Implications of Cohesion for Binary Asteroid Formation

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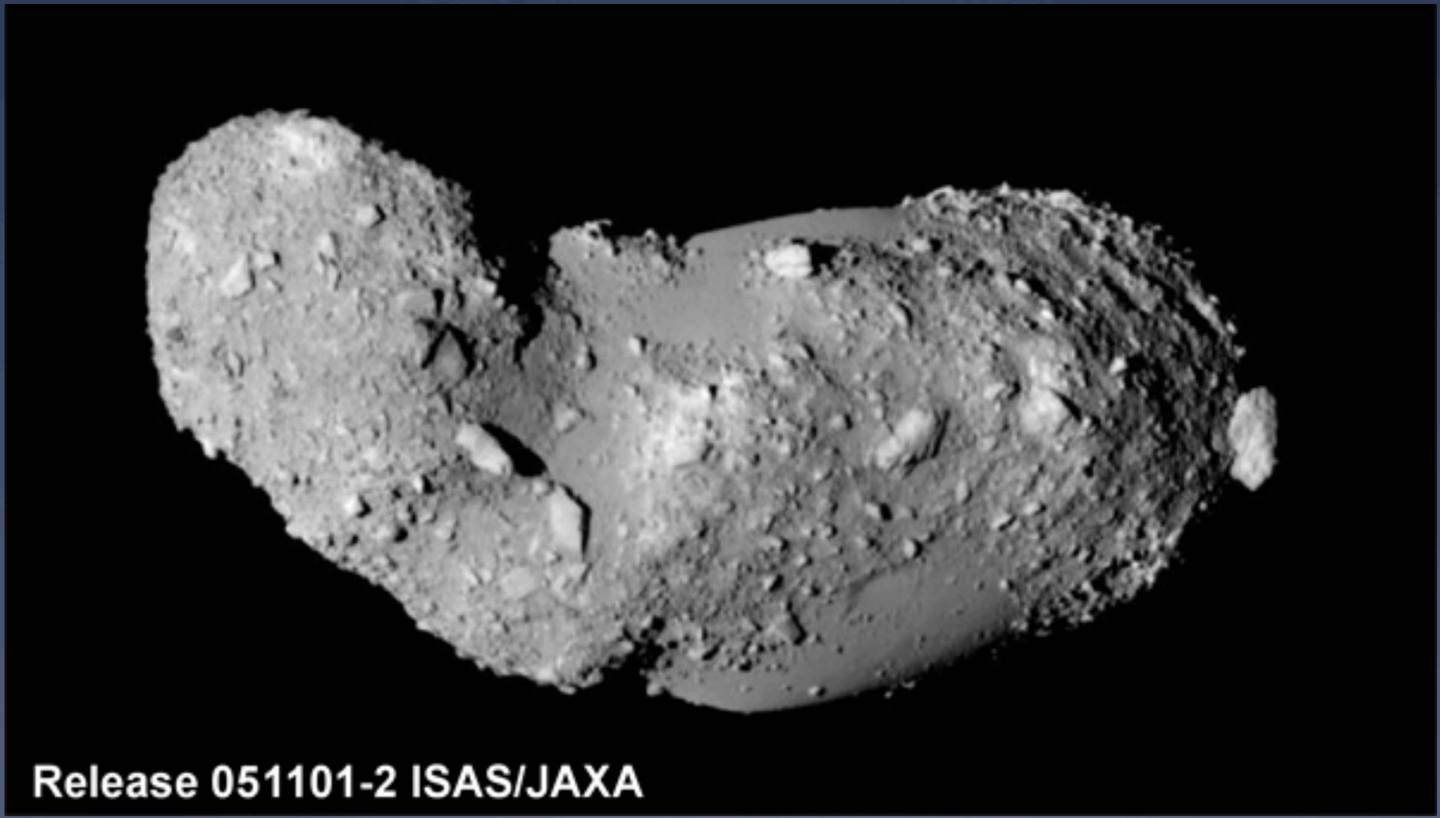
Overview

* Gravitational aggregates
* The code
* Cohesionless models
* Rigid models
* Models with variable cohesion
* The future

Gravitational Aggregates



Gravitational Aggregates

* …are bodies made up of multiple components and having low relative tensile strength (RTS).
  * RTS = (body tensile strength)/(component strength).
  * Zero RTS ➔ rubble pile.
* Gravity dominates over material strength.
* May still have *shear* strength.
  * Ability to hold non-ideal-fluid-equilibrium shape.
* Growing evidence for gravitational aggregates.

