of Eq. (4.1.2) can be easily directly calculated from each WVR channel, as well as most of the quantities at the RHS. An exception is the $T_{atm}(f)$ – if we use the ATM model for its estimation with the mean PWV for the MS (4.6mm), we get different values of $dw(t)$ from different WVR channels 1 – 3. The channel 0 is basically useless because it is intact to anything – both PWV changes or increase/decrease of solar illumination due to antenna off-source/on-source re-pointing do not make any change, it (most likely) just follows the actual air temperature in the low atmosphere around the dish.

Anyway, we can express Eq. (4.1.2) for two different WVR channels and make their ratio. This excludes $dw(t)$ from the expression and predicts that the variations of the $T_{wvr}(f, t)$, i.e. after subtraction of its mean $\langle T_{wvr}(f) \rangle$, as expressed in the LHS of Eq. (4.1.2), are mutually linearly correlated for different WVR channels. Indeed, the scatter plots of $\delta T_{wvr}(f, t) = T_{wvr}(f, t) - \langle T_{wvr}(f) \rangle$ for two different frequencies show pretty linear correlation, at least for the DV antennas. We, however, had to exclude

the channel 0 from this analysis, because it has almost zero temperature variations, so its sensitivity to PWV variation is very small

Albeit we are not capable to enumerate the factor standing in front of $dw(t)$ on the RHS of Eq. (4.1.2), still we can use Eq. (4.1.2) for the qualitative display of the $dw(t)$ curve, just without its absolute scaling. This is useful, as long as we would like to compare it with the indirect/proxy method for (also differential) $dw(t)$ based on the data from the science SPWs (see sections 2.2 and 2.3).

2.4.1.2 Results 1: Differential approach

As written above, we may do qualitative comparison between the PWV variations $dw(t)$ as derived directly from the WVR channel data using Eq. (4.1.2) above, with the dPWV-proxy calculated from the science SPWs. Because, lacking knowledge of $T_{atm}(f)$ in Eq. (4.1.2), we do not have absolute scaling for the WVR-based $dw(t)$, we scale its values arbitrarily, such a way that the value-ranges for both quantities (WVR-based $dw(t)$ and dPWV(t) proxy) are the same. We did it for all antennas equipped with WVR, i.e. except of the CM antennas. The result examples are shown in Figs. 8 and 9, the full ensemble is accessible on-line as *Annex 5*.

Figs. 67 shows the $dw(t)$ data calculated from Eq. (4.1.2) in arbitrary scaling as a blue curve. It is the same in both left and right panels, which are displaying XX and YY pols, what, however, has a meaning only for the dPWV proxy calculated from the science SPWs (WVRs have no polarization distinction). The red curves are the corresponding dPWV proxy calculated by a simplified procedure using a single SPW signal, according to Eq. (2.2.14). The figure is produced from the data of antenna DV08. The y-axis is in mm, the time at x-axis is in MJDsec. Apparently, the (qualitative) correlation is quite good even though the PWV is changing ‘wildly’.

Fig. 68 displays the same result, but now for antenna DA42. In this case, the correlation is much worse, one can barely find any similar features between the red (dPWV-proxy) and blue (WVR-based) curves. If one sees the full ensemble of results, there is striking difference between the DA and DV antennas: While the DV antennas show, in general, very good amount of correlation, the DA antennas are much weaker – Figs. 67 and 68 here are just illustrative examples, but they express the general trend.

Since only apparent difference in construction between DA and DV antennas lies in the number and positions of the supports that hold the sub-reflector, we are thinking about additional scattering (or shadowing) of the sunlight at these constructions, which, as the Sun moves on the sky, may be varying with time as the antennas have alt-azimutal mounts. Likelihood of this explanation has been confirmed during informal discussion with our advisory team on occasion of the test-observation preparation in January 2026. The main consequence is that the assumption of constant contribution by the solar illumination to the WVR temperatures is no longer valid and we had to find another approach.

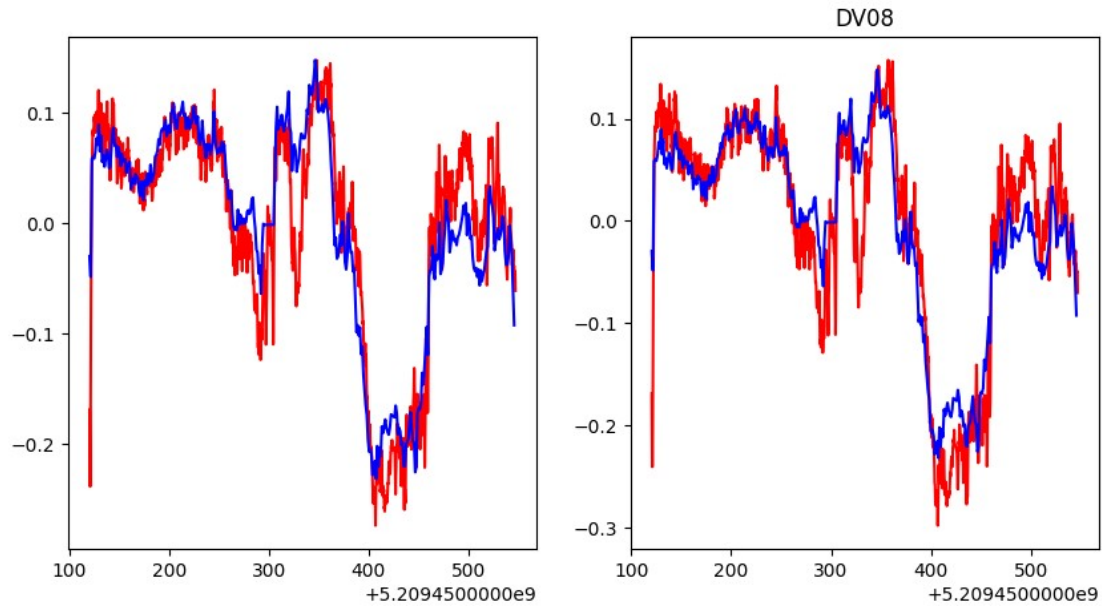


Figure 67: WVR-based PWV variations around its mean (blue, arbitrary scaling) vs. the dPWV proxy calculated from absorption in science SPWs (red, in mm) for antenna DV08. Left/right: dPWV proxy for XX/YY pol.

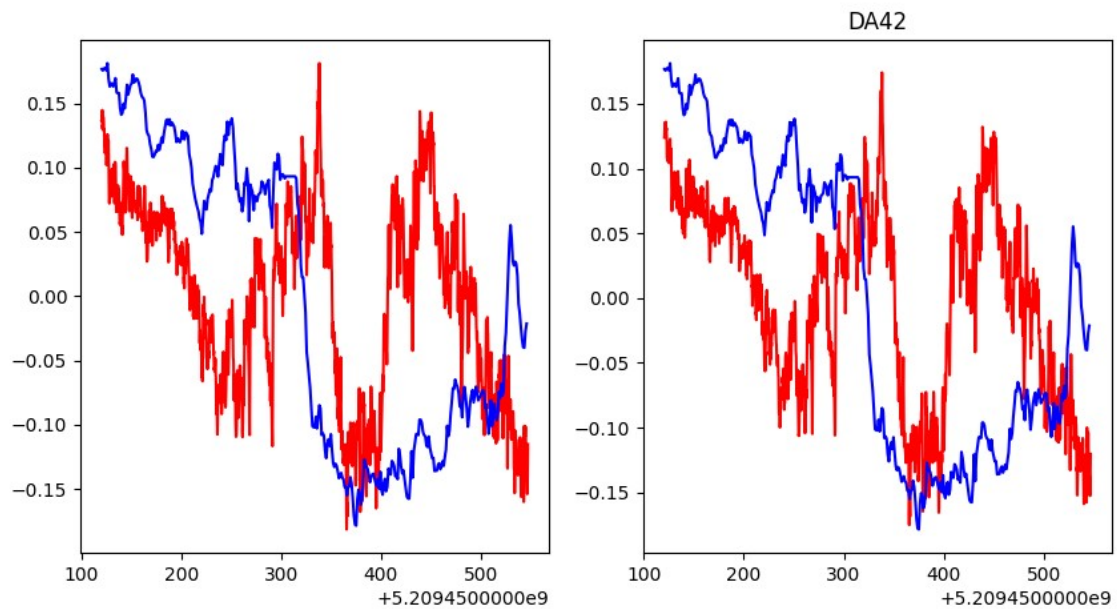


Figure 68: The same as in Fig. 67, now for antenna DA42.

2.4.1.3 Theory 2: Non-linear RT equations for full PWV(t) and L(t)

With having another free/uncontrolled variable in the system it looked like hopeless to try extract the net PWV contribution from the solar-illumination-contaminated raw WVR data. Anyway, our success with decomposition of the (apparent) source brightness dynamics from the science-SPW signal as presented in Section 2.3, inspired us to attempt here the similar task.

First, we have to allow the total solar illumination L_{sun} to be now a function of time, because the fraction r of the solar radiation, which enters the WVR aperture, may vary with time due to uncontrolled reflections/scattering at the antenna construction. Second, the concept of effective *kinetic* temperature of the atmosphere ($T_{atm}(f)$ in Eq. (4.1.2)) has been found as problematic, because we have no means how to evaluate it. For that reasons, the basic Eq. (4.1.1) above is rewritten as

$$T_{WVR}(f, t) = T_{BS}[f, w(t)] + L_{sol}(t) \cdot tr[f, w(t)] \quad (4.1.3)$$

Here, the $T_{BS}[f, w(t)]$ is the sky brightness temperature, which can be approximated by the ATM model via `at.getTebbSky(wh2o=w)` method, parametrically, for user-defined PWV. In Eq. (4.1.3) we explicitly indicate that both the $T_{BS}()$ and transmissivity $tr()$ depend as functions on time not directly, but by means of time-varying PWV, the $w(t)$.

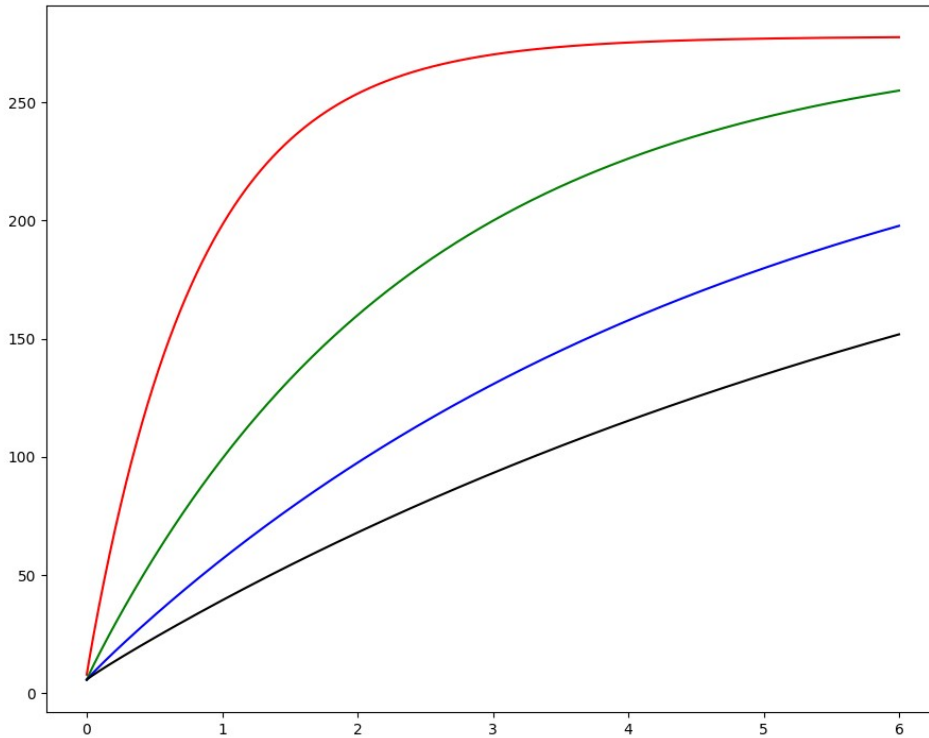


Figure 69: Sky brightness temperature (y-axis, in K) for varying PWV (x-axis, in mm) calculated via ATM model accessed by `at.getTebbSky()`. The red, green, blue, and black curves are for frequencies corresponding to the WVR channels 0,1,2, and 3, respectively.

We have calculated the sky brightness temperature T_{BS} , approximated by `at.getTebbSkySpec()` as a function of PWV for the 4 WVR channels. The ATM model was matched to the real atmospheric conditions by meteo parameters obtained from ALMA Meteo Central station, the airmass has been set by the elevation of the Sun – see `setAtmTool()` in `sol_wvrgcal.py` (*Annex 4*). The result is shown in Fig. 69. If there was no solar contamination, the second term in Eq. (4.1.3) would disappear and we could match the measured WVR temperatures in SPW 4 to the curves in Fig. 69 to fit as much as possible the values at the curves and get this way the actual PWV. The curves look like close to exponentials approaching some limiting value for PWV going to infinity (found to be 277.5 K for scan no. 9), what can be easily shown when instead of T_{BS} as a function of w we plot the sky brightness temperatures in another representation: $\ln(T_{BS}(\infty) - T_{BS}(w))$ as a function of w . This is shown in Fig. 70, and indeed, except of the channel 0, which we had to drop anyway (because of saturation), the curves are linear within a wide range of PWVs.

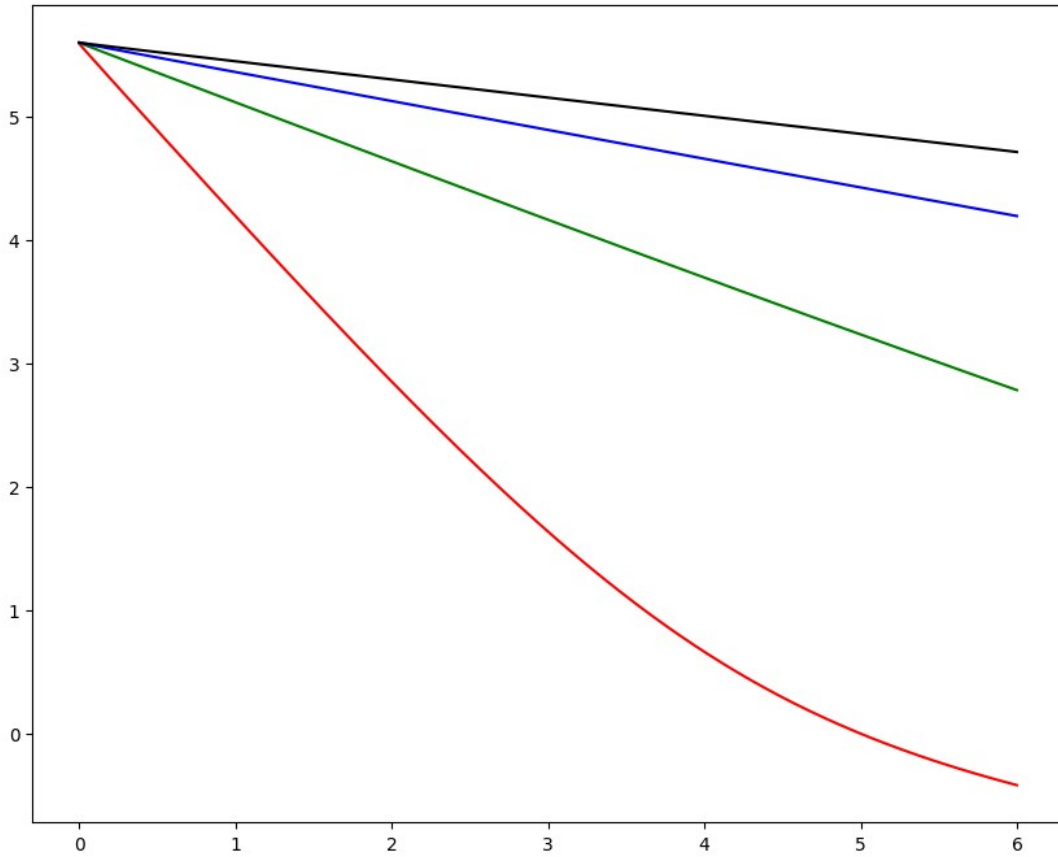


Figure 70: Like in Fig. 69, but now, instead of brightness temperature, we present its logarithm after subtraction from the limiting value of 277.5 K which is reached for infinity PWV (see text).

The logarithmic representation facilitates coupling of the actual T_{wvr} temperatures in channels 1 – 3 to the $T_{BS}(w)$ calculated from the ATM model, because of (almost perfect) linearity. The linearity assures that if we know the $T_{BS}(w_o)$ at some ‘working/operating point’ $PWV=w_o$, we can easily calculate it in another point $PWV=w$ via relation

$$\ln(T_{BS}(\infty) - T_{BS}(w)) - \ln(T_{BS}(\infty) - T_{BS}(w_o)) = -\tau_1^{eff} \cdot (w - w_o) \quad (4.1.4)$$

where we designated the slope of the straight lines in Fig. 11 as $-\tau_1^{eff}$ because, as it will come clear from further, it has a meaning of effective wet opacity per 1 mm of PWV, scaled by air-mass for the current scan. If the relations displayed in Fig. 70 were perfectly linear, τ_1^{eff} would be constant. Because there is apparently some non-linearity caused by dependence of the optical depth on w (so that with changing w we see a bit different layer of the atmosphere with different temperature), τ_1^{eff} is actually very weakly dependent on w , too. However this dependence becomes prominent only in the channel 0 for PWVs higher then ~ 3 mm (see a knee of the red curve in Fig. 70). Anyway, we are searching the coupling between measured T_{wvr} temperatures and brightness sky temperatures T_{BS} calculated from the ATM model iteratively, via Newton-Raphson (NR) method (see *dWdLs()* in *sol_wvrgcal.py* – Annex 4) and calculate the slope numerically in the vicinity of the current ‘operating point’, so its weak dependence on w does not play a role. Moreover, the almost-linear logarithmic representation facilitates convergence of NR iterations a lot.

Eq. (4.1.4) can be rewritten into form

$$\frac{T_{BS}(w) - T_{BS}(\infty)}{T_{BS}(w_o) - T_{BS}(\infty)} = e^{-\tau_1^{eff} \cdot (w - w_o)}$$

from which $T_{BS}(w)$ can be expressed as

$$T_{BS}(w) = -\left[e^{\tau_1^{eff} \cdot w_o} (T_{BS}(\infty) - T_{BS}(w_o)) \right] \cdot e^{-\tau_1^{eff} \cdot w} + T_{BS}(\infty) \quad (4.1.5)$$

Formally it complies with the expression

$$T_{BS}(w) = T_{atm} \cdot (1 - tr(w)) = T_{atm} \cdot (1 - e^{-\tau(w)}) = -[T_{atm} e^{\tau_d}] \cdot e^{-\tau_1 \cdot w} + T_{atm} \quad (4.1.6)$$

so it is nothing to wonder about and also it explains, why we call the slope of straight lines in Fig. 70 as $-\tau_1^{eff}$. On the other hand, since the concept of T_{atm} does not work well (when matching Eqs. (4.1.5) to (4.1.6), it appears that the T_{atm} in the coefficient in front of the exponential need not to have the same value as the second T_{atm} in the last, additive term in Eq. (4.1.6); moreover the effective wet opacity τ_1^{eff} is just close but not exactly the same as τ_1 scaled by airmass), we better stay with the ‘experimentally’ found (i.e. via analyzing the exact ATM solution for *TebbSky*) parameters in the Eq. (4.1.5).

The $T_{BS}(w)$ from Eq. (4.1.5) can be inserted, together with the transmissivity definition, into Eq. (4.1.3). If we assume that we are moving in the linear PWV space close around the ‘operating point’ w_o , which can be conveniently selected as the PWV mean averaged in time over a scan (i.e. $w_o = \langle w \rangle$), then $dw = w - w_o$ is going to be small, the appropriate exponential factor may be linearised and we eventually receive relation

$$\begin{aligned}
T_{wvr}(f, t) = & T_{BS}(f, w_o) + \langle L_{sun} \rangle \cdot tr(f, w_o) \\
+ & \left[\tau_1^{eff}(f) \cdot (T_{BS}(\infty) - T_{BS}(w_o)) - \langle L_{sun} \rangle \cdot tr(f, w_o) \cdot \tau_1(f) \right] \cdot dw(t) \\
& + tr(f, w_o) \cdot dL_{sun}(t)
\end{aligned} \tag{4.1.7}$$

Here we again decompose the solar illumination into stationary part $\langle L_{sun} \rangle$ and its variation $dL_{sun}(t)$ and neglected the mixed term of the second order $dw(t) \cdot dL_{sun}(t)$. The frequency dependence indicates that Eq. (4.1.7) can be formed for each of the WVR channels extra. Actually, for the aforementioned reasons, we omit the saturated channel 0 and use just channels 1 – 3. Anyway, even with the three channels we are able to form over-determined system of three linear equations for two unknown variables $dw(t)$ and $dL_{sun}(t)$.

