

Study of the Difference Between Accumulated Phase Corrections from Self-Cal Tables and Measurement Set Phase Differentials Using Directional Phasor Analysis

Abstract

Determining the exact total phase corrections applied during solar interferometric observations is vital for validating self-calibration convergence and imaging dynamic structures. In this study, we analyze the correspondence between two complementary diagnostic routes: (1) compounding phase corrections across successive self-calibration gain tables, and (2) computing visibility phase differentials between the pre-self-calibration data and the final self-calibrated Measurement Set. We detail the physical origins of the discrepancies that emerge when using simple scalar averaging over channel spectra, examine the empirical heuristic routine developed to correct branch-cut phase jumps across spectral windows, and present a rigorous, parameter-free directional phasor methodology based on circular statistics on the complex unit circle S^1 . Verification across antenna baselines confirms complete phase closure down to numerical limits.

1. Introduction and Problem Context

Validating self-calibration performance requires confirming that the phase corrections applied to the visibilities match the underlying gains derived during each calibration cycle. This can be evaluated via two independent diagnostic approaches:

1. **Accumulated Gain Table Solutions:** Compiling antenna phase corrections across consecutive gaincal() execution cycles to determine the net cumulative phase rotation applied to each antenna.
2. **Measurement Set Visibility Differentials:** Calculating the channel-averaged phase difference between the original pre-selfcal data and the final post-selfcal split Measurement Set:

$$\Delta\phi_{\text{MS}}(t) = \phi_{\text{orig}}(t) - \phi_{\text{selfcal}}(t)$$

In an ideal system, the phase difference $\Delta\phi_{\text{MS}}(t)$ on any baseline to the reference antenna must strictly track the cumulative phase rotation $\phi_{\text{accum}}(t)$. However, direct numerical comparisons often reveal large, unphysical discrepancies across extended time ranges. Detailed investigation of the channel-by-channel phase spectra demonstrates that these errors arise from systematic calculation artifacts when simply calculating linear arithmetic averages of phase angles across frequency channels and spectral windows (SPWs).

2. Failure Modes of Linear Phase Averaging

Screening visibility phase data across multiple SPWs reveals four operational failure regimes that degrade linear phase averaging:

- **Case 1 (Branch Cut Bimodal Splitting):** The majority of data points are clustered in a narrow phase range, but channel noise pushes points across the $\pm\pi$ ($\pm 180^\circ$) branch cut. An arithmetic mean of $+179^\circ$ and -179° evaluates to 0° (a 180° phase reversal) and inflates the standard deviation to $\sim 179^\circ$.

- **Case 2 (SPW-Wide 2π Phase Shifts):** Similar to Case 1, but most points in one entire SPW are shifted by 2π relative to adjacent SPWs, resulting in an artificially high standard deviation for that SPW and corrupting the multi-SPW combined mean.
- **Case 3 (Mixed SPW Coherence):** Only one or two SPWs exhibit good channel alignment (low standard deviation), while the remaining SPWs are scattered across the whole phase range.
- **Case 4 (Full Spectral Decoherence):** All SPW data are degraded simultaneously, exhibiting wide dispersion across the whole channels.

3. Heuristic Correction: Dynamic Phase Standard and Variance Rollback

To correct the phase data under the physical constraint that the phase jump between adjacent timestamps should not be large, an empirical correction workflow was established:

3.1 Phase Standard Establishment Algorithm

For each dataset and at every timestamp, edge channels are discarded (trimming the first and last 10 channels of each SPW to avoid bandpass roll-off). A dynamic reference value—the Phase Standard (ϕ_{std})—is determined through three decision rules:

- **Rule A (Dominant Coherent SPW):** Calculate the standard deviation from the trimmed phase data of all SPWs. For the lowest standard deviation, if it is below a preset threshold ($\leq \pi/3 \approx 1.047$ rad), the corresponding SPW is considered relatively good, and the mean value of its trimmed phase data is set as the phase standard.
- **Rule B (Coherence Failure & Temporal Continuity):** If none of the SPWs are good (all $\sigma > \pi/3$), calculate the standard deviation from the phase data within the middle 50 percentile (25th–75th) of all trimmed channels across all SPWs. If this standard deviation $\geq \pi/2 \approx 1.571$ rad, the entire dataset at this timestamp is deemed unsuited for calculating a phase average. Assuming the phase jump between adjacent timestamps is small, the phase standard from the previous timestamp is retained: $\phi_{\text{std}}(t_i) = \phi_{\text{std}}(t_{i-1})$. (For $t = 0$, Rule A is applied ignoring the phase threshold).
- **Rule C (Moderate Multi-SPW Coherence):** For the remaining cases where no single SPW meets Rule A but the standard deviation of the 50 percentile group remains below $\pi/2$, the mean value of this 50-percentile group is set as the phase standard.

3.2 Point-by-Point 2π Modification and Rollback Verification

Once ϕ_{std} is established, channel corrections are executed:

3. **Phase Modification:** Apply phase correction for each data point according to the phase standard by checking the difference $|\theta_k - \phi_{\text{std}}|$. If the absolute value is $\leq \pi$, no change is applied. If the absolute difference is $> \pi$, apply a $\pm 2\pi$ modification accordingly.
4. **Variance Rollback Check:** Recalculate the mean value and standard deviation for each SPW. If the newly modified data has lower or equal standard deviation, the phase change is kept. Otherwise, roll back to the unchanged phase data to avoid over-correction.

4. The Directional Phasor Formulation (Circular Statistics)

While the heuristic correction routine resolves major branch jumps, its dependence on fixed empirical thresholds ($\pi/3$, $\pi/2$) and temporal state carryover prompted the implementation of a mathematically unified, parameter-free approach using directional statistics on the complex unit circle S^1 .

Because phase-only self-calibration maintains $|g| \approx 1.0$, visibility phases θ_k are mapped directly to unit complex phasors:

$$z_k(t) = e^{i\theta_k(t)} = \cos(\theta_k(t)) + i \cdot \sin(\theta_k(t)) \quad (|z_k| = 1)$$

4.1 Resultant Vector and Directional Mean

Averaging over all trimmed channels across all SPWs yields the mean resultant vector:

$$R(t) = (1/N) \sum z_k(t) = C(t) + i \cdot S(t), \quad \theta(t) = \text{atan2}(S(t), C(t))$$

Because vector addition operates directly on Cartesian coordinates, phase wrapping across $\pm\pi$ is resolved naturally:

- **Cases 1 & 2 (Boundary Wrapping):** Phasors at $+179^\circ$ and -179° add constructively along the negative real axis ($C \approx -0.9998$, $S = 0$), immediately yielding $\bar{\theta} = 180^\circ$ (π rad) without unwrapping thresholds.
- **Case 3 (Mixed SPW Coherence):** Incoherent SPWs scatter uniformly over S^1 and cancel vectorially ($\sum e^{i\theta} \approx 0$), allowing the coherent SPWs to dominate $R(t)$ automatically.
- **Case 4 (Full Decoherence):** The resultant magnitude approaches zero ($|R| \rightarrow 0$), correctly signaling total dispersion while introducing zero state contamination into subsequent integrations.

4.2 Circular Dispersion and Multi-Cadence Table Accumulation

The circular standard deviation is defined analytically from the coherence length $|R| \in [0, 1]$:

$$\sigma_{\text{circ}}(t) = \sqrt{-2 \ln |R(t)|} \quad (\text{rad})$$

For self-calibration sequences with decreasing solution intervals (e.g., ['inf', '60s', 'int', ...]), tables are accumulated multiplicatively using nearest-centroid timestamp matching:

$$G_{\text{accum}}(t) = \prod_{m=0}^{M-1} e^{i\phi_m(t)} \implies \phi_{\text{accum}}(t) = \arg(G_{\text{accum}}(t)) \in [-\pi, \pi]$$

5. Results and Residual Phase Closure Verification

Evaluating the modified phase data across antenna baselines confirms that both approaches share an identical physical result: the total phase correction during solar observation. Fig 1 shows the

comparison using the ALMA test data taken on 2026/07/30. The residual phase closure metric: $\delta\phi(t) = \text{wrap}_{-\pi}(\Delta\phi_{\text{MS}}(t) - \phi_{\text{accum}}(t))$ is almost zero which verifies that the accumulated gain tables accurately compile the exact cumulative phase rotation applied to the calibrated Measurement Set.