Modeling Gravitational Aggregates

* Ingredients:
  * Gravity.
  * Collisions (with adjustable dissipation).
  * Component shape effects (shear strength).
  * Variable *cohesion*.

The Code: `pkdgrav`

- *N*-body code that treats gravity and collisions between spheres (or collections of spheres).
- Solves equations of motion for point masses using second-order leapfrog integrator:

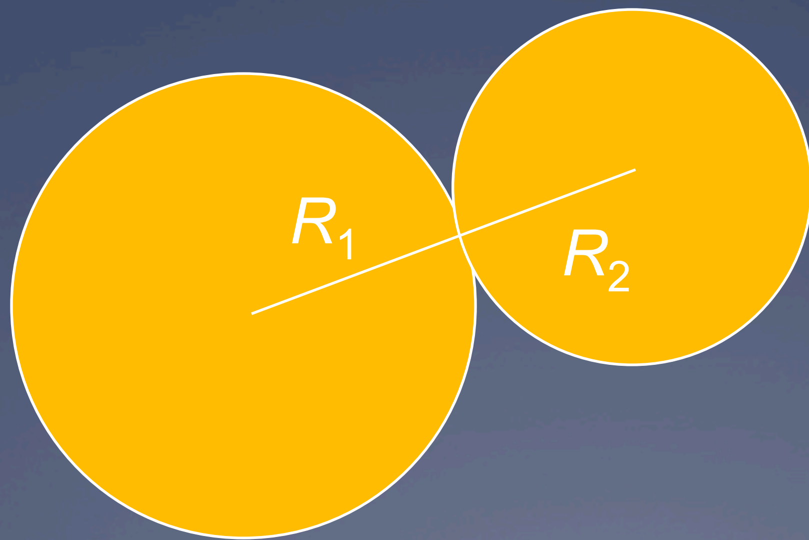
$$\ddot{\mathbf{r}}_i = -\sum_{j \neq i} \frac{G m_j (\mathbf{r}_i - \mathbf{r}_j)}{\left|\mathbf{r}_i - \mathbf{r}_j\right|^3}$$

Code Details

- Based on cosmological code developed by Joachim Stadel and Tom Quinn.
- Uses modified *k*-d tree algorithm (with expansions to hexadecapole) to speed up calculations.
  - Reduces force cost to $O(N \log N)$.
  - Introduces small errors (<< 1%) in force calculation.
- Exploits parallelization to distribute work among available cores.
  - Linear scaling up to hundreds of cores.

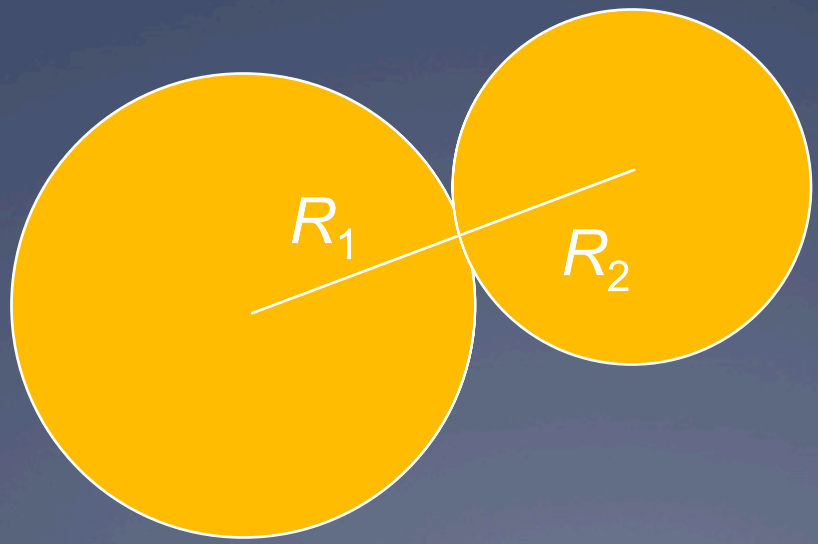
Collision Detection

- Particles collide when separation distance equals sum of radii.
  - Collisions predicted in advance during integration.
  - Uses nearest-neighbor search tree.



Collision Resolution

- Post-impact speed(s) and/or body spin(s) set by sticking/bouncing/splitting rules.
  - Bouncing parameterized by coefficient of restitution (normal & tangential).