The Eq. (4.1.7), respectively the set of its realizations for three independent frequencies of WVR channels 1-3, represents a cornerstone for decomposition of the measured WVR signal T_{wvr} in SPW 4 into PWV variations and variable solar illumination. It is somewhat similar to the Eqs. (3.1.4) and (3.1.5) for decomposition of T_{sky} in science SPWs into PWV-induced absorption-variation and the source brightness dynamics, by which it is inspired. On the other hand, there are two distinctions: We have a stationary part in the 1st line of the RHS in Eq. (4.1.7) and this stationary part itself is unknown – we do not know the scan-averaged PWV value w_o and neither the mean solar illumination $\langle L_{sun} \rangle$. For that reason, even the matrix coefficients in the 2nd and 3rd line of the RHS of Eq. (4.1.7) are not known.

In order to solve the system (4.1.7), we proceed by means of a two-step algorithm, which is based on the following ideas:

1. The unknown variations $dw(t)$ and $dL_{sun}(t)$ are, by design, a small fluctuations around the mean value of the respective quantity. If they have (in the found solution) a non-zero mean, it can be added/transferred to $w_o = \langle w \rangle$ and $\langle L_{sun} \rangle$, respectively.
2. In the corrected system, with the variations $dw(t)$ and $dL_{sun}(t)$ having already zero mean (as per Item 1 above), the stationary part at the 1st row of Eq. (4.1.7) must correspond to the time-averaged mean of the T_{wvr} , i.e. $\langle T_{wvr}(f) \rangle = T_{BS}(f, w_o) + \langle L_{sun} \rangle \cdot tr(f, w_o)$.
3. From the equality for stationary parts in the above Item 2 we can calculate $\langle L_{sun} \rangle$ for any value of w_o using the ATM model (*atmosphere* tool in CASA) – T_{BS} is calculated as *at.getTebbSkySpec(wh2o=w_o)*, the transmissivity via *at* methods for wet and dry opacity. The averaged $\langle T_{wvr} \rangle$ we simply calculate from SPW 4 data, for channels 1 – 3 extra.
4. The point is, that for any input value of w_o we get three values of $\langle L_{sun} \rangle$ – each WVR channel (1 -3) gives us one solution. In fact, we aim at a single value, so we evaluate the STDDEV of those three independent solutions and aim at minimizing the dispersion in iterations. This gives us a good estimate of $w_o = \langle w \rangle$ and $\langle L_{sun} \rangle$. This is the end of the Step 1.
5. Now, with having the stationary part solved, we can calculate the matrix elements (the sensitivity factors for $dw(t)$ and $dL_{sun}(t)$) and the RHS vector (the measured T_{wvr} minus the stationary part at the 1st line of Eq. (4.1.7)). This is beginning of the Step 2.

6. Then we solve the system (4.1.7) and find the time variations $dw(t)$ and $dL_{sun}(t)$. Ideally, they should have zero mean value (time average), but it is not quite true.
7. So, we correct the mean values $w_o = \langle w \rangle$ and $\langle L_{sun} \rangle$ by the means of variations found in the Item 6 and continue again from the Item 5 (rebuilding the linear system). When the means calculated in Item 6 are close to zero, we finish.

In practical arrangement, we start with initial guess of w_o by the method `getPWVforAnt()` from our `sol_wvrgcal.py` collection, which has been added in order to estimate the scan averaged PWV. It uses linear interpolation of PWV values as found in the SYSCAL table, which are ‘bracketing’ the scan (in time – see Fig 23). This, we believe, does better job then `aU.getMedianPWV()`, which provides a single PWV characterization for entire MS (i.e. averaged over entire ALMA array and all scans).

Minimizing of the $\langle L_{sun} \rangle$ dispersion (STDDEV) from the three independent values in the *Item 4* is performed via Newton-Raphson iterations. An illustration, on how the dispersion depends on varying value of w_o is shown in Fig. 71.

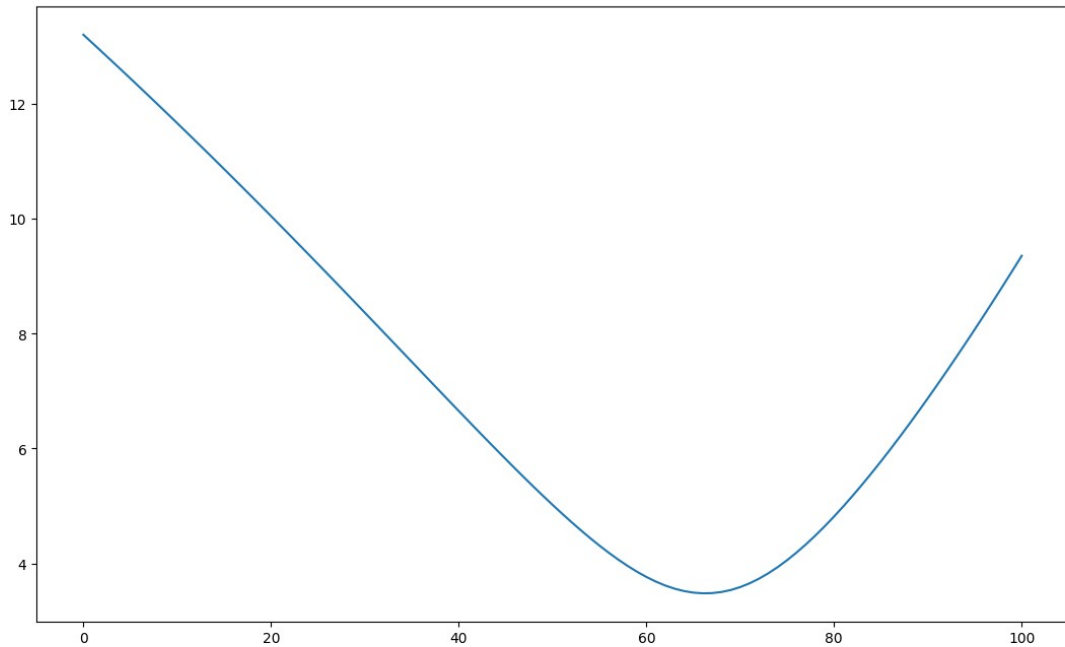


Figure 71: STDDEV of $\langle L_{sun} \rangle$ in Kelvins in a dependence on PWV. The x-axis is expressed in the PWV array index: We have calculated the dispersion for PWVs in the range 4.0 - 5.0 mm, with a step of 0.01mm. Hence, the minimum dispersion of 3.7 K is reached for PWV 4.64 mm (index 65).

The entire solution procedure is implemented as a method `dWdLs()` in `sol_wvrgcal.py` (Annex 4).

2.4.1.4 Results 2: Full PWV(t) and L(t) series

The new approach, which implements solution of the system (4.1.7) via iterative algorithm described in 2.4.1.3 as *dWdLs()* routine, has multiple advantages. First of all, it does for what it has been designed – removes the varying solar illumination from the WVR signal and leaves corrected ‘net’ PWV variations. But there is more in it: Unlike the approach based on Eq. (4.1.2) in Subsection 2.4.1.1, which extracts the fluctuations of PWV up to an unknown multiplicative factor, here we have full scaling and we are able to extract complete PWV, including its mean value. Results of the WVR-signal decomposition to PWV and solar illumination for antennas DA42 and DV08 (we have chosen the same as in *Results 1* for easy comparison) are shown in Figs. 72 – 75.

Fig. 72 shows the full PWV time profile over the on-source sub-scan of the solar scan 9 for DA42 (i.e., one of the ‘problematic’ DA antennas). We have selected just the on-source data in order to do comparison with our dPWV proxy derived from the science SPWs. Already at the first look it is apparent that the profile differs significantly from the blue curve in Fig. 68, which presents the WVR data without correction to variable solar illumination.

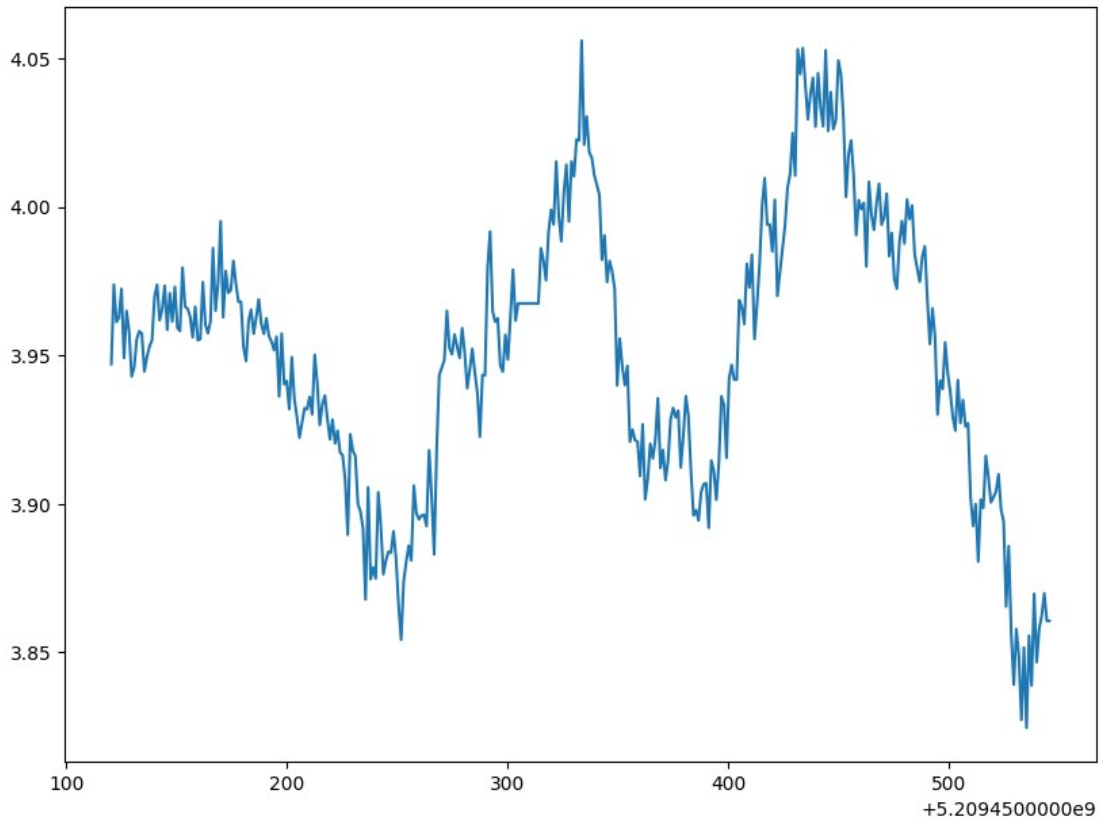


Figure 72: PWV for the antenna DA42 in mm, time axis in MJDsec.

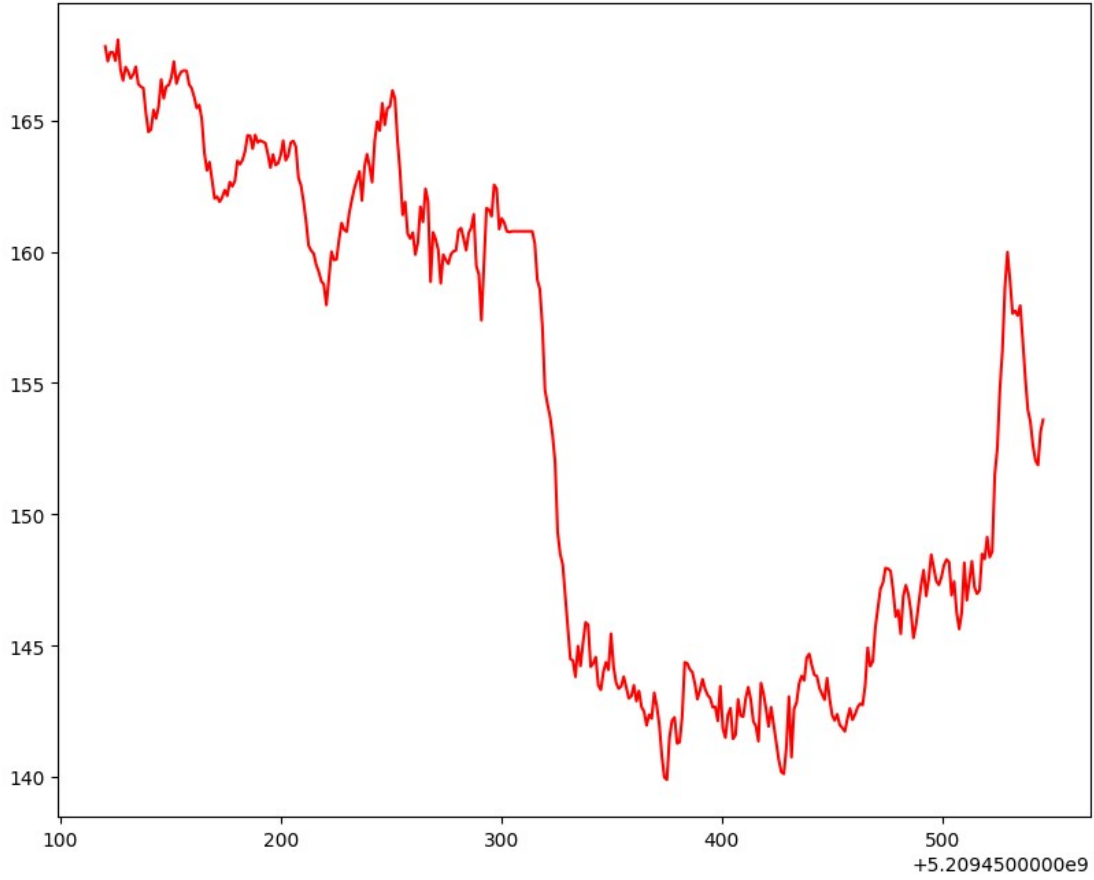


Figure 73: Solar illumination contribution to the WVR signal in Kelvins for antenna DA42. Time in x-axis as MJDsec.

Fig. 73 shows the $L_{sun}(t)$ contribution (solar illumination) to the WVR signal. Note that in line with Eq. (4.1.3) this is the ‘net’ illumination – for calculating its actual contribution to the WVR data it has to be scaled by atmospheric transmissivity that is specific for each WVR channel. The data shows abrupt drop by $\sim 20\text{K}$ after time 300 s (+MJD offset) and then recovery at $\sim 520\text{s}$.

Figs. 74 and 75 present the same as Figs. 72 and 73, respectively, now for the DV08 antenna (which has shown good correlation between WVR and dPWV proxy already in the differential calculation by Eq. (4.1.2) in Fig. 67). As it can be seen in Fig. 74, the PWV profile remain qualitatively the same as in Fig. 67 (blue curve), but now we have its absolute scaling in mm, including the mean-value offset to the fluctuations.

Fig. 75 shows less contribution to the solar illumination for DV08 than Fig. 73 does for antenna DA42, but, most notably, the illumination is much more steady, varying only in units of Kelvins.

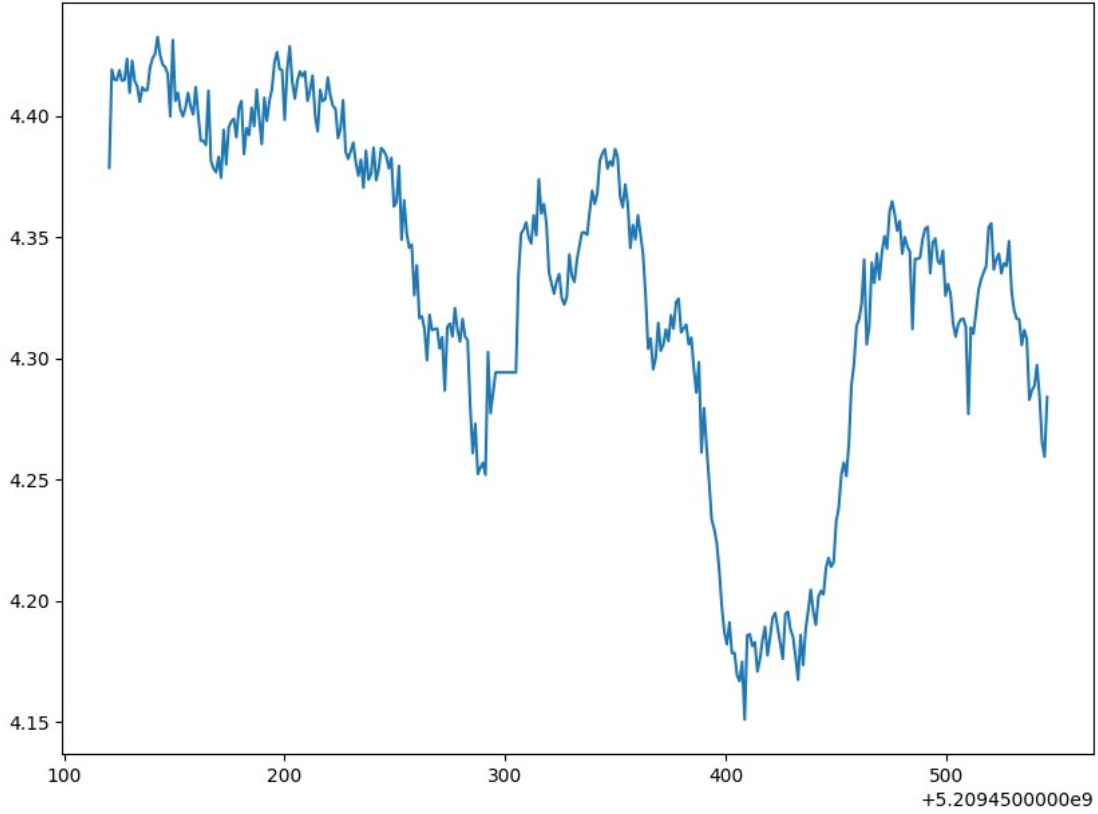


Figure 74: PWV profile for DV08, all notations like in Fig. 72.

The solution of system (4.1.7) – finding of the stationary parts $w_o = \langle w \rangle$ and $\langle L_{sun} \rangle$, and their fluctuations $dw(t)$ and $dL_{sun}(t)$, separately for WV emission and solar illumination, is implemented in $dWdLs()$ per antenna. In order to make comparison of PWV variations $dw(t)$ (just the fluctuating part), corrected by elimination of the variable solar illumination, with the dPWV-proxy calculated by Eq. (3.1.4) from the science SPWs for entire array, we have implemented a method $dpwvXsolWVR_raw()$, which iterates over all antennas (skipping the CM ones) and plots the comparisons the same way as displayed in Figs. 67 and 68 (these, were, actually, produced by an older version of the same method, which internally called solution to Eq. (4.1.2) for the arbitrarily scaled $dw(t)$). Results for antenna DA48 and DV07 are presented in Figs. 76 and 77. We have selected one of the DA antennas, which exhibited very weak or no correlation between WVR-based PWV variations and their science-SPW-based dPWV proxy counterpart in the previous approach (*Results 1*, without the varying solar contamination) and one DV antenna, which were qualitatively fine already with the simpler differential method, but anyway, they lacked the amplitude scaling (which is now solved). We admit that these

examples are one of best cases, but one can easily inspect the full dataset in online *Annex 5* to see that the correlation is now really very good for all antennas.

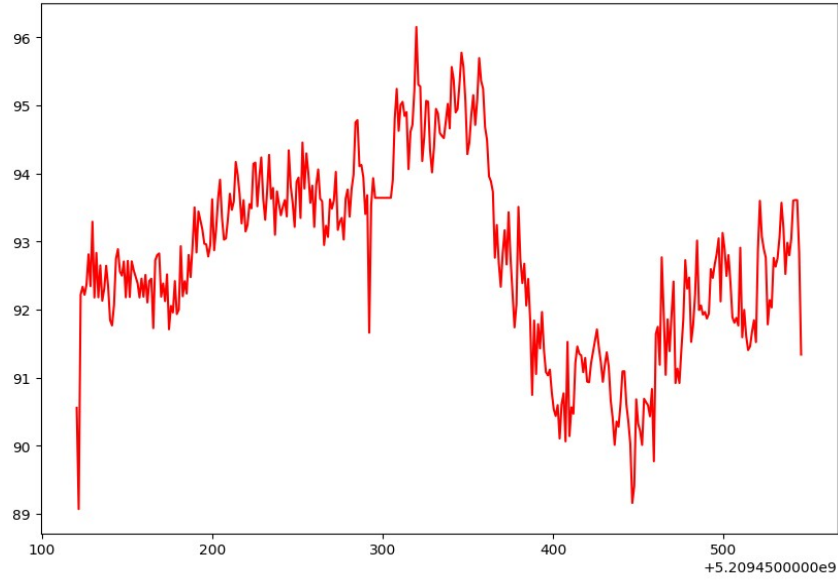


Figure 75: Solar illumination for DV08, all notations like in Fig.73.

