

Dear Editor,

Pravec et al. explore asteroid clusters by conducting photometric observations, by numerically examining the encounters of the candidates of asteroid clusters, and by developing a theory of their evolution. First, they conducted Monte-Carlo simulations to search for the encounters of the members in asteroid clusters. Second, they used their photometric observations to determine the rotational period and sizes of the member of asteroid clusters. Then, they analyzed the mass ratios of the members in asteroid cluster to see if their formation is related to rotational fission. Because this paper gives important insights into the evolution of asteroids due to rotational fission, which is a critical issue in planetary science, I recommend this study for publication in Icarus.

I do not have any critical issues with this manuscript, but I found that the manuscript might have to be polished a bit so that readers could understand this study clearly. Some locations might miss descriptions of notations, equations, plots, and so on. Additional explanations help readers understand this important manuscript clearly.

I also found that the authors consider “cohesionless” materials; however, in this argument, they do not use structural analyses (they just extended Pravec et al., 2010). If they want to argue whether or not materials are cohesionless, they have to formulate it in their model. **Please note that recent progress in the literature show that materials must have cohesive strength in the present problem; the authors may realize that because of the current assumption of the initial shape, which is an ellipsoid, the theory regarding the equilibrium shape problem can directly be applied to the present problem.** However, I believe the point of this paper is different from the argument about if materials are cohesionless; therefore, I do not ask them for this point. I simply suggest that the authors remove the statements regarding “cohesionless” from this paper.

Lastly, I have a quick comment on the assumption of the bulk density. The authors assume the bulk density to be 2 g/cm³. I thought this assumption is conservative because the observed spin periods are about 3.6 hr and the derived aspect ratios are about 1.2 – 1.4, according to Section 3. Should it be a lower value?

Here, I conduct a simple analysis to argue that the 2 g/cm³ bulk density may be high to match the observed spin periods of the primaries. The gravity potential of a biaxial ellipsoid in the Cartesian coordinate frame (x, y, z) is given as

$$U = \pi\rho G \left(-A_0 + A_x x^2 + A_y (y^2 + z^2) \right),$$

where ρ is the bulk density, G is the gravity constant, and A_i ($i = x, y$) are the constants. If a small element is resting on the tip of the minimum principal axis, this element will be lofted due to the centrifugal force at a certain spin period. This condition can be obtained by equating the gravity force and centrifugal force acting on this element. The spin period at this condition can be given as

$$T = \frac{1}{3600} \sqrt{\frac{2 \pi}{\rho G A_x}}.$$

Given a bulk density of 2 g/cm³ and an aspect ratio, I derive the spin period at which a small element is about to be lofted (see the dashed line in Figure 1).

To compute the condition at which the biaxial ellipsoid and the small element completely split each other, I have to consider the total energy of the system (as the authors considered in the manuscript). If the element is small enough, the transition between the rotational energy of the biaxial ellipsoid and the orbital energy of the small body is negligible. In this case, it is necessary to consider an additional energy that allows the small element to escape the gravity field of the system. This means that the biaxial ellipsoid might have to spin faster than the condition at which the small element is just about to be lofted. The solid line in Figure 1 describes this condition.

The two lines are obviously faster than the observed rotation periods. To argue that rotational fission occurs at the observed rotation periods, the authors might have to consider the bulk density setting. I suggest that the authors consider the use of the current bulk density assumption or simply describe this discrepancy in the manuscript. I agree that the current spin periods of the primaries would be affected by YORP or small impacts; however, this statement obviously weakens the statements regarding Figure 15 in this manuscript.