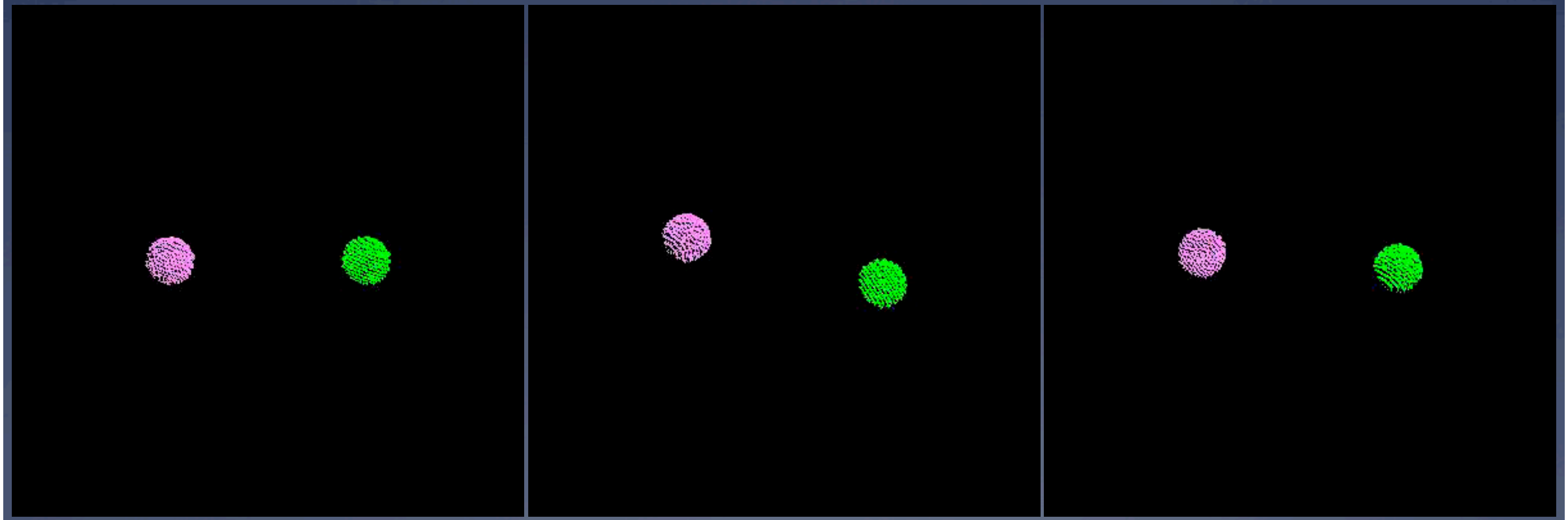


Cohesionless Models

- Idealized rubble piles (perfect, solid spheres; bouncing, no sticking or splitting).
- Many uses. Basic assumption: gravity more important than material properties.