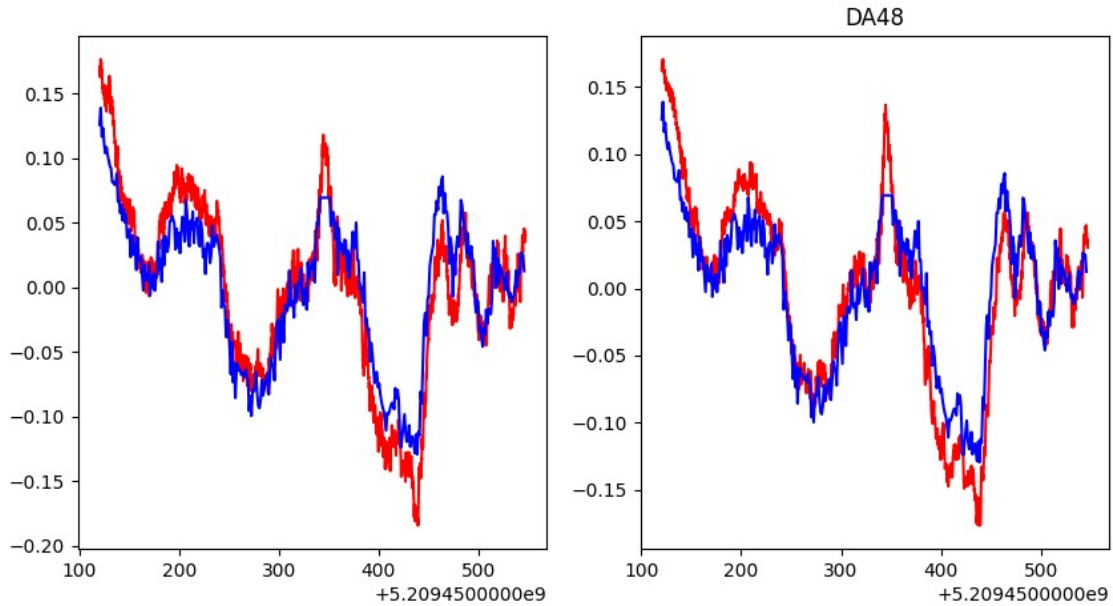


Figure 76: Antenna DA48: PWV fluctuations $dW(t)$ as per raw WVR data (blue curve, mm) and their $dPWV(t)$ proxy from the science SPWs (red, mm). The $dPWV$ proxy has XX and YY pols, for that sake we have two panels. Basically the same display as in Fig. 67, but now the blue curve: (i) has corrected solar variable illumination, and (ii) need not to be scaled to range, it has absolute scaling in mm.

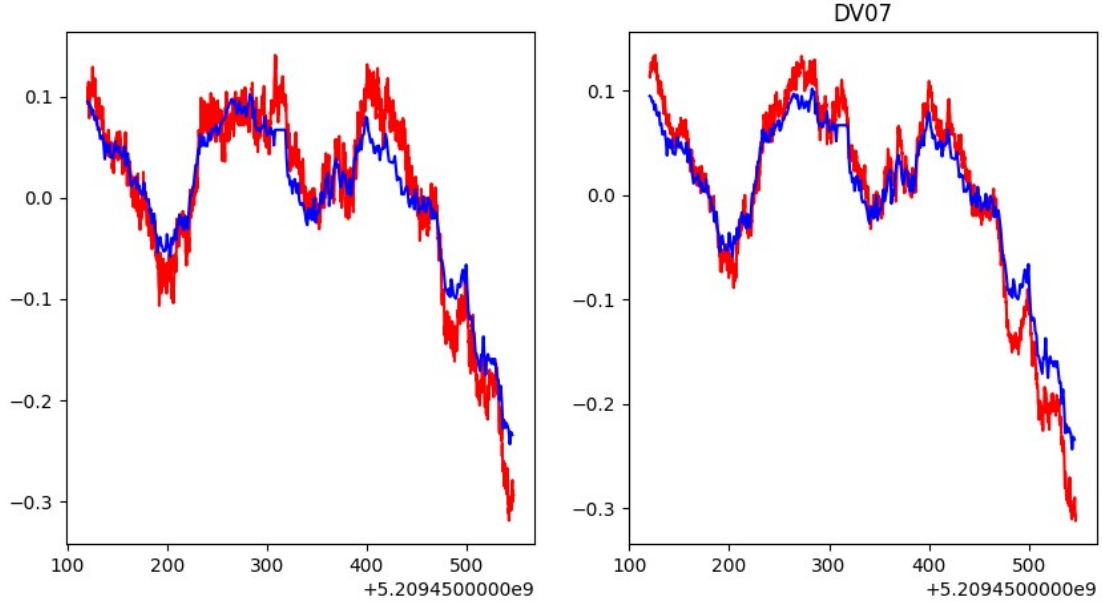


Figure 77: Like Fig. 76, now for antenna DV07.

2.4.1.5 Testing/Verification

In order to verify the method, we did some ‘extreme’ tests. First, we tried to detect the solar-contamination signal in the non-solar scan no. 5 (which observes the BP calibrator), where we, naturally, expect no solar contamination. Results for antennas DV08 and DV42 are shown in Figs. 78 and 79, respectively.

In particular, the DV08 result is nice, with very low mean around 3K. The DA42 antenna shows a bit higher contribution (instrumental?) around 18K, but anyway, both are pretty stationary with very low variations.

As another test we tried to make the solar illumination extraction from the WVR data for *entire* solar scan, i.e. including the first and last sub-scans that are off-source, and the including re-pointing of antennas between off- and on-source and back. This is actually beyond the assumptions for Eq. (4.1.7), because it was derived with neglected second order term $dw(t).dL_{sun}(t)$. This is not true in this case because the change in solar contamination is actually quite large (~ 90 K) during the re-pointing as can be expected already from the jumps in Fig. 73. The results are shown in Figs. 80 and 81. Fig. 80 shows clear rapid increase (and drop at the end of scan) of the solar illumination contribution, exactly as expected. Its value in the off-position is ~ 5 K. Whether it is an error or there is still some minimal solar illumination that affects the WVR signal (we are 2 degs from the solar disc center, i.e., 1.75 deg from the limb) is a question. Anyway, in aspect of identification of what part of the total WVR signal is the solar contamination, the method performs well. Fig. 81 then shows the corrected net PWV variation after the solar-contamination removal. In ideal case it would be a smooth (well, up to ordinary

fluctuations around 0.05 mm) curve, with no jumps. This is not really true – the re-pointing of the antenna is still visible in the data by the jumps of about 0.2mm PWV in the times when the pointing has changed. Anyway, taking into account that we are beyond the assumptions for the approach, it is still very good result – vast majority of the raw WVR signal has been removed into solar illumination jumps, without that, the PWV would drop by tens or hundreds of mm (with a usual coupling of the raw WVR data to ATM model, neglecting the variable solar contamination).

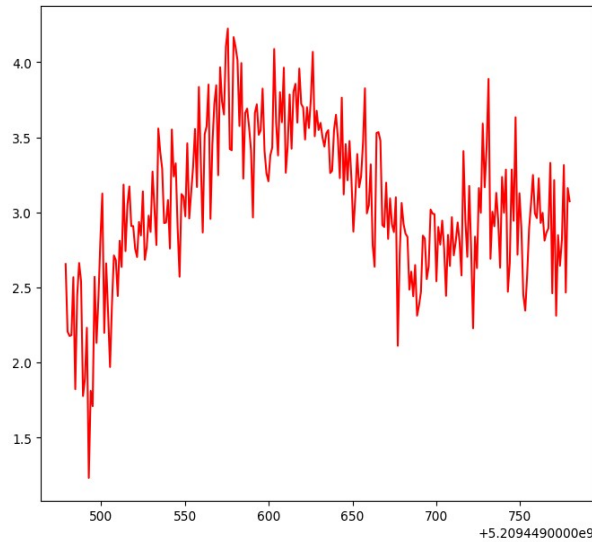


Figure 78: "Solar contamination" for the non-solar scan no. 5 (BP calibrator) at antenna DV08. Y-axis in K, time in x-axis as MJDsecs.

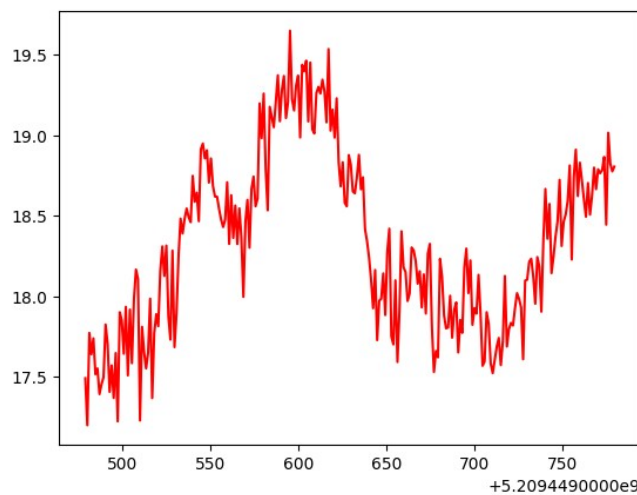


Figure 79: The same like in Fig. 78, now for DA42.

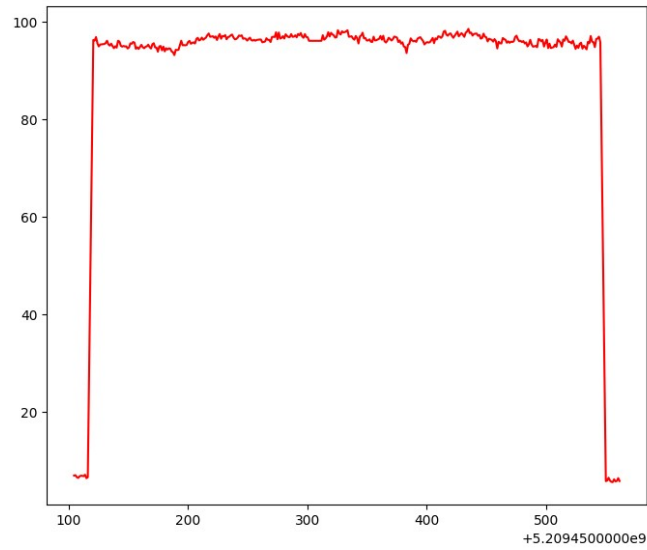


Figure 80: Solar illumination in Kelvins for entire solar scan 9 at antenna DV17. The jumps reflect antenna re-pointing between on-source and off-source positions.

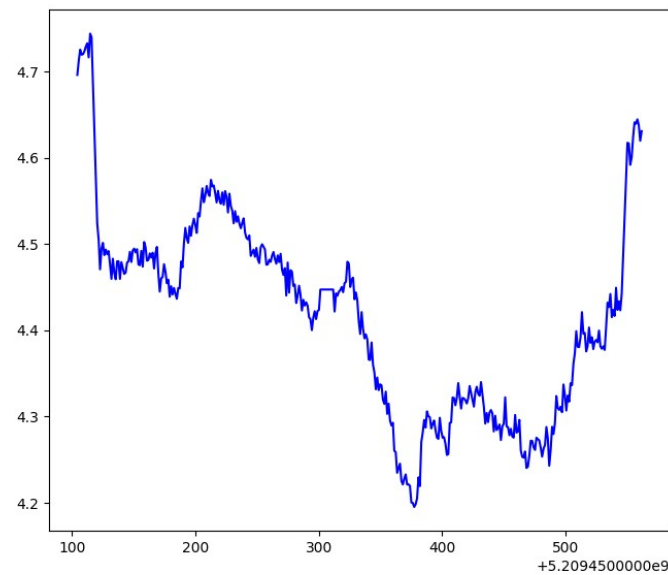


Figure 81: Net PWV time profile (in mm) at antenna DV17 for entire solar scan 9, after removing most of the raw WVR-signal jump into the solar illumination part in Fig. 80.

2.4.1.6 Our vs. SYSCAL PWV values

With the apparatus derived in Section 2.4.1.3 (“Theory 2”) we are essentially able to calculate the PWV values at any time, for which we have the raw WVR data. For the scans not directed towards the Sun we may neglect the solar contamination L_{sun} and use direct coupling between the $T_{wvr}(f)$ temperatures, as measured in channels of SPW 4, and the sky brightness temperature $T_{BS}(f, w)$ as calculated by the ATM model for various WVR channel frequencies f and varying PWV value w . Conveniently, we can do it in logarithmic representation of $T_{BS}(f, w)$ and use Eq. (4.1.4) in an iterative way, to find the PWV, where $T_{BS}(f, w)$ matches $T_{wvr}(f)$ in each WVR channel f separately. The three values of PWV from the WVR channels 1 - 3 (we dropped the saturated channel 0) are then averaged to obtain a single PWV value in any time instant. This is implemented in routine `pwwCalc()` in the `sol_wvr_cal.py` collection (Annex 4). We assume that the standard ALMA procedure for obtaining the PWV values from the raw WVR temperatures is similar, but due to lack of open documentation we are not sure about the details.

In addition to our PWV calculations, there are SYSCAL-provided PWV values available at some sparse time instants over the MS. They are accessible from the MS/ASDM_CAWVR table.

Since we are interested in relation between the raw WVR data and SYSCAL PWV, we have over-plotted their values over entire period of execution block into one display in Fig. 82.

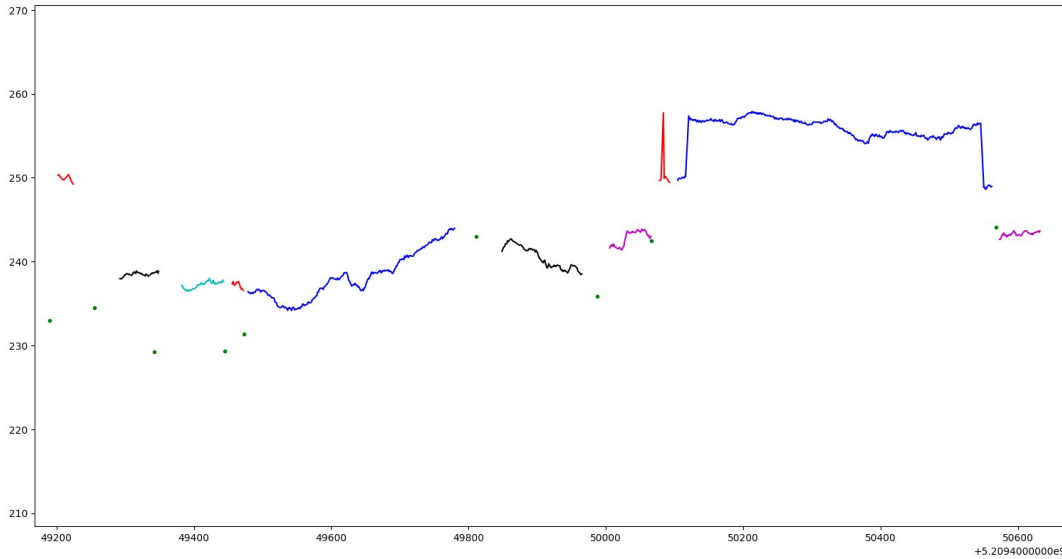


Figure 82: Raw WVR channel 1 temperatures (in K) as continuous curves of various colors for various scans (red - ATM scan, blue - BP and target, magenta - PhaseCal,...) over-plotted by arbitrarily scaled values of PWV, as read-out from the ASDM_CALWVR table – the green dots. All for antenna DV17.

We do not have ambition to use Fig. 82 for quantitative correlation of those two quantities, we are aware that other factors like air-mass (different for different scans) play role here. So we use the single

WVR channel (channel 1) and scaled the PWV data from the SYSCAL table by arbitrary factor (50x), just to boost them in the same range of ‘temperatures’ (around 240 K), having the green points in Fig. 82 close (in y-dimension) to the WVR curves. The purpose was to see, at what times (relative to WVR scans) are the SYSCAL PWVs evaluated. Apparently, they are evaluated at the edges of the scans or even in between them (which is a bit surprising for us).

Anyway, we have implemented a method *getPWVforAntenna(scan, antID)* in our collection (*Annex 4*), which for a given *scan* and *antID* takes the two SYSCAL-based PWVs that are ‘bracketing’ the scan in time, and using linear interpolation it calculates the PWV value interpolated to the central time of that scan. Then, we can use our method *pwvCalc()*, which calculates PWV from the raw WVR data at any time instant and extract the time average (over the scan) of those WVR-based PWVs for the same scan. This way we have approximately synchronized the SYSCAL-based and ours WVR-based PWVs, they are both located in the central time of the scan.

We can do it for all antennas (except of CM) and all scans that have non-contaminated WVR (all but the solar scan 9) and make a scatter plot between ‘our’ PWVs and the system-calculated (SYSCAL).

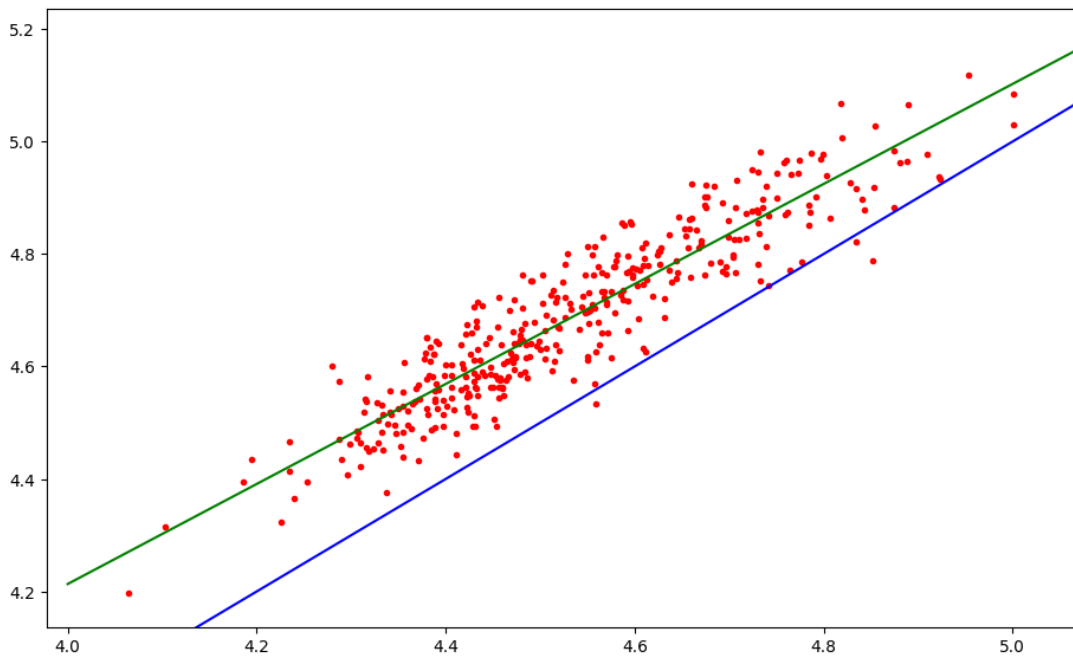


Figure 83: The red dots: PWV values obtained from ASDM_CALWVR (x-axis) vs. PWVs calculated by our method from the raw WVR data (y-axis), both in mm. The blue line represents ideal balance $y=x$, the green line is the best linear fit $y=0.89x+0.79$

This procedure has been implemented in *pwvCalcVerify()* and its result is shown in Fig. 24, together with the ideal-status line $y=x$ (blue) and actual best fit (green). It is clear that our PWV values are systematically overestimated (w.r.t. the system-defined PWVs) by $\sim 0.2\text{mm}$, but at the same moment our slope is lower (~ 0.9) which means that larger fluctuations of T_{wvr} would map to lesser fluctuation in PWV than if the system was capable to measure them for any time instant as well.