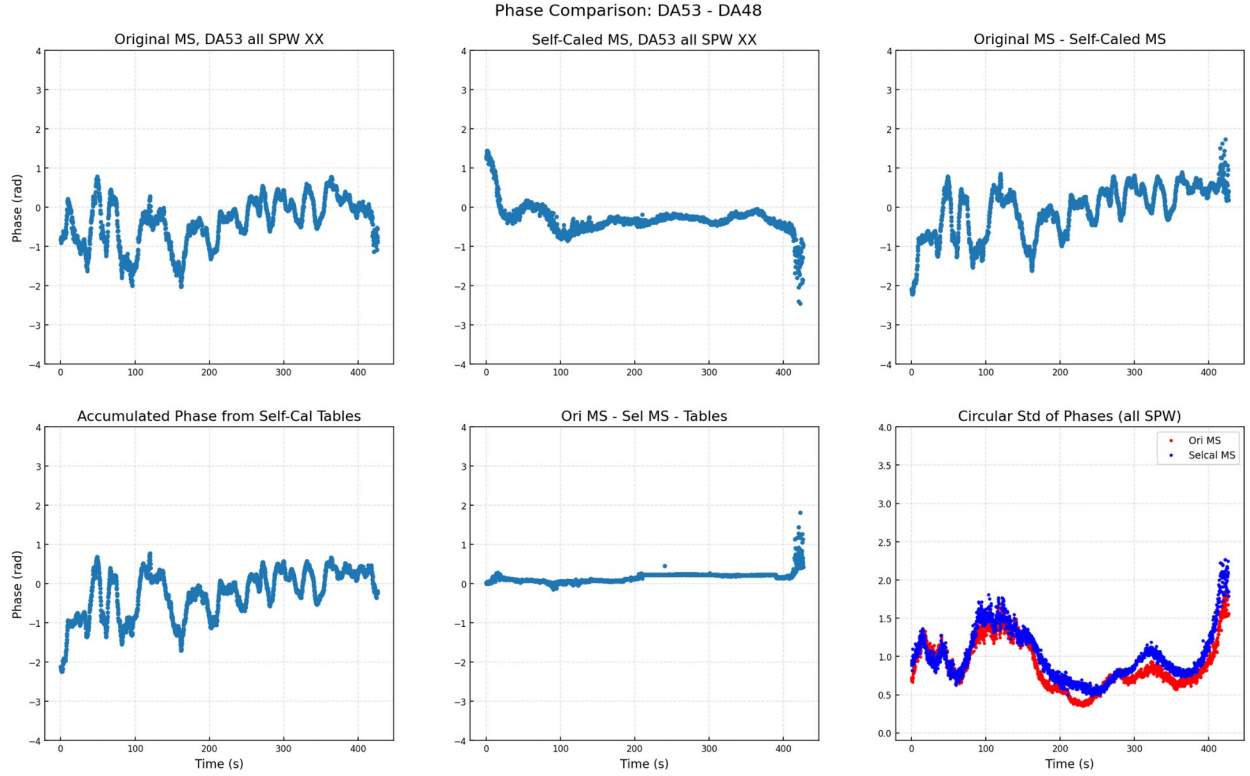


Fig 1. Phase comparison between accumulated phase gain table from self-calibration and phase differential between the unself-calibrated data and the final self-calibrated measurement set. The circular standard deviations of phases at all spectrum windows from both measurement sets are shown by the blue and red curve in the last panel.

6. Conclusion

The phase discrepancies observed between gain tables and visibility differentials were caused by linear phase averaging artifacts over branch cuts. The heuristic phase standard selection, 2π modification, and rollback verification confirmed that the visibility phase difference and the accumulated calibration solutions represent the exact same total phase correction. The directional phasor formulation provides a robust, parameter-free foundation that automates this analysis across all antennas and datasets without heuristic tuning.