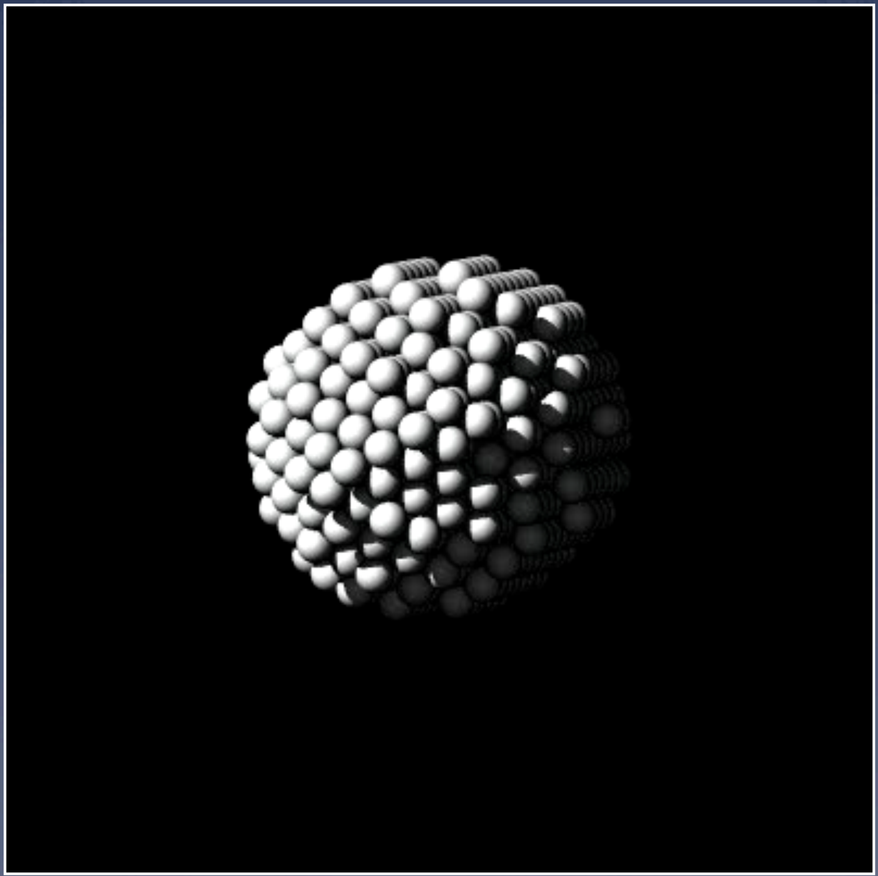
Cohesionless Models

Rubble pile collisions



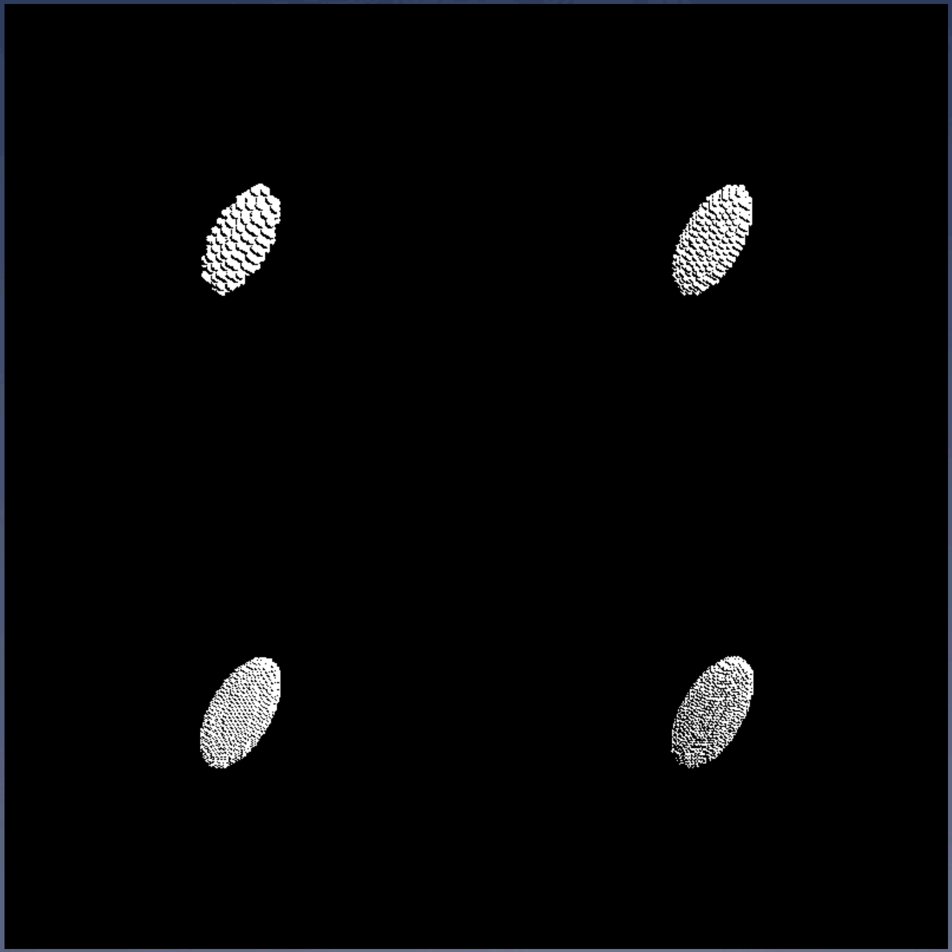
Cohesionless Models

Tidal disruption