We were a bit dis-certained by this finding and have tried explanation for it. We have re-checked the procedure and found three minor issues in our methodology:

1. We have left the default value $H_w=1.0\text{km}$ for the height scale, with which water-vapor concentration decreases with height. After checking literature (Stirling et al. (2006): ALMA Memo 541) we have corrected it to more typical value for Chajnantor $H_w=1.5\text{km}$.
2. We have noticed that the four WVR channels in SPW 4 are of uneven widths (ranging from 1.5 to 2.5 GHz, which we have not realized before). We have corrected the ATM model for SPW 4 and the procedure *pwvCalc()* accordingly.
3. We should include the forward- and rear- spillover antenna efficiencies for the WVR SPW 4 data into radiative-transfer model, as we do for the science SPWs.

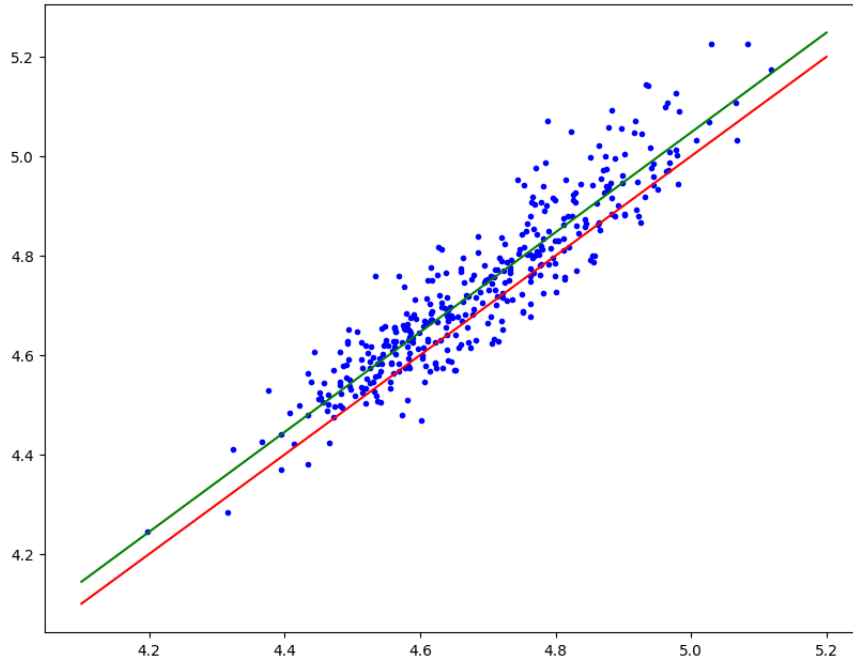


Figure 84: Improved correlation between PWV values (i) obtained by *pwvCalc()* from the WVR data in SPW 4 and (ii) values read out from *ASDM_CALWVR* table after corrections in procedure. Blue dots: our WVR-based PWVs (y) vs. SYSCAL PWVs (x), red line: $y=1.0 \cdot x$, green line: regression with $K=1.004$, $Q=0.027$.

As can be seen from Fig. 84, the correlation between the PWVs, as measured by our method implemented in *pwwCalc()* after incorporation of the fixes 1 – 3 identified above, and the SYSCAL-provided values, has improved a lot and basically the values are statistically the same: The relation is expressed by linear regression $PWV_{wvr} = K \cdot PWV_{syscal} + Q$, with regression coefficients $K=1.004$ and $Q=0.027$ (reminder: before these corrections, the regression coefficients were $K=0.89$ and $Q=0.79$).

2.4.1.7 Further development – methodological convergence to the PWV-proxy calculations

As it can be seen from governing equations for the PWV-proxy calculations and for real PWV extraction from the raw WVR data contaminated by solar illumination – i.e., Eqs. 3.3.4 and 4.1.7, both approaches are formally actually very similar. Hence, inspired by the PWV(t)-proxy calculations, developed to a higher level in Section 2.3, we have implemented similar upgrades also to the extraction of PWV from the raw WVR data. Namely, the procedure *dWdLs()* has been consecutively improved by inclusion of

- non-linear solver via Jacobian matrix (mathematically rigorous approach),
- antenna efficiencies, and
- correction of calibration – with having the WVR data already in brightness temperature scale, the instrumental factors – the gain G and instrumental offset P_{inst} in WVR equation analogical to Eq. (3.3.17) are to be set to $G=1.0$ and $P_{inst}=0.0$ in “ACD calibration equations”. We may, however, relax from having exactly those values and let the instrumental parameters slightly varying.

Resulting implementations (via consecutive upgrades) are in routines *dWdLs2()* and *dWdLs3()*, the latter one including the idea of “floating calibration”. Essentially, they do not differ much from the *dWdBdG()* used for science-SPW PWV(t)-proxy calculations – the main difference is in just 4 WVR channels without sub-division (no counterpart to 128-channel AC SPW) and lack of polarization information. We may say that the mathematical procedure for finding PWV(t) in presence of varying source brightness (whether it is real variation, scintillation or time-dependent scattering at antenna construction) from science SPWs and from the WVR raw data converged together.

2.4.2. WVR vs. sci SPW-based PWV: Specifically designed test observation

As mentioned in the introduction to this part of the study, in order to verify the methodology for PWV(t) proxy-calculations from the science SPWs against the standard WVR data in solar case we need a specific test observation, where part of the antennas are observing slightly off-source in order to enable unsaturated WVR data for the solar scan. The main advantage that motivated design of this test lies in possibility to avoid calculation of the solar illumination/contamination, which brings additional uncertainty to the entire procedure as presented in the previous Section 2.4.1. We start with observation design, and then we are going to describe the actual performance and results.

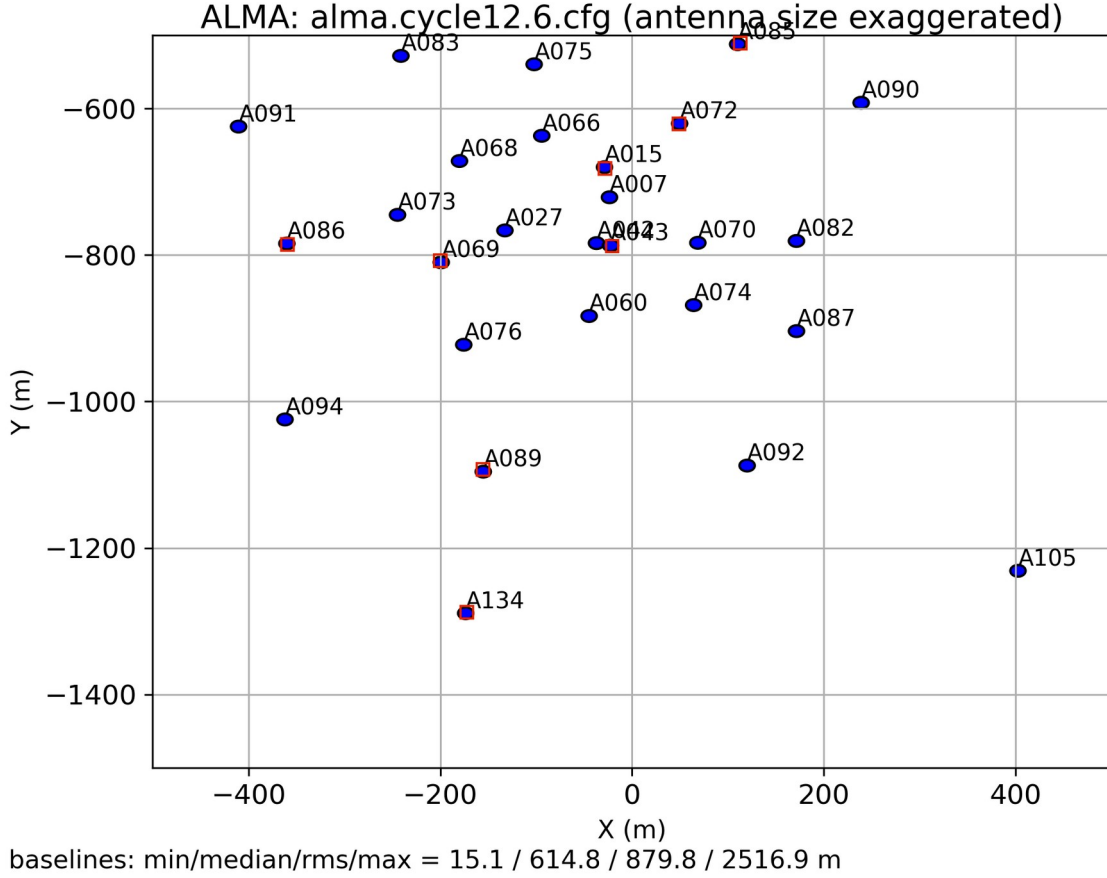


Figure 85: Zoom to the central part of the ALMA 12m array in configuration C-6. Red squares show the antennas that are not present for solar imaging – either deliberately, in order to make off-source WVR measurement or because of technical issues/flagging.

2.4.2.1 Setup of the observation

In order to assess performance of the PWV proxy for predicting the actual water-vapor variations in solar case, we need to compare them to the directly measured WVR signal. Both on-source and off-source signals need to be collected ideally from the same air column. This is not possible to be done exactly, but in order to approach the ideal situation, we identify close antenna pairs in the 12m array, which are located rather in the central part, as we do not want to lose our high resolution. One antenna in the pair is observing the Sun, the second is pointed to the sky a bit off-source. There is also one antenna pair, where both antennas point off-source but into different places at the sky. Still they keep pointing separation the same as the Sun-sky antenna pairs do. This is required as a ‘control sample’ for precluding the false-negative result: If PWV-proxy signal is very different from the WVR in the Sun-sky pair, the reason may be both in the fact that the PWV-proxy approach fails for the solar observation, but alternatively also because the exactly same air column (which we cannot have) is necessary for reproducing the WVR signal. If the control sky-sky pair shows (approximately) the same

WVR signal in spite of antenna-position and target-pointing separations, and the Sun-sky pair heavy discrepancy between PWV-proxy (on-source) and WVR (off-source), the situation is clear – only the first option (i.e. fail of the approach for the solar scan) remains valid.

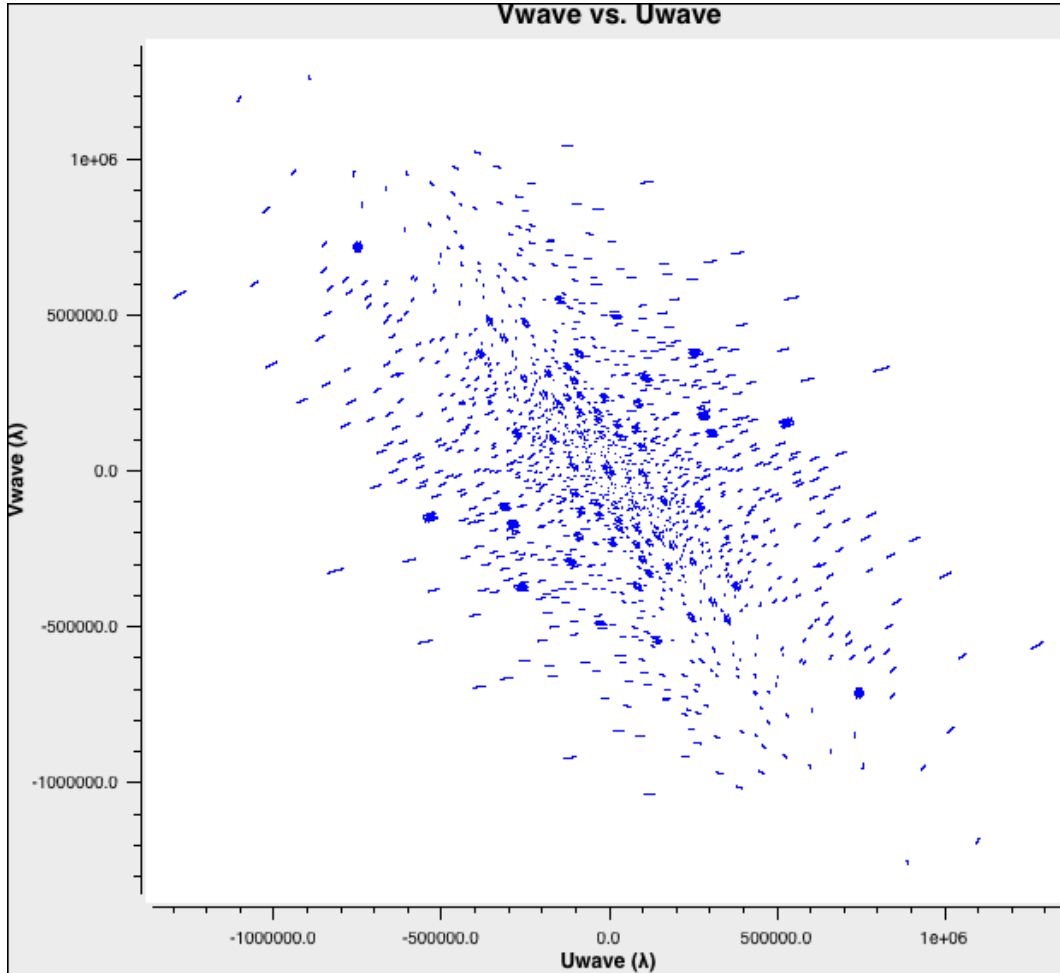


Figure 86: The uv coverage for the C-6 array combined with the ACA, including the mixed baselines. The antennas marked at the previous figure have been removed from calculation.

We would like to have at least two Sun-sky pairs for the case that one antenna fails – even bad T_{sys} is killing for correct finding of the dPWV, as explained above. On the other hand, the high-resolution observations are demanding on the total antenna count as the uv coverage is the critical factor for successful imaging – our simulations presented in the MTR have shown that if array configuration is extended just by one step above the limits found in this study (e.g., from C-6 to C-7 for Band 5), we are not able to reconstruct the image even for the case of zero-pressure atmosphere. In order to assess, how many antennas may be taken from the imaging and used for the off-source WVR measurement, we performed simulation as in the Section 2.1 above, but now with reduced set of antennas (the ‘WVR antennas’ taken away).

The first run has been done with total 8 antennas removed. This is the total ‘non-presence’ count, for any reason – WVR measurement as described here, but includes also antenna-not-ready or flagged cases. We used configuration C-6 and the antenna positions are identified by their pads name. The

antennas removed from the set has been located at pads A015, A043, A069, A072, A085, A086, A089, and A134.

Figure 85 shows a zoom-in picture of the 12-m main array configuration. Since we want to keep long baseline for the uv -coverage, the antenna picked-out are residing in the central part and they are marked by a red square.

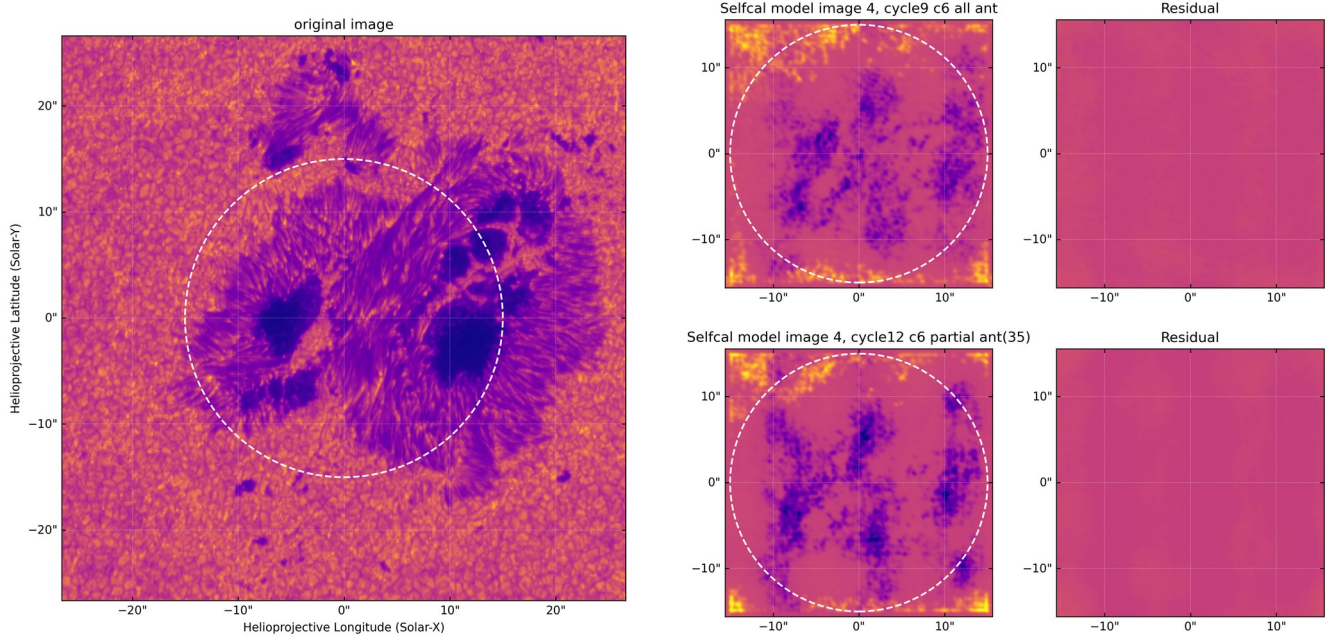


Figure 87: Simulation of the solar imaging in Band 5 in complete vs. reduced configuration C-6 using the phase-screen model for the atmospheric turbulence. The RMS over 1 min reference interval has been set again to 30 deg, as in the Section 2.1. The bottom right panels show reconstructed image and its residual in the reduced array (8 antennas missing) in comparison to their counterparts in the complete array. The ‘ground truth’ initial model is in left.

The corresponding uv coverage for C-6 configuration, including the combined 12m-7m baselines, but lacking the removed 8 antennas on the pads listed above, is shown in Fig. 86.

Finally, Fig. 87 shows the comparison between the fully self-calibrated images from this simulation and the one that uses the entire array (Cycle 9 configuration C-6). It can be seen that the quality of the image dropped significantly with the 8 antennas removed in comparison with the full array. In order to find still acceptable balance between the imaging quality and sufficient number of antennas we have run another simulation with just 5 antennas removed. This is more or less marginal case as we need at least 3 antennas for the WVR observation as described above and 1 – 2 antennas are typically flagged (for various reasons) in the solar datasets. As can be seen in Fig. 88, the image quality is, nevertheless, much better in this case.

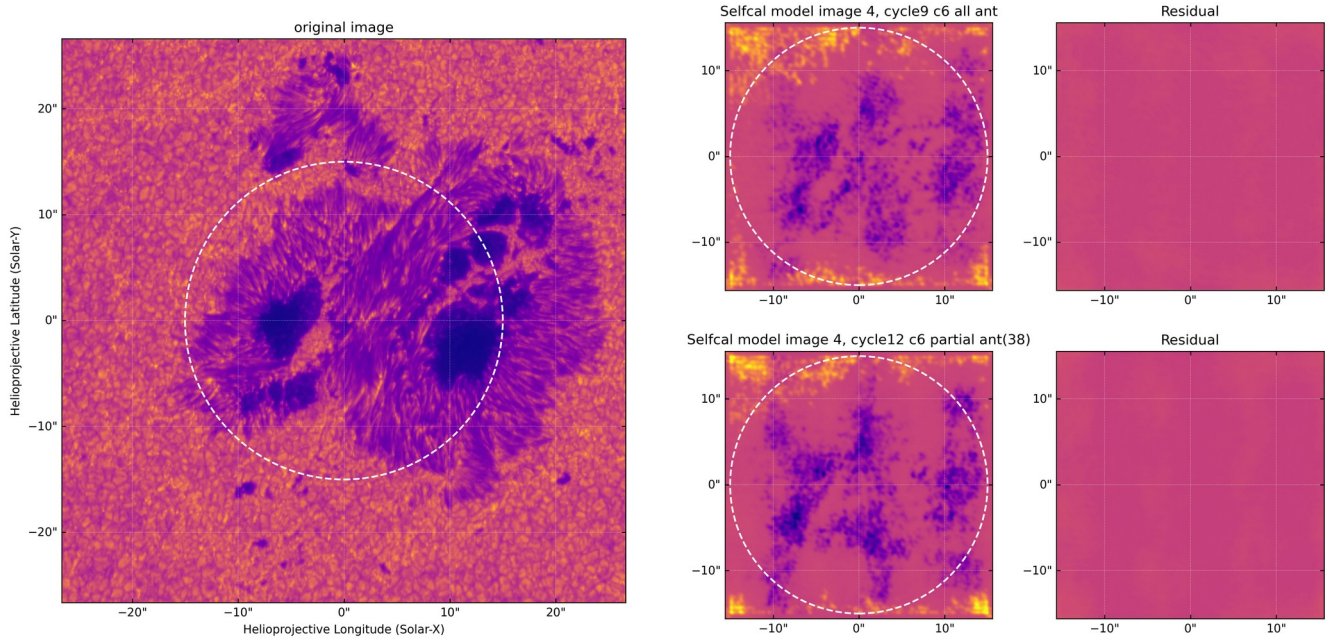


Figure 88: Like at the figure above, now with just five antennas removed from the main array.