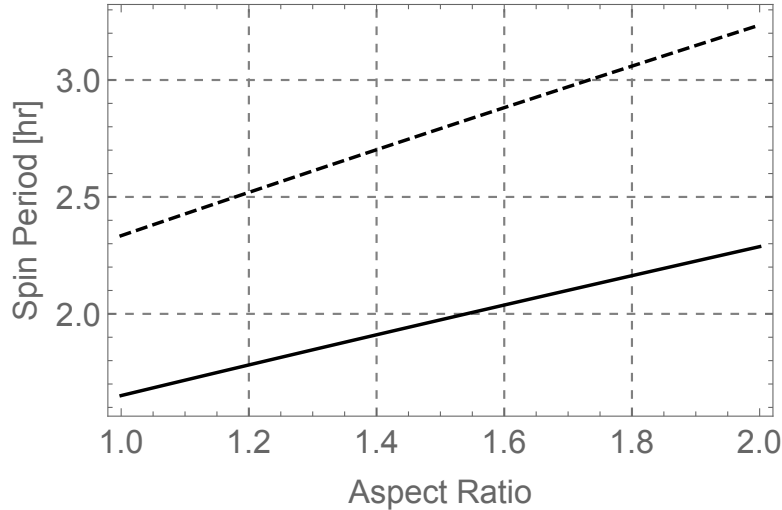


Figure 1. Critical spin period as a function of the aspect ratio of a biaxial ellipsoid under the assumption that the bulk density is 2 g/cm³. The dashed line shows the condition at which a small element resting on the tip along the minimum principal axis is about to lift off. In other words, the element should experience zero force, and thus the dynamical equilibrium point is about to touch the surface of the body. The solid line describes the element resting on the tip

along the minimum principal axis has enough energy to escape the gravity of the biaxial ellipsoid.

Below are minor suggestions (for the main text):

1. In abstract in page 3: I think “(ibid)” should be the year.
2. “where n and $a \dots$ ” after Equation (1) in page 5: “is” should be “are”.
3. Equations, $r \leq 5 - 10 R_{\text{Hill}}$, $v \leq 1 - 2 v_{\text{esc}}$ in page 5: I believe that r and v are normalized. Please describe how to derive them. If these parameters are dimensional, please describes their units.
4. Definition of d_{mean} , Page 5: I recommend that the authors show simple review of what this parameter means. I know that Pravec and Vokrouhlický (2009) also discuss this parameter. However, showing some examples, e.g., what values effectively indicate the encounter, would be helpful for readers to understand the arguments here.
5. The second to the last paragraph in Section 2 in page 6: please define \dot{a}_{max} in the sentence (“It was chosen from the range ...”) in page 5.
6. In Section 2.4: for the two members that did not show orbital convergence, do you think they would depart from other members, instead of the primary?
7. Line 2 in Page 13: please remove either “this” or “2” from “this Section 2”.
8. The sentence after Equation (4) in page 14: “calculate also” should be “also calculate”.
9. ρ_s in Equations (8) and (9) in page 22: please use a different notation to describe the location of a mass element as ρ is already used as the bulk density.
10. $v_{\infty ij}$ in Equation (12) in page 23: please define that this quantity is the relative velocity of element i with respect to element j when r_{ij} becomes infinity.
11. Definition of ω_c , ω , and ω_j in page 23: Please define these spin rates clearly.
12. Equation (13) in page 24: please explain the right-hand side means.
13. “We also note that ...” before Equation (14) in page 24: please remove “that”.
14. Equation (17) in page 25: although this equation is reasonable, I recommend that the authors describe that the initial body is assumed to be rigid, and therefore the distances between the primary and the secondaries do not change before rotational fission.

15. Equation (21) in page 26: please describe that the current study assumes that the initial body is a biaxial ellipsoid, and therefore $b_1 = c_1$. If not, please define c_1 appropriately.
16. General: Please define the terms, primary and secondaries. General readers might be concerned which components are either primary or secondaries.
17. Figures for the histograms of the distribution of encounters: I suggest the authors describe how to this these figures in the captions (at least in Figure 1). For example, for Figure 1, each histogram shows different encounters of the cluster of Irvine. The explanation of the numbers (e.g., 14797, 180233, ...) will make the figure clearer. Also, I did not understand what the numbers of the encounters, which are given in the histogram figures, physically means. I believe the authors conducted Monte-Carlo simulations with uncertainties. In page 5, they mention that they integrated a set of geometric clones (500 clones for each asteroids) with ... Please clarify.

I also read the supplemental material, and it looks fine. But, I have a quick question about Lucascavin (Supplemental Figure 15). The signature of this object looks complex, do you expect this feature results from the shape?