2.4.2.2 The observations

The observation setup has been prepared in cooperation with our advisory team, P2Gs and AoDs and submitted as the project 2025.1.0005.CSV. The documents for scientific reasoning for the test, the AOT files, ephemerides, etc., are available either as the online *Annex 5* in the Development study data repository at http://wave.asu.cas.cz/shared/ESO/final_report or at the JIRA ticket for the specific test project 2025.1.0005.CSV at <https://jira.alma.cl/browse/CSV-4018>.

The first attempt has been done in Winter 2025 during configuration C-5. Nevertheless, setting up the sub-arrays for synchronous observations took some time (and failed attempts). The procedure is demanding for coordination because the solar-target ephemerides are only short-living and we would like to thanks once more the P2G team and AoDs for kind cooperation even during the Christmas break. Eventually, all has been ready in January 2026, when ALMA has already moved to more compact configuration C-3 and moreover, the weather was really bad. Anyway, the test has been performed as ‘technical’ under non-observing conditions (PWV 10 – 12 mm, rain) just in order to demonstrate viability of the co-temporal observations of two different targets in two sub-arrays.

The test performed well and the procedure has become the basis for the already scientific-merit test planned for next C-6 configuration in Summer 2026. What is remarkable, that even though the non-observable weather with the rain (we were able to detect it as increased absorption gradually covering the array as time progressed), we were still able to get the images of the Sun! As it is not of direct interest for this Study, we present the results just on-line – interested reader may find a brief descriptive text, QA2 report, and the calibrated (just standard way) and self-calibrated movies from this observation in Repository http://wave.asu.cas.cz/shared/ESO/final_report (*Annex 5*).

The second execution of the project (2 concurrent SBs – one for the solar target, one for the WVR off-the-Sun sub-array) took place on July 30th, 2026 under much better conditions. We have reached ALMA in desired configuration C-6, the weather conditions were excellent: PWV 0.6 – 0.9mm. The *listobs()* output and QA2 report can be found in the Repository http://wave.asu.cas.cz/shared/ESO/final_report, again.

In order to keep as much close air-mass as possible between the on-source and off-the-Sun (WVR) measurements (we wish to minimize the influence of different air column between the two synchronously executed SBs), specific ephemeris for the WVR SB has been prepared with offset of 2 degs from the Sun solely in azimuthal direction. For that sake, a routine *off_ephem.py* for generation of this specific ephemeris has been devised (with use of CASA *measures* toolkit and *Analysis Utils*). It uses the solar-disc center ephemeris from the *Solar Ephemeris Generator* (Skokic et al., 2015) as an input, transforms to the local-ALMA Az/El frame, adds the offset in azimuth, and transforms back to RA/Dec. It can be found in the code listings attached as *Annex 4*.

2.4.2.3 Results of the July 2026 observations

The July 30 observation has been executed under exceptionally (for the solar observing standards) good weather conditions with PWV in a range 0.6 – 0.9mm. Even with a diminished number of antennas (47 in 12m+7m mixed array, one of them being flagged during calibration/QA2) the imaging went smoothly and self-calibration loop ran with minimum failed *gaincal()* solutions. The results - QA2 report, MS listings, and, above all, self-calibrated solar movie with high resolution – are available in the online Repository at http://wave.asu.cas.cz/shared/ESO/final_report. Having in mind the main purpose of the test, we focus here, in the main text of the report, to the question of how much the PWV(t)-proxy time series (calculated from signal in science SPWs) correspond to PWV derived from the direct WVR measurements.

Comparison of direct and indirect PWV measurements using two concurrent subarrays

The key purpose of this test was to investigate, whether our PWV proxy as calculated from differential absorption of signal in the science SPWs (reminder: the most sophisticated method for this purpose, *dWdBdG2()*, decomposes the signal variations to parts attributed to PWV variations, scintillation, and intrinsic change of solar brightness temperature, and is even capable of few-percent corrections of the instrumental gains and power offsets) indeed corresponds to the PWV directly measured by WVRs. Here we present the results of direct comparison between direct WVR measurements obtained by off-the-Sun observation with Array 2 (antennas DA50, DA52, DA62, and DV03) with the PWV-proxy measured by Array 1 during the on-the-Sun target observation. Moreover, we add the WVR measurements of Array 1 for the (sub)scans, when the Array 1 was not pointed to the solar target – namely leading and trailing phase-cal scan, ATM scan preceding the solar scan, and the two short target sub-scans at the very beginning and very end of the solar scan, which are pointed 2 degs off-the-Sun.

First, we had to handle the problem with different than expected configuration (population of the antenna pads with antennas differs from our expectation in 2.4.2.1) of the Array 1. To that end, we have prepared an Excell table (available online in *Annex 5*) with antenna positions as per *listobs()*. The four antennas of Array 2 (off-Sun WVR measurement) are in the upper part, the lower part is composed by Array 1. For each antenna of Array 2 we have calculated distance to each antenna in Array 1 (right

columns) and perform sorting by that distance. Tables of the antennas of Array 1 closest to each antenna of Array 2 are in a separate sheet (per Array2 antenna) and the respective column of mutual distances is highlighted. This way we have found that, for example, the closest on-Sun antennas (Array 1) to the antenna DV03 in Array 2 (off-Sun WVR) are antennas DA42 (separated from DV03 by 38 meters) and DV01 (at 45 m distance).

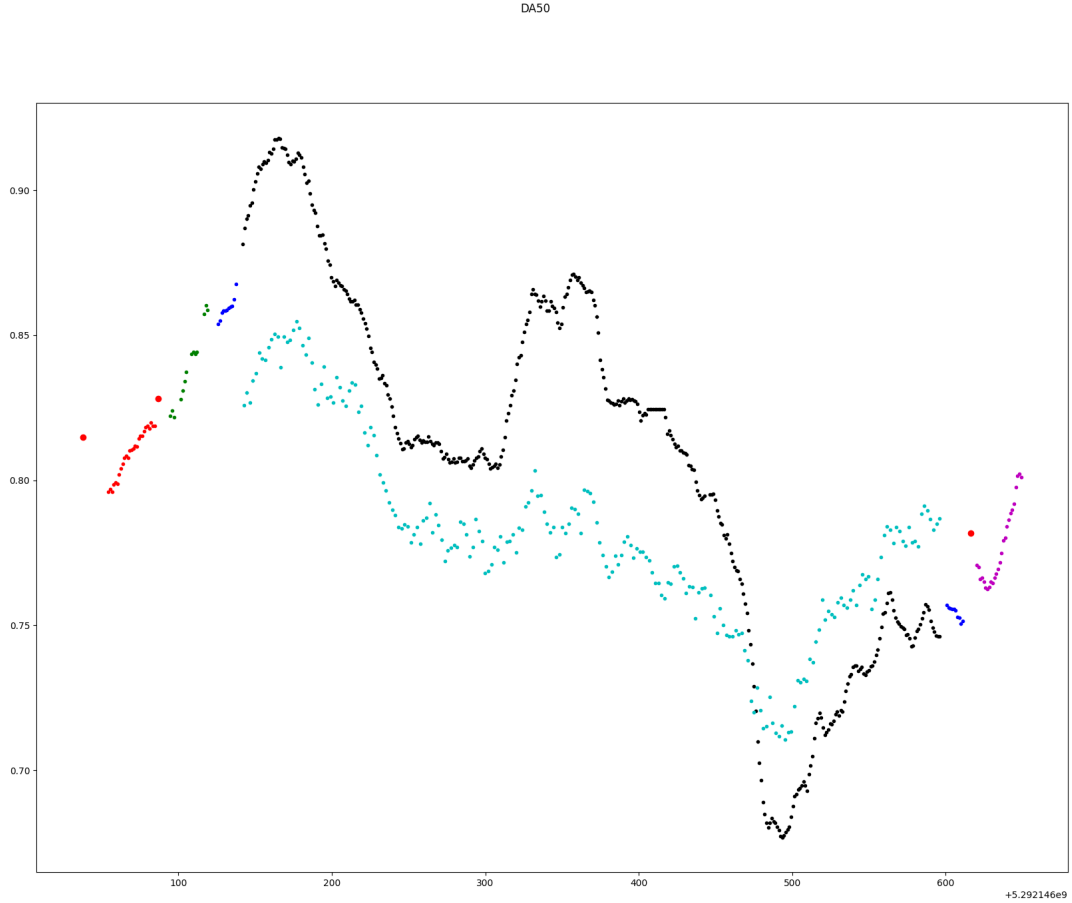


Figure 89: PWV curve (time in MJDsecs, PWV in mm) for four consecutive scans around the target scan. See description in the text.

In the following we present comparisons between selected Array1 – Array 2 pairs. Fig. 89 shows the PWV as calculated by *dWdLs3()* method in our collection (see *Annex 4*) from direct WVR measurement at antenna DA50 in the Array 2 – i.e. pointing 2 degs in azimuth off-the-Sun. The *dWdLs3()* method represents an improved procedure for inferring the PWV value from the raw WVR temperatures, which (inspired by our proxy calculation *dWdBdG()*) accounts for possible little (small units of percents) inaccuracies in determination of WVR brightness temperatures from the very primitive WVR data. The curves shown by different color codes as small filled circles are PWVs (in mm) for the following (sub)scans, respectively: Red = leading phase-cal scan, green = the ATM scan, blue = leading and trailing off-source sub-scans of the target scan (specific for solar observations),

black = the on-source sub-scans of the target scan (reminder: the target ‘on-source’ for Array 2 is an ‘empty space’ 2 degs apart of the Sun), and magenta = the trailing phase-cal scan. Time in the x-axis is in MJDsecs. The total PWV curve over all four scans is nicely (almost) continuous, the very little jumps may be attributed to the fact that we are pointing to slightly different positions at the sky (phase-cal is ~ 5 deg separated from the Sun). The three distinct bigger red circles (at times approx. 50, 80 and 620) are PWVs as read directly from the ASDM_CALWVR table and are added for comparison.

Even though we are pointing out of the Sun, we can still use our proxy method $dWdBdG()$ for estimation of PWV from the science SPWs: We did the same job at the early phases of this study with band-pass calibrator data. In this case, the water vapor manifests itself by emission and because the science SPWs are rather far from the WV line center, the estimation has bigger error (as explained in Section 2.2). This proxy-based PWV curve, still for the same antenna DV50, during the ‘on-source’ sub-scans is presented in the same figure as the cyan curve.

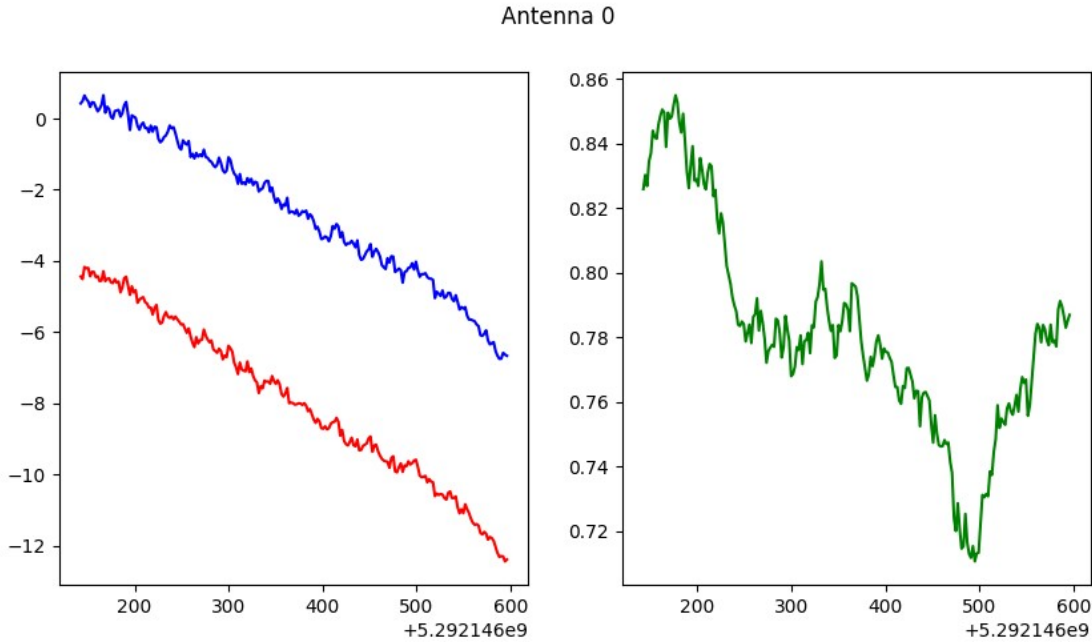


Figure 90: DA 50 (Array 2/off-the-Sun). Right: PWV (in mm), determined by our proxy method from the science SPWs, as a function of time (MJDSec). The green curve is identical with the cyan curve in Fig. 89. Left: The ‘by-product’ of the proxy method $dWdBdG()$ is the decomposed solar contribution to the total signal. It is close to zero, as expected for the off-the-Sun observation by antenna DA50, despite we started our iterative solution with the normal solar temperature 6700K. Curves: red XX, blue YY polarization, values in Kelvins.

Albeit the large uncertainties in calculation of the PWV from the emission in science SPWs (we subtract two almost identical numbers, loaded by errors, and finally make a ratio of two such uncertain differences), the cyan proxy-based curve reproduces the WVR-based PWVs (black dots) surprisingly well. In order to check, how much our proxy method $dWdBdG()$ is robust, we show another representation of the PWV-proxy curve, now alongside with the decomposed varying solar-brightness

contribution. The results for the same antenna DA50 are shown in the next Fig. 90. The right panel just reproduces the cyan curve from Fig. 89. On the left we see the decomposed solar-brightness contribution. As we are pointing 2 degs off the Sun, we expect it should be close to zero. And, indeed, it is. What is noteworthy on the result? We have started our iterative solution with expected solar brightness temperature of 6700K. Anyway it converged close to zero, demonstrating how robust the procedure is.

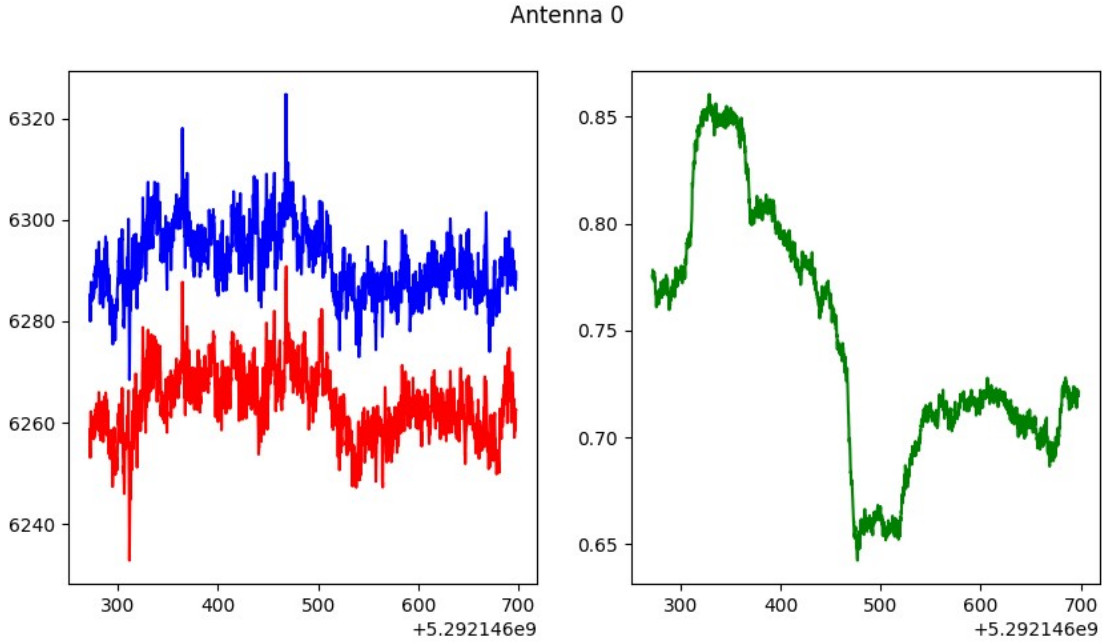


Figure 91: Like in Fig. 90, now for on-the Sun pointing antenna CM01. PWV-proxy (the green curve) corresponds quite well to direct WVR measurement in neighboring antenna DA50 – see the black curve in Fig. 89 – when the black curve is watched at the same time (start at approx. 270 MJDsecs).

Now, we can compare these PWVs obtained directly from WVR at antenna DA50 pointing off-the-Sun with the close antenna in Array 1, pointing to the Sun. Per Excell table sheet DA50 (*Annex 5*), the closest antenna in the Array 1 is CM01, separated from DA50 by 31 m. It does not have WVR measurement, but for our purpose it is irrelevant – we are comparing the real WVR from DA50 with sci-SPW PWV-proxy of the antenna CM01. The proxy-based PWV (right panel) together with decomposed solar contribution to the total sci-SPW signal (left) is shown in Fig. 91. Now we are pointing to the Sun, so the target brightness temperature is correctly $\sim 6300\text{K}$, varying on short time-scale most likely because of (tropospheric) scintillation. As it can be seen, the green PWV (proxy @CM01) curve shows, indeed, similar features as the black PWV curve (direct WVR @DA50), when we take into account the time-shift $\sim 2\text{min}$ between execution of the SB1 and SB2 for on- and off- the Sun observations.

In order to see the correlation yet better, we put both measurements into a single figure in Fig. 92. The filled-circle curves (red, green, blue, black, and magenta) are – as in Fig. 89 – the PWVs as per direct

WVR @DA50 (off the Sun). Empty cyan squares represent sciSPW-based PWV proxy @CM01 (on the Sun; basically the green curve from Fig. 91). Finally, the yellow empty squares show sciSPW-based PWV proxy as (inaccurately) estimated from WV emission @DA50 (off-the-Sun; basically the green curve from Fig. 90). Correlation or at least strong similarity between the curves is obvious.

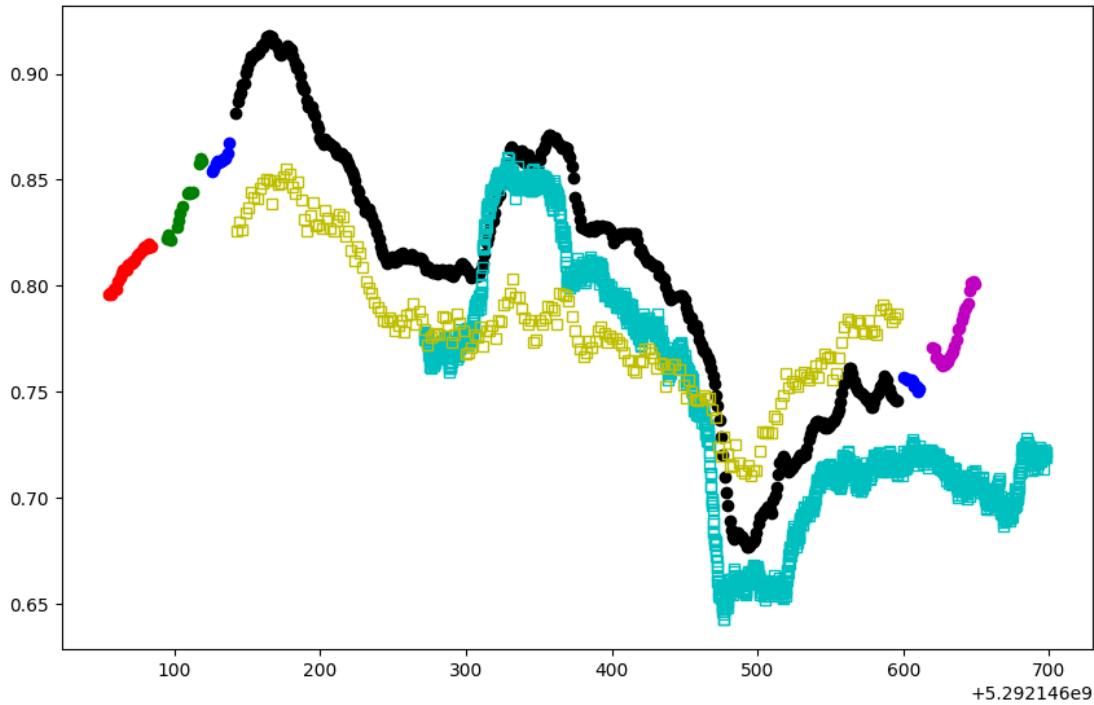


Figure 92: Composed Figs. 89 and 91 (right panel): Comparison between WVRs @DA50 (off Sun) and PWV proxy @CM01 (on Sun). See details in text.

In order to assess the difference between the WVR at Array 2 (off-Sun) antenna and PWV-proxy at its partner in Array 1 (on-Sun), e.g., the difference between the black and cyan curves in Fig. 92, we would like to address yet another question: To what extent the PWV curves between the two antennas in a pair are different as a consequence of the antenna mutual separation. To that sake we need a close pair of antennas that are both equipped with WVRs. The Excell table with antenna positions shows closest pair of such antennas to be formed by DV03 (Array 2, off-Sun) and DA42 (Array 1, on-Sun), separated by 38m. We have done very similar composed figure to Fig. 92, but now it is enriched by the WVR measurements of the DA42 antenna for all the scans and sub-scans, which are not pointed to the Sun. The results are presented in Fig. 93.

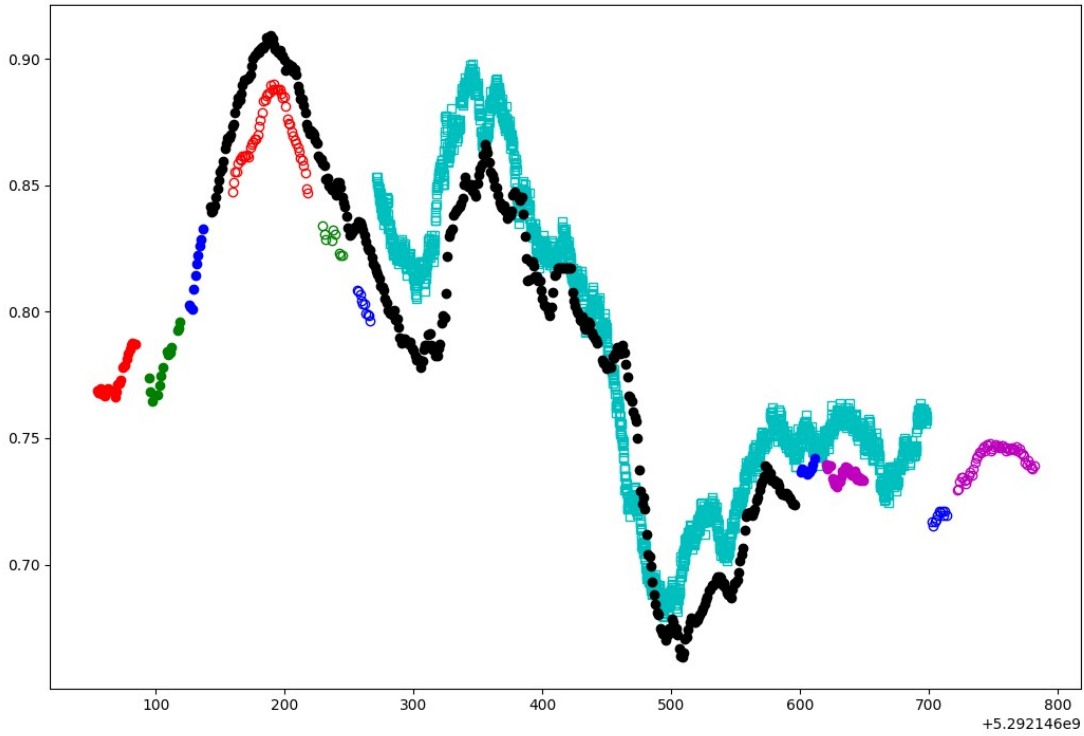


Figure 93: Comparison of WVRs and PWV-proxy for antennas DV03: off-the Sun , filled symbols and DA42: on-the-Sun, empty symbols. See the details in the text.

Meaning of the curves is as in Fig. 89, but now we have two sets: Filled symbols are for antenna DV03 (off-Sun) and empty symbols for antenna DA42 (on-Sun). The red, green, blue, black and magenta are, again, WVR-based PWVs for leading phase-cal, ATM, tgt-off-source, tgt-on-source, and trailing phase-cal scans/subscans. For antenna DA42 (on-Sun) we clearly do not have black (empty) symbols, because the on-source subscan has saturated WVRs. The empty-squares cyan curve is the PWV proxy calculated from WV absorption in the science SPWs at antenna DA42. Again, similarity of the curves is striking. Moreover, now we have time overlap between direct WVR measurements by both antennas DV03 and DA42 in the period of $\sim 150 - 270$ MJDsecs, so we can compare the difference between the PWVs to be attributed not to the different methodology (direct WVR vs. sciSPW proxy) but to different antennas, and, namely, their separation (different atmosphere in the ray-path). Basically, they are of the order of ~ 0.05 mm of PWV, which is very similar to the difference between the black (WVR @DV03) and cyan (PWV-proxy @DA42) curves. So, in this sense, the proxy gives the same result as WVR within the uncertainty caused by scanning different atmospheric mass.

Summary: Results of the test demonstrate reliability and robustness of the sciSPW-based proxy method for the PWV calculation. We got similar PWV curves by both methods (direct WVR vs. proxy), the difference between them is not significantly higher than the difference between two direct WVR measurements at two mutually separated antennas. The correlation works surprisingly well even in this

extremely (for the solar observations) low PWV dataset (with median PWV less than 0.8mm). Together with our older comparison of PWV proxy vs. direct WVR carried out at the Sun-contaminated but still valid WVR data of the December 2023 observation (with – in contrast to the current dataset – quite a high PWV) we may say that the method is usable along a broad span of PWVs (0.8 – 5.0mm).

2.5 Application

In the previous sections we have developed method for calculation of the PWV(t) time series from the signal present in science SPWs in Band 5, implemented it as a library of Python routines, and – in our humble opinion – quite convincingly demonstrated that the PWV calculated by this method is tightly correlated to the PWV inferred from the WVR measurements. Now, we are just the last step from reaching the final goal – making use of the results for the phase corrections, i.e. , providing the solar counterpart to *wvrgcal()* gain-calibration method. We are going to make this last step in this section.

2.5.1 Calculation of the optical-path dependence on PWV for different frequencies (dispersion).

Just a brief re-cap: Insofar we have used the factor of -25.0 for recalculation of the PWV variations in mm into phases in radians (see, e.g., Section 2.2 above). It has come from the factor between PWV and the additional optical-path: 6.7 – this value has been reported by Richard Hills in his original intro material (Annex to MTR). It is then scaled by relation between the optical path and the phase, which for the wavelength about 1.57 mm (190GHz), gives, indeed, a combined factor of approx. value of -25.

We would like to first verify that the scaling factor is correct and sufficiently general over entire pass-band of the Band 5, and second, provide more exact routine for recalculation of the PWV to the phase delay. To that end we have implemented a new routine *wv2path()* in *sol_wvrgcal.py* collection. The method internally calls *at.getDispersivePathLength()* and *at.getNonDispersivePathLength()* in a loop for all channels of our science SPWs. We have been in particular interested in the dispersive part – i.e. whether the factor 6.7 can be really considered as a constant over frequency domain of Band 5 – the general knowledge says that the refractive index of dielectrics has an irregularity at ‘resonant’ frequencies, where the absorption occurs – known as Kramers-Kronig (KK) relations, so our intuition tends to contradict the statement.

Result: Indeed, the non-dispersive part is constant over SPW 5 and its value is 6.677e-3 m per 1 mm of PWV. The dispersive part is very slightly changing around its average value of 1.0e-4 m per 1 mm PWV and thus, represents merely a negligible correction. We are probably too far from the line center so that the KK-relations have no significant effect on the refractive index. Both paths (dispersive and non-dispersive) have been found to depend linearly on the PWV, so their effective (per 1 mm PWV) value does not depend on the PWV parameter in the *wv2path()* routine. To sum up: A constant factor 6.7 as a scaling between PWV and additional optical path is appropriate.

Anyway, for the correct calculations we switch to the more exact approach, which respects (a small, but anyway some) dispersivity and also the dependence of the phase not only on the optical paths but also on the wavelength (which, is, again, quite small as the total bandwidth of Band 5 covers 16 GHz; at

$f_{LO}=198\text{GHz}$ it represents variation in the interval $\pm 4\%$). Namely, we use our *wv2path()* method for calculation of the phase delays for 1mm of PWV and then we sum both contributions (dispersive + non-dispersive) to find the frequency-dependent phase factor $PF^{spw}(f)$ defined in Eq. 20. Its value, indeed, varies between -25.0 and -29.0 over entire range of all four SPWs, so the initial approximation -25 is not as bad. Because *applycal()* takes care about frequency dependence of phases within a single SPW, the gain calibration tables (*G*- or *T*-type) are not frequency dependent (just on the SPW, but not on their channels). For that reason we calculate the $PF^{spw}(f=f_c)$ at the central frequency of each SPW, having just a single number representing the SPW.

These *PF* factors are subsequently multiplied by the actual PWV(t) as obtained by solution of the RT problem using the most sophisticated *dWdBdG()* method. This way we can calculate the absolute atmospheric phase delay for each antenna

$$\varphi(\text{ant}, \text{spw}, t) = PF(\text{spw}) \cdot PWV(\text{ant}, t) \quad (20)$$

If we subtract the value for the reference antenna, we get the relative phase delay w.r.t. the reference. This value can be in the form of complex gain $\exp[i(\varphi_{ant} - \varphi_{ref})]$ already written into the calibration table.

2.5.2 sol_pwvGcal(): The solar counterpart to wvrgcal()

The above-described procedure is implemented in the new routine *sol_pwvGcal()*, which may be considered as the replacement of *wvrgcal()* for the solar scans. It copies the header and entire structure of the WVRGCAL table without its data (using *norows=True* in *tb.copy()*) and subsequently fills its columns by relevant data – times, antenna_1 IDs (antenna_2 is the refant) and the complex gains calculated as complex units (i.e. just the phase calibration) $\exp(i\varphi)$, where φ is determined by Eq. (20). For SNR column we adopted the WVRGCAL strategy and set value of 5.0, preventing thus flagging in subsequent *applycal()* due to SNR below 3.0 (the default threshold).

We were interested, what is the relation of our time-dependent WV-calibration to the standard phase calibration applied to the target, i.e. the linear-interpolated phase solution between the two adjacent phase-calibrator scans. The results are shown in Figs. 94 and 95. Fig. 94 shows the phase solution calculated directly by Eq. (20), i.e. including the possible wrap-overs over 360deg as blue curve, and already wrapped-up linear interpolation for the scan-based (*solint='inf'*) phase solutions for the phase calibrator for antenna 7 (CM10) in SPW 11. Fig. 95 shows the same for antenna 20 (DA55) in SPW 7. The red and green lines are phase-cal interpolation for XX and YY polarizations. It seems that the phase-calibrator solutions interpolated in time between the adjacent phase-cal scans (before and after the target scan) at least mimic the trend of the WV-based phase delays. However, for the more distant antenna DA55 the phase solution is clearly wrapped-over.

Because the linear trend seems to be contained in the WV-calibration, one may think about dropping the standard phase-cal interpolated solution. However, we cannot fully abandon it, because it contains info about relative phase delay between XX and YY polarizations which is, likely, of instrumental nature. So, we have to somehow combine both solutions before applying them to the solar target – we discuss this in further text.

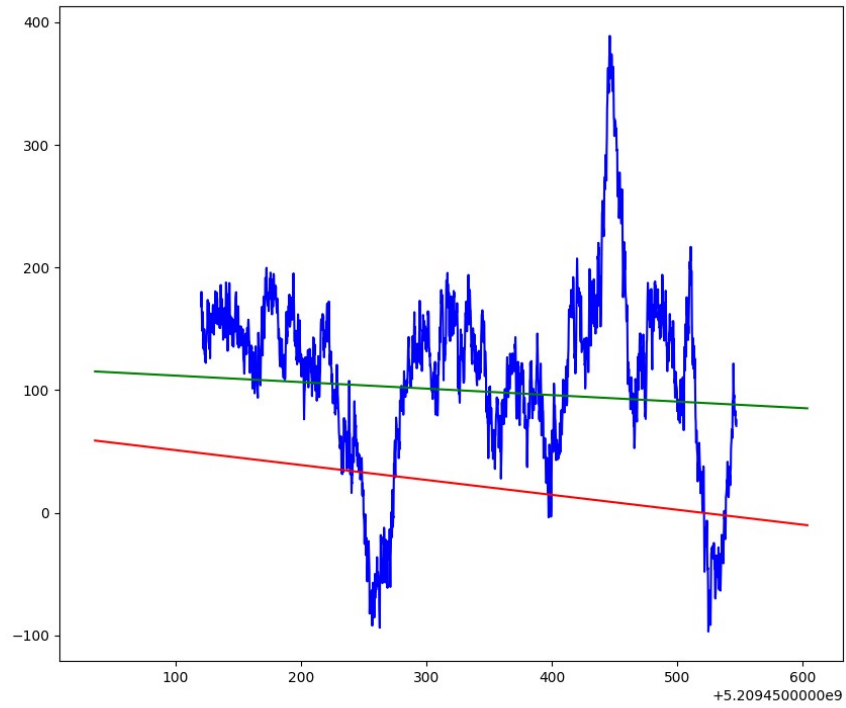


Figure 94: Comparison of the WV-based phase delay relative to refAnt (blue) with XX (red) and YY (green) linear interpolations for phaseCal solutions in the adjacent phaseCal scans for CM10 in SPW11. Phase delays are in degs.

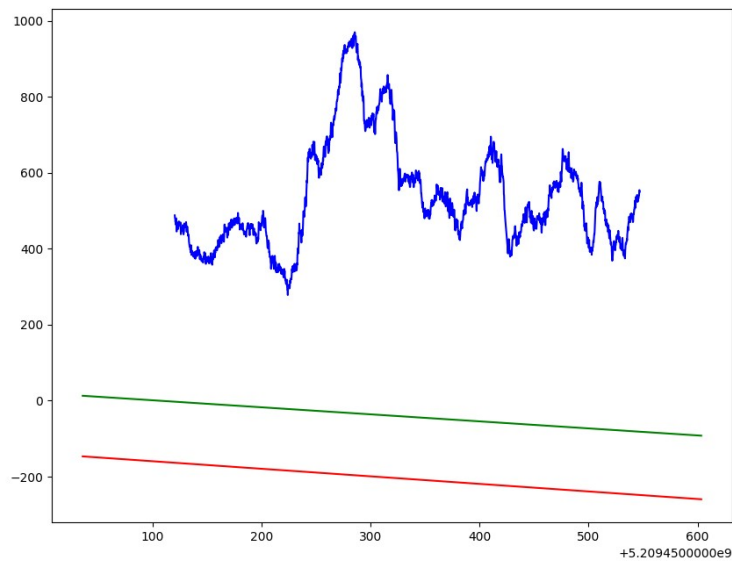


Figure 95: Like in Fig. 94 , now for DA55 in SPW7.

As the last result we present comparison of the phase delays extracted from the gaintable calculated by `sol_pwvGcal()` with the self-cal gaintable for the solar target. This is the most relevant result as it aims to direct application of the entire procedure for solar target phase corrections. Results are shown in Figs. 96 (for antenna id3) and 97 (for antenna id44). We are showing the phases extracted from the complex gains contained in gain caltables using our helper method `getPhase()`, so the initial phases for both `sol_pwvGcal()` (a single solution displayed as the green curve) and `gaincal()` (two solutions XX and YY displayed as read and blue curves respectively) are already wrapped to $-\pi$ to $+\pi$ interval. However, as we use `np.unwrap()` the phases may (and namely in case of self-cal gain caltable they do) leave that interval as time proceeds.

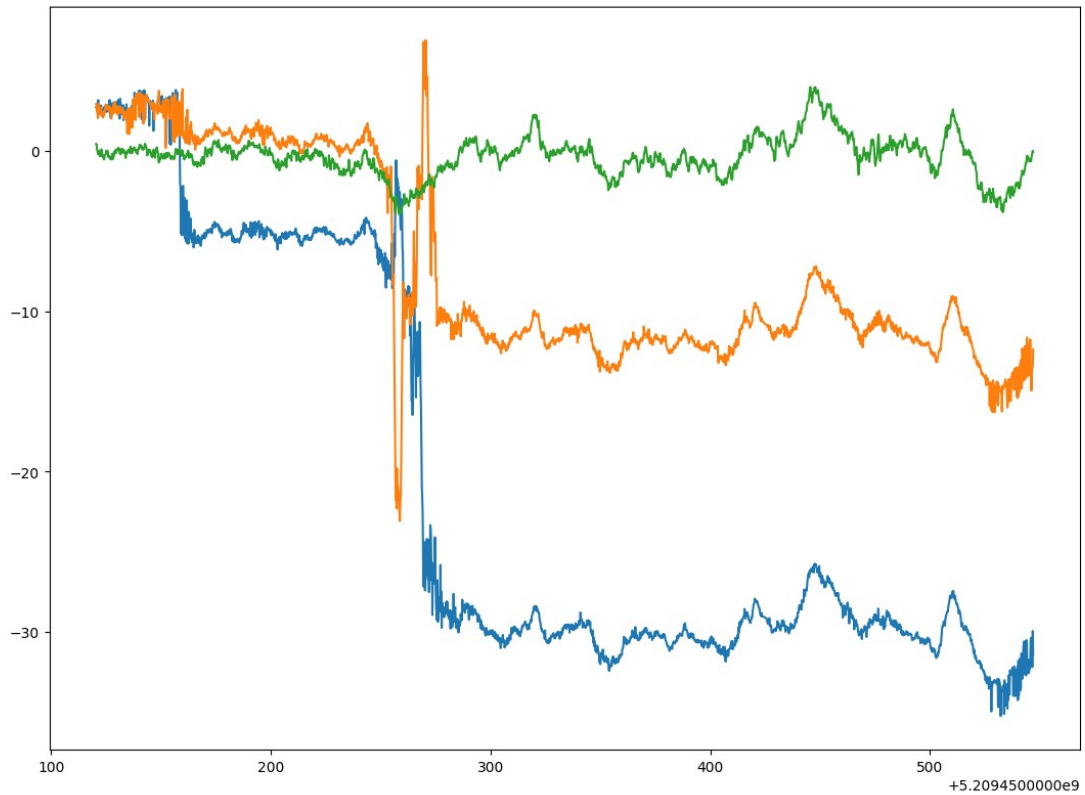


Figure 96: Phases of the complex gains from the WV-gcal table (green) and self-cal gaintable – for XX (red) and YY (blue) polarizations as a function of time (MJD Secs). The phases are displayed in radians(!). Curves are for antenna id3.

If one neglects the jumps in self-cal gaintable phases caused by failing self-cal solutions at specific time instants, qualitative similarity between the curves is clearly apparent.

If the phases for more quiet (with less self-cal jumps) antenna id44 are subtracted, i.e., WV-gcal minus self-cal is plotted, we get a basically straight line with some oscillations – see Fig. 98. It looks like the slope of this line exactly corresponds to the linear trend for the adjacent phase-calibrator scans. The remaining oscillations may be attributed to intrinsic dynamics of the solar atmosphere, the sudden short jumps originate in ‘jumpy’ self-cal solution. The red and blue curves are for XX and YY (respectively) polarizations of the phase-differences calculated as PWV-based minus self-cal-based solutions, the accompanying straight lines are the linear interpolations of the phase-cal INF solution in the adjacent phase scans (as in Fig. 95). Clearly, the self-cal solution has been performed at the ‘standard’-calibrated data, so the instrumental XX – YY difference is already removed (that is why the red and blue curves are almost identical). What remains – in addition to intrinsic solar dynamics, which changes the phases, too (and we do not see it in our self-cal table that is based on the entire-scan, i.e. INF, image model) – is the linear trend: In the solar self-cal solution it has been removed by call to *applycal(interp='linear')* for the phase-calibrator INF gain caltable in the standard calibration, while the PWV-based phase solution contains the full phase variation, including the trend.

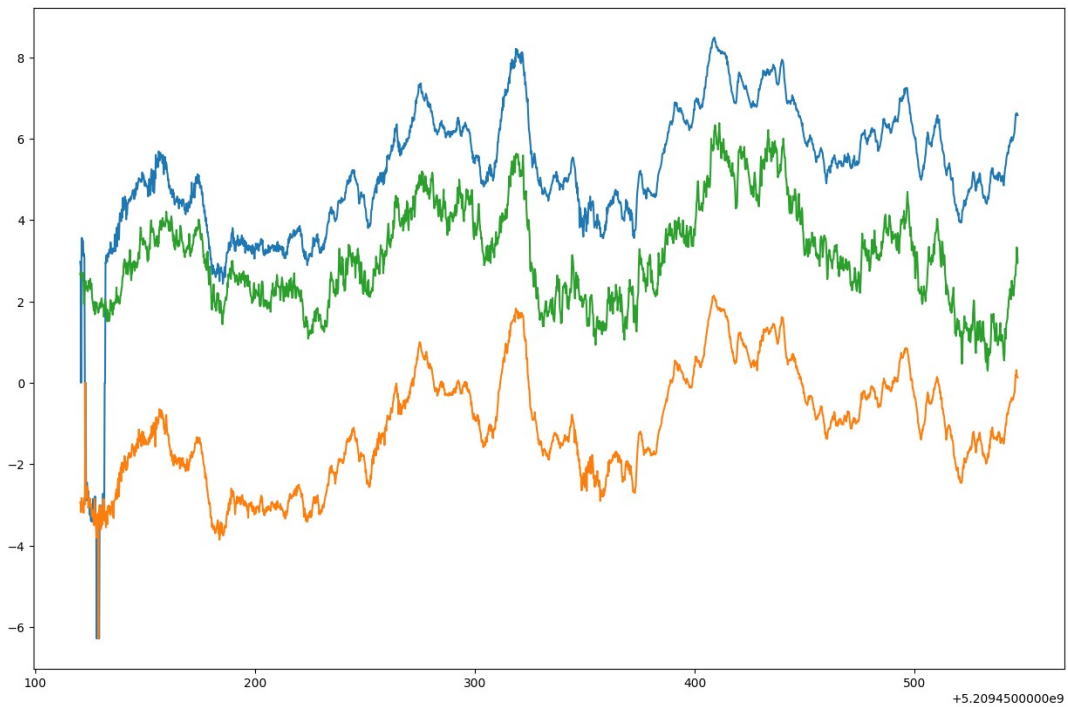


Figure 97: Like in Fig. 14, now for antenna 44.

Summary: We are able to produce solar equivalent of the WVRGCAL table and use it for corrections (via the standard *applycal()* taks) of the phase delays in the solar scan. Still some work is needed in

combining the standard linear-interpolated phase-calibrator solutions (phase-inf gain caltable from the standard calibration procedure) and our WV-gcal caltable. Clearly, if we apply the WV-gcal calibration on top of the fully calibrated (without self-cal) MS, which has the phase-inf gain caltable applied already, we apply the trend twice. (Note that this does not happen in non-solar data: There we apply the *wvrgcal()* solution both to target and phase calibrator, so the trend compensates; in solar case we apply the solar WV-gcal to the solar data only, the phase calibrator does not use *wvrgcal()* in solar datasets). This double-application of the trend is visible in the difference between WV-gcal and self-cal gaincal(). On the other hand, we cannot drop the standard phase-calibrator based solution completely, as it contains also info about relative instrumental delay between XX and YY polarizations. Merging of both calibration will come in next section.

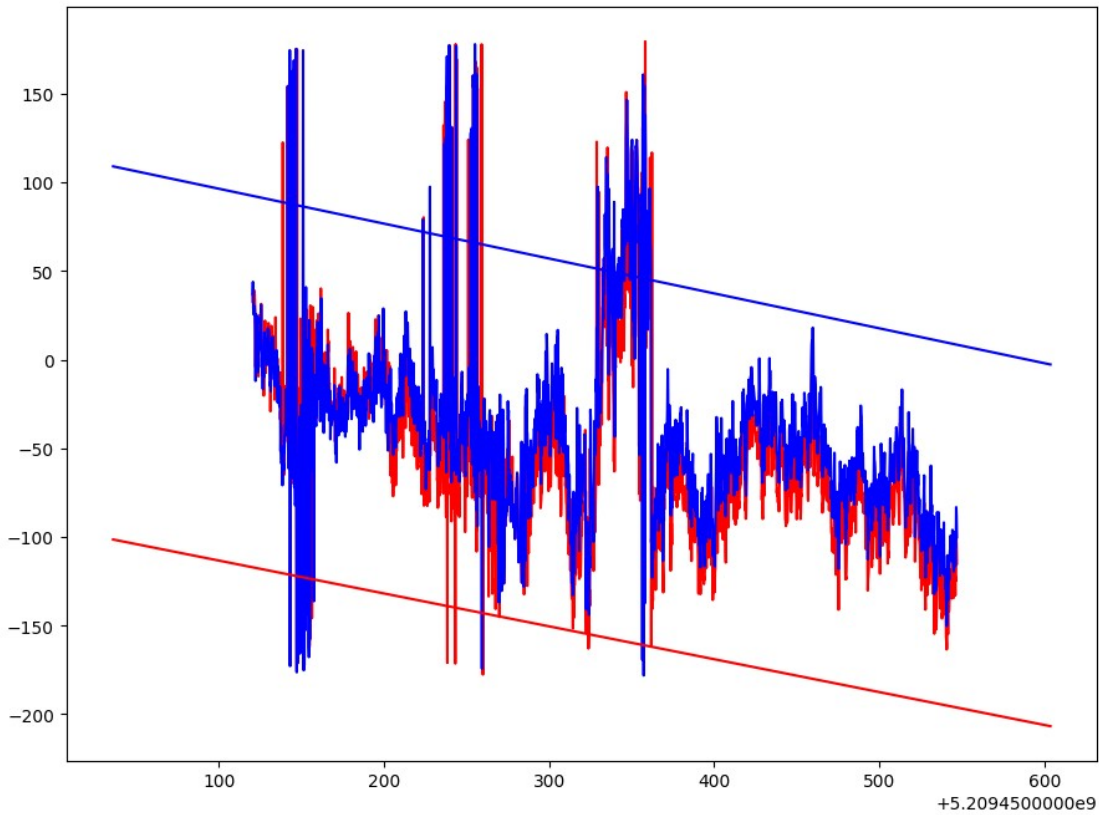


Figure 98: Solar phase solutions (in degs): PWV-based minus self-cal-based (the curves), overplotted by linear interpolation of the phase-calibrator-solution (straight lines). Red: XX, blue YY polarization, all for antenna id 44 and SPW 5. The difference between PWV and self-cal have been wrapped into -180 – +180 deg interval in order to prevent phase jumps, which are present as a consequence of self-cal troubles at specific time instants.

2.5.3 Final improvements

Receiving the second, recent, test dataset from July 30th 2026 has appealed on further possible improvements of the method. They are motivated by Fig. 65, which displays the scan-average solar brightness temperatures in dependence on antenna ID. The impression from that figure says that there may be systematic dependence of T_B on the antenna type, most likely related to their different efficiencies. Just for recap: We are using the default values of 0.95 and 0.90 for rear- and forward-efficiency of all antennas in the set, respectively, in line with White et al. (2017). As the July 2026 data show similar distribution, we decided to make two further improvements of the method, related to this observation: Defining the specific antenna efficiencies for different antenna categories, and, after this correction, make use of the solar brightness temperature averaged not only over the scan, but also over the ALMA array. Details are described in the next two sub-sections. They are followed the last subsection on this topic – description of the promised improved *sol_pwvGcal2()* method, which combines the atmospheric and instrumental (polarization-dependent) delays into a single gain cal table.

2.5.3.1 Improved antenna efficiencies

We have done similar analysis as is for the 2023 dataset presented in Fig. 65 also for the new data from July 2026. In spite the arrays for both observations are not identical, vast majority of the antennas are present in both observations, so we can compare their ‘performance’. In the XLS file *project22_25_Tb.ods* located in the on-line repository http://wave.asu.cas.cz/shared/ESO/final_report (Annex 5), we have separate sheets for 2023 data, 2026 data, and their combination. We are calculating relative normalized efficiency factors for each antenna based on its scan-averaged solar T_B value divided by expected theoretical T_B for the observation (this depend namely on the position angle – see the formula by Alissandrakis et al., 2020). It can be seen that the grouping of antennas by their type (CM, DA, and DV) is present in both datasets. Moreover, for specific antennas DA64 and DV08, which are present in both datasets, we have visible ‘under-performance’: Their normalized T_B s are significantly smaller in both datasets and both polarizations than the rest of the antennas. The other antennas also exhibit some variations, but they are randomly up- and down- for different dataset and/or polarization, while DA64 and DV08 shows sort of systematic behavior (if one dares to speak about it in case of just 4 data series).

We have excluded clear outliers and calculated the correction factors for efficiencies for all mentioned categories (CM, DA, DV, and the two outlying antennas). The results are summarized in the following table:

Antenna	η_l corrected	η_F corrected
CM	0.941	0.891
DA	0.947	0.897
DV	0.956	0.905
DA64	0.929	0.880
DV08	0.943	0.894

We count with this new values in our improved routines for both sci-SPW and WVR-based PWV(t) calculations *dWdBdG2()* and *dWdLs4()*.

2.5.3.2 Making use of array-averaged T_B

Related to the previous topic, we came to an idea to use another ‘boundary condition’ for our optimization (least-square) system: Once the antenna efficiencies are corrected, the Fig. 65 recalculated with improved efficiencies should give more-or less flat distribution of T_B . In other words, the solar brightness temperature may vary from one antenna to the other on short timescales (because of the scintillation revealed in Section 2.3), but there is no reason why the scan-averaged values should differ – all antennas should measure the same value. In order to enforce such requirement, we postulate requirement

$$\langle T_B(t) \rangle = \frac{1}{NS} \sum_{t=0}^{NS-1} T_B(t) = T_{tgt} \quad (5.3.1)$$

where NS is the number of time samples and T_{tgt} represents the target (the Sun) brightness temperature averaged over *antenna array*. We now add this equation to the already extended grand-system formed by set of equations (3.3.17) and (3.3.18), which represent the radiative-transfer equations and ACD calibration equations, respectively. Visually it means that the matrix in Fig. 54 gets extended by further lower belt composed by identical simple 3x2 matrices of the form (0,1,0/0,0,1) – in the Python implementation we call them *M-blocks* (from array Mean) – which, again may repeat, in order to increase their weight in the system; the repetition factor is called *mWeight*. The matrix $J^T \times J$ from Fig. 55 gets a bit more complicated by this additional equations, namely the upper-left main N-block receives also off-diagonal components (insofar they have been zero). Nevertheless, still it is possible to make very easy and low-CPU-demand inversion (again found with help of the Gemini AI) and thus solve the system block-wise.

This trick has been incorporated to methods for calculation of both science-SPW based PWV(t) – we have implemented a new routine *dWdBdG2()* for that sake – as well as for calculations of PWV directly from the WVR raw data (a new method *dWdLs4()*). While for the sci-SPW PWV calculation we set the array-mean target temperature at the RHS of Eq. (5.3.1) to the averaged solar brightness temperature over entire array, in case of WVR calculations via *dWdLs4()* we force zero target temperature in case of WVRs pointing well off the Sun.

We shall present the results obtained with this upgraded procedures in the next subsection, where we have accumulated one more minor improvement.

2.5.3.3 Correct weights for the grand-system of equations

During implementation of the last described improvement we have revised the weights, with which the individual equations enter the system. In course of this procedure we have realized that the SQLD SPWs, which have much higher cadence (we have 0.192 s for the AC spws, while 12x higher cadence in SQLDs), have also correspondingly smaller received powers, both in radiative-transfer equations (3.3.17) and the ACD calibration equations (3.3.18). Hence, they enter the system with lower weights than the equations for the AC SPWs. There is a simple cure to that issue: We scale all equations of the combined set (3.3.17) and (3.3.18) by inverse gain. This way, all the equations are expressed in equivalent temperatures, which are mutually co-mensurable (all radiative-transfer eqs. have their RHS around 6000K, as well as the all array-mean equations, the ACD calibration equations move around 300 K). This upgrade comes only to the procedure *dWdBdG2()*, as the raw WVR data, handled in *dWdLs4()*, are already scaled (calibrated) to temperatures.

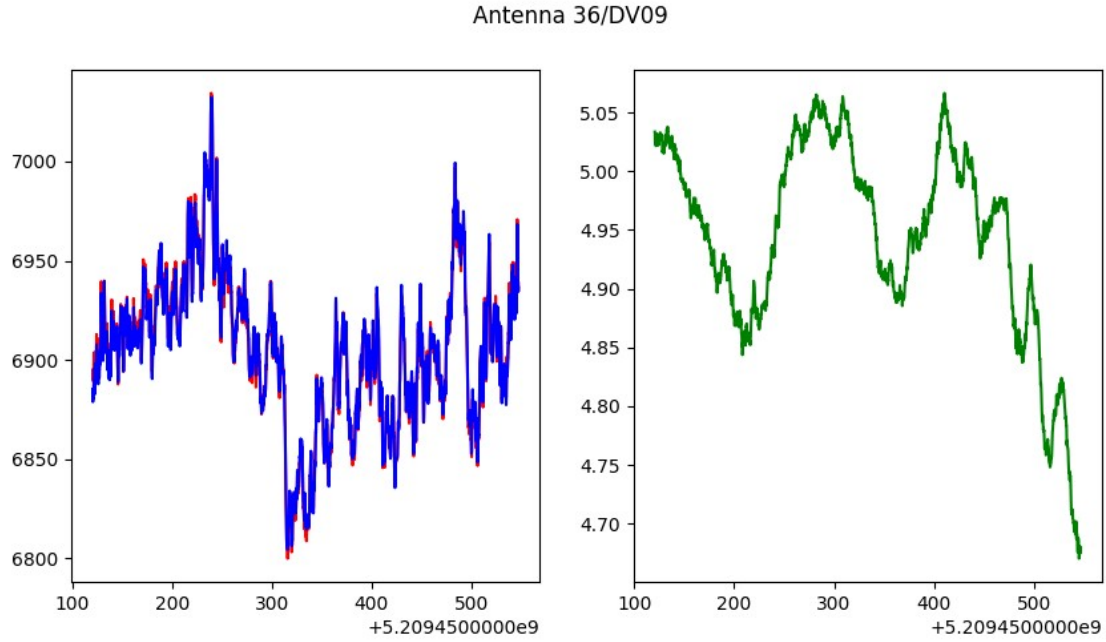


Figure 99: Left: Solar (apparent/scintillating) brightness temperature (K) in XX/YY (red/blue) for 2023 dataset and antenna DV09. Right PWV (mm) for the same data. Time is expressed as MJDsecs.

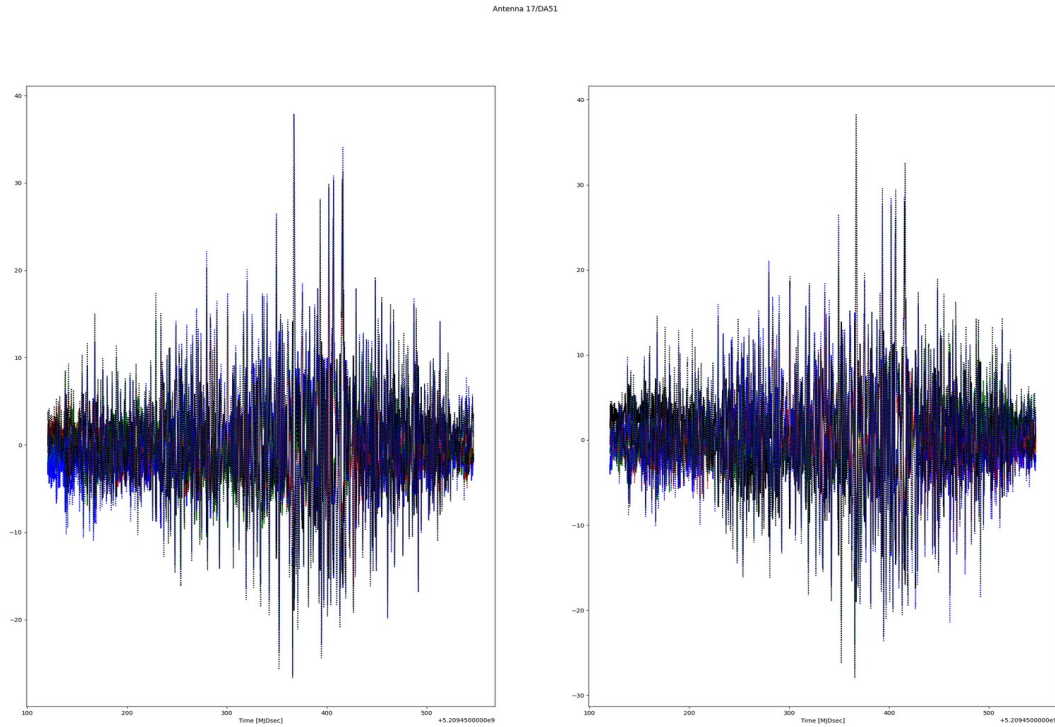


Figure 100: Residuals/errors as equivalent temperature in Kelvins for 2023 dataset and antennaDA51. Left: XX, right: YY polarizations, the individual SPWs are color-coded like in Fig. 31. For most antennas the RMS is well within 10 K, which represents an improvement.

We present here some selected results of the improved procedures. The full set is available online in the Repository (*Annex 5*). Actually, impact of the upgrades is significant namely what concerns the $T_B(t)$ profiles for XX and YY polarizations: These are now very close each other and oscillate close to the array-mean T_B value. This can be seen in Fig. 99, which shows the $T_B(t)$ for XX/YY polarizations for the solar scan of the 2023 dataset for antenna DV09 as red/blue curves in the left panel and PWV(t) time series in the right.

_43

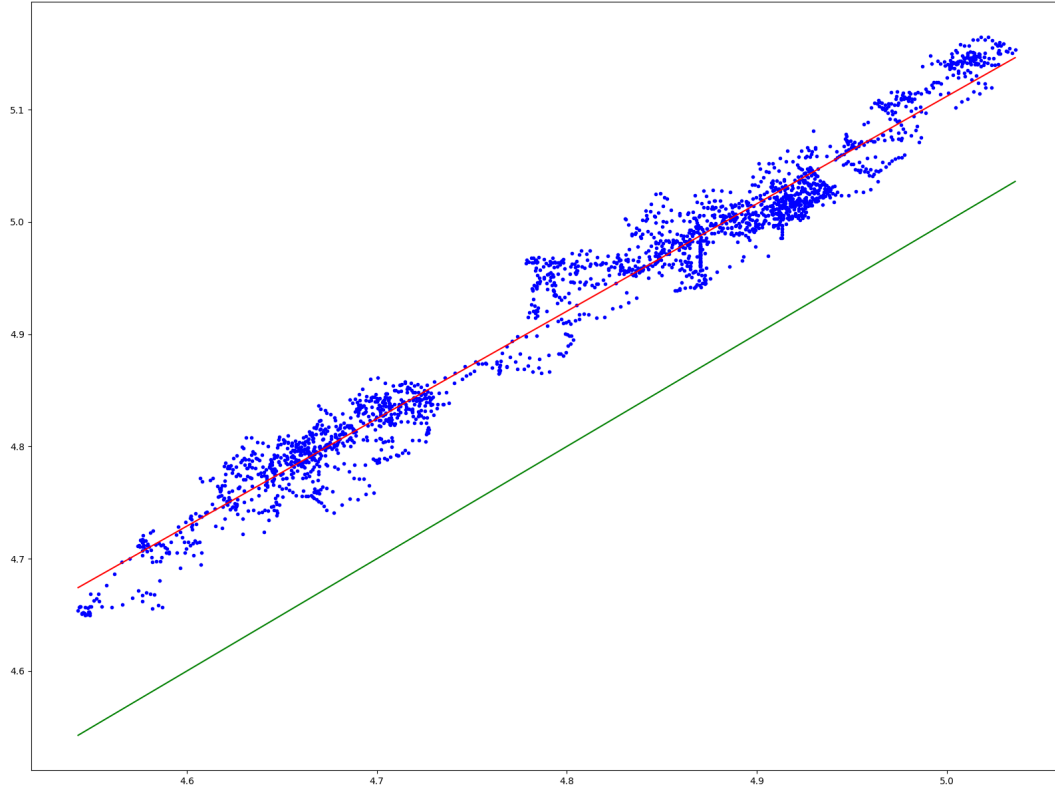


Figure 101: PWV in mm as obtained from (de-contaminated) WVR data - at x-axis, vs. the same quantity as calculated from science SPW signal (y-axis). The red line correspond to the best fit, the green one represents the ideal correlation $y=x$. For the antenna id 43 and scan 9 in the 2023 dataset.

Further improvement is visible also in the residuals/errors of the least-square solutions expressed in term of effective temperatures. This is shown in next Fig. 100, which may be compared with figures displaying the same quantity in the gradual improvement of the method form Section 2.3, for example with Fig. 49. The residuals significantly diminished and are typically with RMS less then 10 K.

Finally, in Fig. 101 we present cross-correlation between the PWV(t) as obtained from the science SPW signal via our upgraded $dWdBdG2()$ method, and the same quantity derived from the (solar-contaminated) WVR signal and extracted (disentangled from the variable solar illumination) by also

upgraded *dWdLs4()* method. The data are for antenna id 43 (DV17) recording the scan 9 (the Sun) in the 2023 dataset. In general, the correlation between PWVs derived by those two different methods improved, most of the linear regression coefficients is in the interval 0.95 – 1.05, with small systematic offset $\sim 0.2\text{mm}$ – the science SPW-based PWVs are higher by that value. Full database of cross-correlation results is available as XLS table (and also as figures similar to Fig. 101) in the online Repository (*Annex 5*).

2.5.3.4 Combined instrumental and atmospheric delays

In Section 2.5.2 we faced the problem of combination of the calibration using our PWV(t) and its recalculation to the phase delays with the ‘standard’ phase-calibrator based *gaincal()*. The point is that the phase-calibrator gaintable applies correction for (i) instrumental, polarization dependent delays together with (ii) linear trend of the atmosphere-related delays. Our *sol_pwvGcal()* routine, which is an analog to *wvrgcal()* for the solar data, creates a gaintable, that fixes only the atmosphere-based phase distortions. Now, if we do the ‘standard’ calibration, i.e. applying the PWV-precalibration first (like application of the *wvrgcal()* gaintable does), followed by the phase-calibrator based *gaincal()*, the atmospheric average delay and its linear trend are applied twice to the data. As we have already mentioned, it is not a problem in standard *wvrgcal()* in non-solar data, because the atmospheric delay is applied to the phase calibrator, too, before it is even used for *gaincal()*. Hence, this application of the atmospheric part to the delay compensates.

With the solar data we need to do something similar. To that end, we use *dWdLs4()* for calculation of the PWV(t) series not just for the solar scan, but also (and namely! – recall, it was just the matter of good luck that we have WVR data for the solar on-source subscan) for the surrounding scans: The ATM scan preceding the solar-target, and the leading and trailing phase-calibrator scans. Once we have PWV(t) calculated for the phase-cal scan (via *dWdLs4()* from the WVR data) and for the Sun (via *dWdBdG2()* from sci SPW signal), we may apply correction for the atmospheric part of the delay both to the phase calibrator and the Sun, exactly as *wvrgcal()* does. This extended procedure is implemented in the routine *sun_pwvGcal2()* in our collection (listings in *Annex 4*, source code as a file in online repo as *Annex 5*). We follow the standard *wvrgcal()*-based procedure and apply the *sun_pwvGcal2()* fix to the phase calibrator *prior* it is used for generating gaintable, i.e. in early steps of the calibration script, so that the applied correction gets exposed via *split(datacolumn='corrected')*.

An example of such combined time series of PWV(t) in the scans surrounding the solar scan in 2023 dataset, as calculated with upgraded routines *dWdLs4()* and *dWdBdG2()*, are shown in Fig. 102.

To sum up, the strategy for “Solar WVR gain calibration” is as follows:

1. In the *Step 4* of the standard solar calibration script (i.e., the same place, where non-solar calibration script for manual data reduction calls *wvrgcal()*) we call *sun_pwvGcal2()*.
2. This routine prepares a standard *T*-type (i.e., gain with a single polarization channel) calibration table *sun_pwv_gcal.W*.

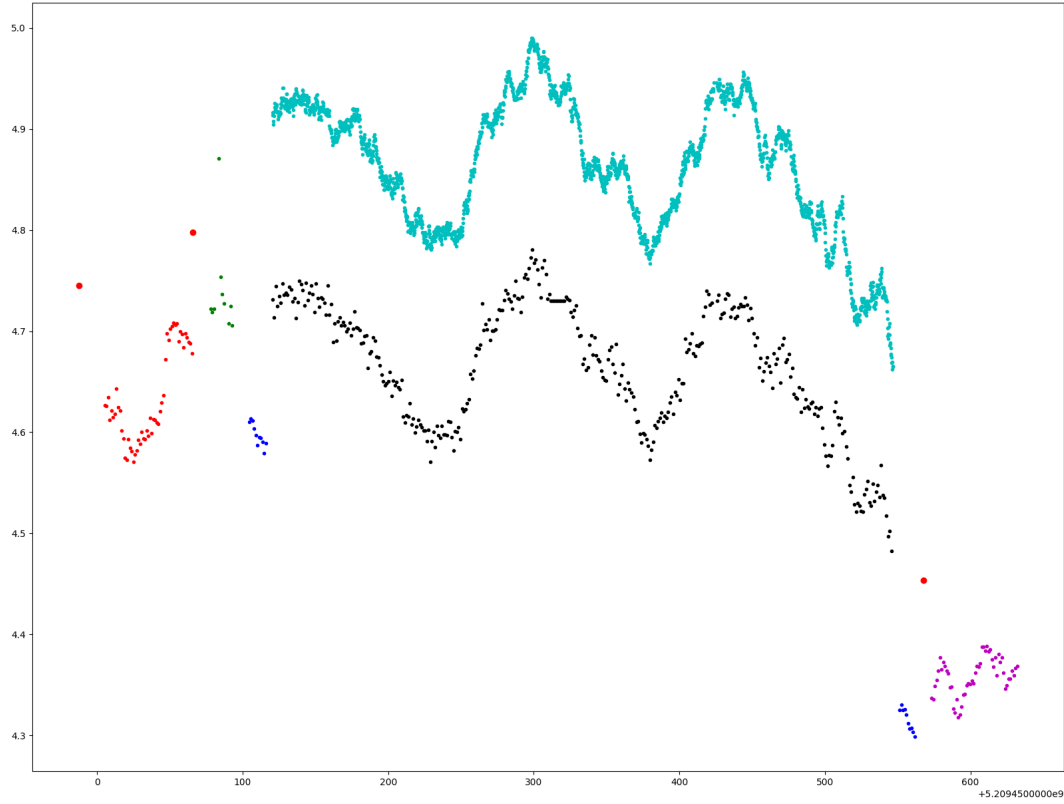


Figure 102: Calculated PWV in mm as function of time (MJDsecs) for the 2023 dataset for antenna DV03. The little dots in red, green, blue, black, blue, and magenta represent the PWV as calculated from the WVR data using latest method `dWdLs4()` in leading phase-cal scan, ATM scan, solar-off leading subscan, solar on-source subscans, solar-off trailing subscan, and trailing phase-cal scan, respectively. The cyan dots represent the PWV(t) as calculated from science SPWs in solar on-source subscans via upgraded `dWdBdG2()` method. The larger red circles are PWV values as directly read from the SYSCAL table. The same data for the new July 2026 dataset are shown in Fig. 93.

3. This T-type gaintable table (the same type as `wvrscal()` produces for non-solar data) contains data – complex gains with atmosphere induced phase delays – just for two fields: The phase calibrator and the Sun.
4. The atmosphere-induced phase delays for the Sun are internally calculated by the method `dWdBdG2()` from the signal in science SPW. The atmospheric delays for the phase calibrator are internally calculated by `dWdLs4()`. [Note: The current routines names, burdened by legacy of the development process, might get renamed in the fully operation version of the software package deployed to `sun_reductionUtils()` library.]

5. The `sun_pwv_gcal.W` gain table is – together with `Tsys+` `Tant` and `Tantpos` tables – applied to the phase calibrator and the Sun in the *Step 6* of solar calibration script, i.e. at the same moment, where `wvrgcal()`-generated gaintable is being applied in non-solar data.
6. The call to `MS split(datacolumn = 'corrected')` in *Step 8* assures that all the corrections are involved in the further calibration steps.

2.5.4 Results of applied PWV-based gain corrections

We have tested application of the corrections from the gain-table generated by `sun_pwvGcal2()` by direct comparison between movies of the data calibrated just by the standard procedure, and those corrected for the atmospheric phase variations via `sun_pwvGcal2()`. The movies for 2023 dataset are in the on-line data repository and the better quality of the phase-corrected movie is visible at first glance. Even in the case of this relatively high PWV dataset ($\sim 5\text{mm}$) the correction removed most of the fast (frame-to-frame) flickering, leaving just the intrinsic dynamics of the solar atmosphere at its much slower timescale. The PWV-based correction may be compared also with the movie obtained by the *Approach 1*, i.e. standard calibration followed by time-scale decomposed self-calibration. From qualitative point of view it seems that the self-calibration, in comparison with the PWV-based correction, tends to reduce the dynamic features that are most likely related to intrinsic solar evolution.

The data from July 30th 2026 are yet under processing and will be delivered to the on-line repository ASAP.

3. Conclusions, suggestions for implementation, and outlook

Instead of concluding remarks, we would like to briefly recapitulate the main results obtained by carrying out this project/study and, based on them, formulate a set of recommendations, or rather suggestions or a “wish-list”, as we may likely lack the technical knowledge, which is necessary to assess the feasibility of implementations of our suggestions at engineering level. As we feel further potential for improvement, we finish the report with outlook for post-study development in this broader topic.

3.1 A brief recap and summary of obtained results

Before reporting the specific outcomes, let us mention the list of analytical tools implemented as software utilities or libraries (see the docs in *Annex 3*), which has been developed in order to provide answers to the questions that initiated this Study:

- The SW package for solar ALMA simulations, which, for the first time, uses realistic setup for solar observations involving
 - mixed 12m+7m arrays in all realistic configurations (up to C-8),
 - atmospheric corruption of the visibilities using two different advanced models, namely the phase-screen.
- Procedure for self-calibration of solar observations with very high cadence (0.192s) based on time-scale separation between the fast atmosphere-induced phase variations and slower intrinsic solar dynamics. Implemented as a set of scripts (implementation of *Approach 1* of this Study).
- Analytical tools and metrics for quality assessment of the self-calibration solutions (see *Annex 6*).
- A broad collection of routines for analysis of the WV emission and absorption features in the science SPW and WVR channels and their impact on the science data.
- As a by-product of the study, we have developed the first working version of *Solar Script Generator* (SSG, see the MTR in *Annex 1*) for assisted (highly automated) generation of solar calibration scripts. After its extension by further features (polarization, single-dish full-disc and regional-map) by Masumi Shimojo, the SSG has become *de facto* standard in the community of manual data reducers for solar data.

With the above tools we have come to the following main outcomes:

- Self-calibration/imaging loop that utilizes the time-scale separation between apparent (atmospheric) and intrinsic dynamics in the solar images – *Approach 1* of this study – was capable to process Band 3 data observed in C-7 configuration and Band 5 data in C-6. This boosts the insofar resolution almost five times! Successful completion has been demonstrated

on three observing datasets (one Band 3 from December 2023 and two Band 5: one from 2023 and second from July 2026).

- The advanced solar simulator indicates that this is actually the hard limit – reconstructed images for simulated observation in Band 3 in configuration C-8 have been already of poor quality even with atmospheric perturbation switched off. Unless an alternative to standard *tclean()* will be employed (e.g. *BRAIN/resolve()*), the low *uv* density in combination with rather fast time-variability of the solar images becomes the hard-stop limiting factor for the maximum usable configuration.
- We have clearly proved that atmospheric WV and its variations are clearly pronounced in the science SPW data and can be used for calculation of the atmosphere-induced phase delays. We have developed and tested advanced mathematical model (implemented as an extensive set of Python routines – *Annex 4*) for extraction of PWV(t) time profiles from the science SPWs. and These high-cadence PWV(t) time series are subsequently directly used for atmospheric phase-delays calculations – *Approach 2* of this study. The model is based on an extended optimization problem, which involves, in addition to science SPW data, also ACD calibration equations and allows thus for fine tuning of the instrumental parameters: gains and instrumental power-offsets. This methodology for better-fitted gains has potential for application in non-solar data, too.
- In case of the calibrators (BP, phase) as targets, the WV variations are seen as a weak variable emission in the science data, for the Sun as much stronger absorption signal. In spite the weak WV sci-SPW emission in the calibrator scans, not reaching much above the instrumental noise (increased T_{RX} in the solar MD mode), we were able to find striking correlation between the WV emission variations and the calibrator’s phase fluctuation for most antennas.
- For the Sun the WV absorption signal is much stronger and even though we do not have the ‘ground truth’ for bench-marking of our calculated WV-induced phase variations (except of the ‘soft’ self-cal based solution), we have found strong correlation between our PWV(t) series with the PWV calculated from the direct WVR measurements. This comparison has been achieved thanks to specifically designed test observation in July 2026 (two sub-arrays) and confirmed even for the 2023 test dataset thanks to incredibly good luck (WVR pointed a bit off-the-Sun during solar scan near the limb).
- The above findings have been incorporated into routine *sun_pwvGcal2()*, which represents 1-to-1 counterpart to *wvrGcal()* for the solar datasets.
- As a sort of by-product of the PWV(t) analysis we have revealed the effect of tropospheric scintillation in solar mm data: Variations of the signal (auto-correlations and SQLDs) in science SPW cannot be attributed solely to WV fluctuations but it has another, faster component most likely connected with lensing (focusing/defocusing) effects of the tropospheric bubbles/eddies. Our methodology count with this effect and the routines are designed to separate both components (i.e. varying PWV and scintillation). This finding may have broader impact out of solar ALMA observing.

3.2 Suggestions for implementation of high-resolution solar ALMA Observing mode

Based on the results summarized in the previous section, we suggest the following implementation changes that for the solar ALMA observing mode. Some of them are directly related to the new mode of high-resolution observations, but some represent general improvements that may help already in the current setup. We start with the high-resolution mode suggestions:

- Both simulations and the test data show that ALMA configuration may be extended to
 - C-7 in Band 3
 - C-6 in Band 5

We suggest to start high-resolution solar observation in this setup.

- Simulations show that for other current or future Bands used for solar observing the maximum limit is given by the uv dilution. Consequently, the limiting configuration is given proportionally to the above findings for Bands 3 and 5 by relation for the longest baseline $L_{\max} < 1500$ k λ . If possible, we would recommend to prove the limiting configuration in the insofar untested Bands also by a short test dataset, before starting scientific observation.
- Both the simulations and test observation demonstrate that very high cadence is vital for time-scale separation between solar intrinsic dynamics and faster apparent changes in the image caused by atmospheric turbulence. The high cadence is anyway a longer-term demand in the community (see, e.g., previous *Solar Development Study* by Wedemeyer et al, 2020, or projects for ALMA 2040 vision). The cost is in higher data-rate, but it is, perhaps, still acceptable having in mind low spectral resolution of TDM mode. We suggest to use 0.192s cadence for high-resolution observations as default. For case of the ‘standard’ solar observations (i.e., e.g., Band 3 up to C-4 configuration) the higher cadence could be possible on PIs request.
- Simulations with phase-screen perturbation indicate that the phase-RMS on the long time scale, used traditionally as go/no-go test criterion, has a little relevance for the solar observations – the self-cal procedure is capable of reconstruction as long as high-cadence time sampling is performed. We suggest to skip/ignore this test for high-cadence solar data.
- We have, as a result of this study, a well tested procedure for deriving PWV(t) time series from science SPWs and their subsequent application to atmospheric phase delay corrections, very analogous to the standard *wvrgcal()* procedure. We suggest to have the *sun_pwvGcal()* routine (and its sub-routines), which plays the role of *wvrgcal()* in solar datasets, as a part of standardized *sun_reductionUtils.py* library. We are ready to help with integration and deployment.
- The three datasets observed for the tests in December 2023 and July 2026 have been fully calibrated and self-calibrated and contain high-quality and unique data (there is no previous ALMA solar observation, which would at least approach such a high spatial and temporal

resolution). The data are likely to be of high interest of scientific solar community and may revive general interest (now a bit decaying, due to frustration from low success rate) for using ALMA in solar research. We suggest to release them as CSV datasets (or similar open format) and we are ready to assist with actions that are necessary for that (metadata creation, documentation preparation, etc.).

During carrying out the Study, we have found few other, more general (not directly related to high-resolution mode) items, whose implementation could improve performance of solar calibration or observations:

- The standard routine *sol_ampCal2()* from *sun_reductionUtils.py* ignores the opacity factor that multiplies the term $T_{\text{sys}}+T_{\text{ant}}$ (e.g. White et al., 2017). For most of observations it is not a problem as the dry opacity is very small and, consequently, the correction approaches 1.0. For Band 5, this is, however, not correct. Our $T_{\text{sys}}+T_{\text{ant}}$ values in SPW 5 (next to the WV line center) for 2023 test data with quite a high PWV~5mm are around 3000K, i.e. clearly diminished by absorption. We suggest to include the opacity term in calculations and are ready to implement it.
- For the solar on-source sub-scans the solar mode uses higher attenuation settings. The attenuation factor between low and high attenuation is for calibration purposes derived from SQLD spws as a ratio $AF=(P_{\text{sky}}-P_o)/(P_{\text{off}}-P_o)$, where P_{sky} is the power measured in the ATM scan in the low-attenuation regime, P_{off} is the power measured in the solar off-source sub-scans (there are two – leading and trailing the solar on-source subscan) in the high-attenuation regime, and P_o is the instrumental (zero-signal) offset in the SQLD spw. This method has an issue caused by the fact, that P_{off} may vary in time as the atmospheric emission varies. Namely, in Band 5, where WV opacity/emissivity is strong, the emissivity variance corresponding to quite fast changes of PWV can have strong impact, as we have seen in our 2023 dataset. For that reason we started using just the leading off-source sub-scan, which follows immediately after the ATM scan in our study, in order to avoid larger change of atmospheric emissivity during the long (~7min) on-source subscan. Note, however, that the *sol_ampCal2()* in *sun_reductionUtils.py* uses the average of leading and trailing subscans for the calculation of the attenuation factor. As a cure, we suggest the following change of the procedure: During the ATM scan, there are present also ACD hot- and ambient-load subscans. We suggest to have the hot-load subscan repeated, the second run to be performed with the high-attenuation (solar) regime. First, the hot load has higher temperature than sky, so the determination of the attenuation factor as

$$AF=(P_{\text{hot}}^{LA}-P_o)/(P_{\text{hot}}^{HA}-P_o),$$

where *LA* and *HA* superscripts denote the low- and high-attenuation regimes, should be more precise, second, the hot-load temperature does not vary significantly with time.

- By processing the ‘technical’ test dataset – Band 5 observations in the sub-array regime of configuration C-3 under non-observing condition (PWV 10 – 12, rain) we have found that by

applying our self-cal procedure, we are still able to extract somewhat valuable solar movie (see online *Annex 5*). Based on that experience and in the view of solar community frustrated by enormously small number of accepted solar proposals, we would like to suggest consideration of creating specific back-up observing queue for bad-weather tolerant projects. Both the solar community and ALMA observatory may benefit from such setup: Solar community would get their proposals observed, ALMA could increase the number of on-source hours per year – the bad weather conditions form the most significant reason for no-observation.

3.3. Future directions of further development

The Study is formally concluded by this report, but its results immediately initiate considerations on further improvements. We are keen on to continue work in this direction in the post-project period and contribute further to soar ALMA ObsMode improvement. We are fully aware that the results of our study – even if fully accepted – will not come into implementation until the current major WSU update is finished. Taking this fact positively, we would like to use this time between the project delivery and implementation of its results for further improvement. Here are few ideas for continuation of the work:

- The PWV(t) series calculations and their utilization for phase corrections is not limited to extended array configuration – it can be tested in any solar Band 5 data. We plan to do so in order to increase credibility of our findings by application to the larger dataset.
- Related to previous: At the two test Band 5 dataset used in our Study we have found likely systematic variations of averaged solar brightness temperatures across antennas. This effect has been ascribed to per-antenna varying antenna efficiencies and we have used it for prerapring a table of corrected antenna efficiencies by the antenna type (+ two outliers). Again, such a task is not limited to long-baseline configuration and any solar Band 5 dataset can be used for that.
- During discussions at the very end of our project we have touched an issue of possible side-band ratio effect to our calculations of PWV(t). Namely, whether the side-band ratio may influence the spectral slope of data between USB and LSB spws. The current result of consultations is that there should be none or negligible influence. Anyway, we would like to check it more in detail.
- The simulations shows that the diluted *uv* coverage, in conjunction with solar dynamics at rather fast timescale, is the blocker for pushing the bar of more extended array configurations further. Nevertheless, there may be solution in using something different than classical *tclean()*. We have started consultation with Fabrizia Guglielmini (ESO) and Thorsten Ensslin (DZA) on possible utilization of *BRAIN/resolve()* cleaner for the solar data. This is going to be a larger project, but we hope to have it finished before the solar high-resolution mode is approved and deployed.

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Annex 1

Complete Mid-term report, including all its annexes