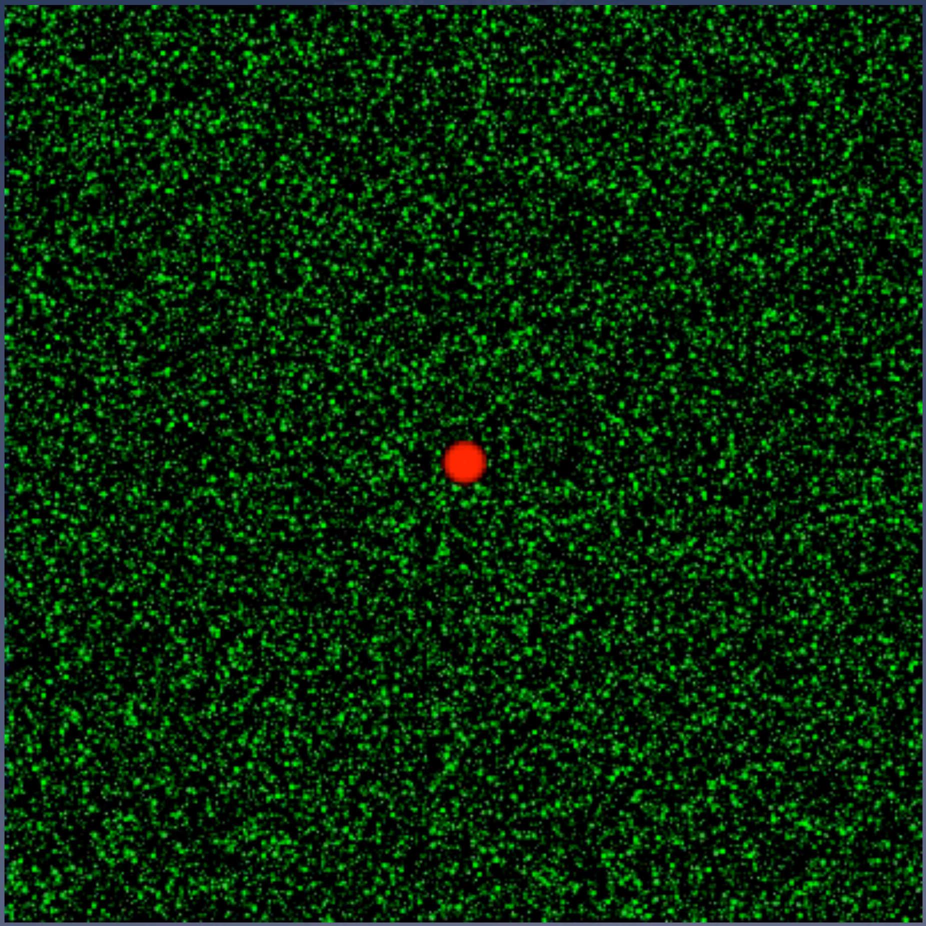
Cohesionless Models

Effect of resolution



Cohesionless Models

Planetary rings

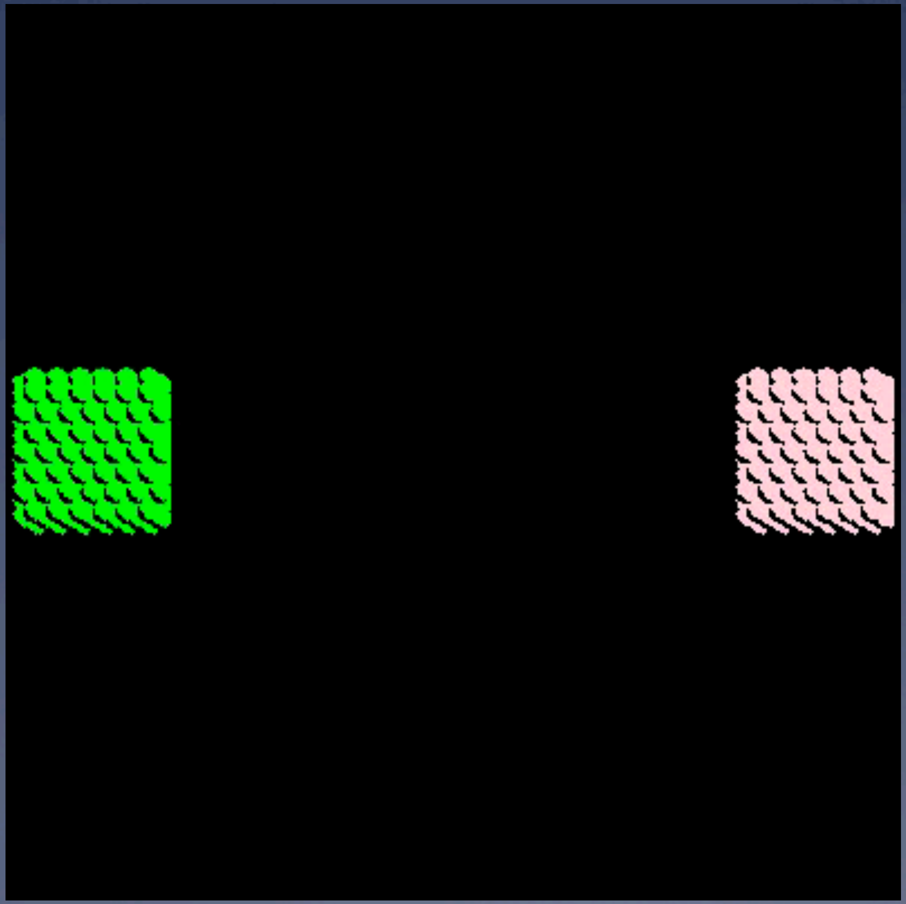


Shear Strength

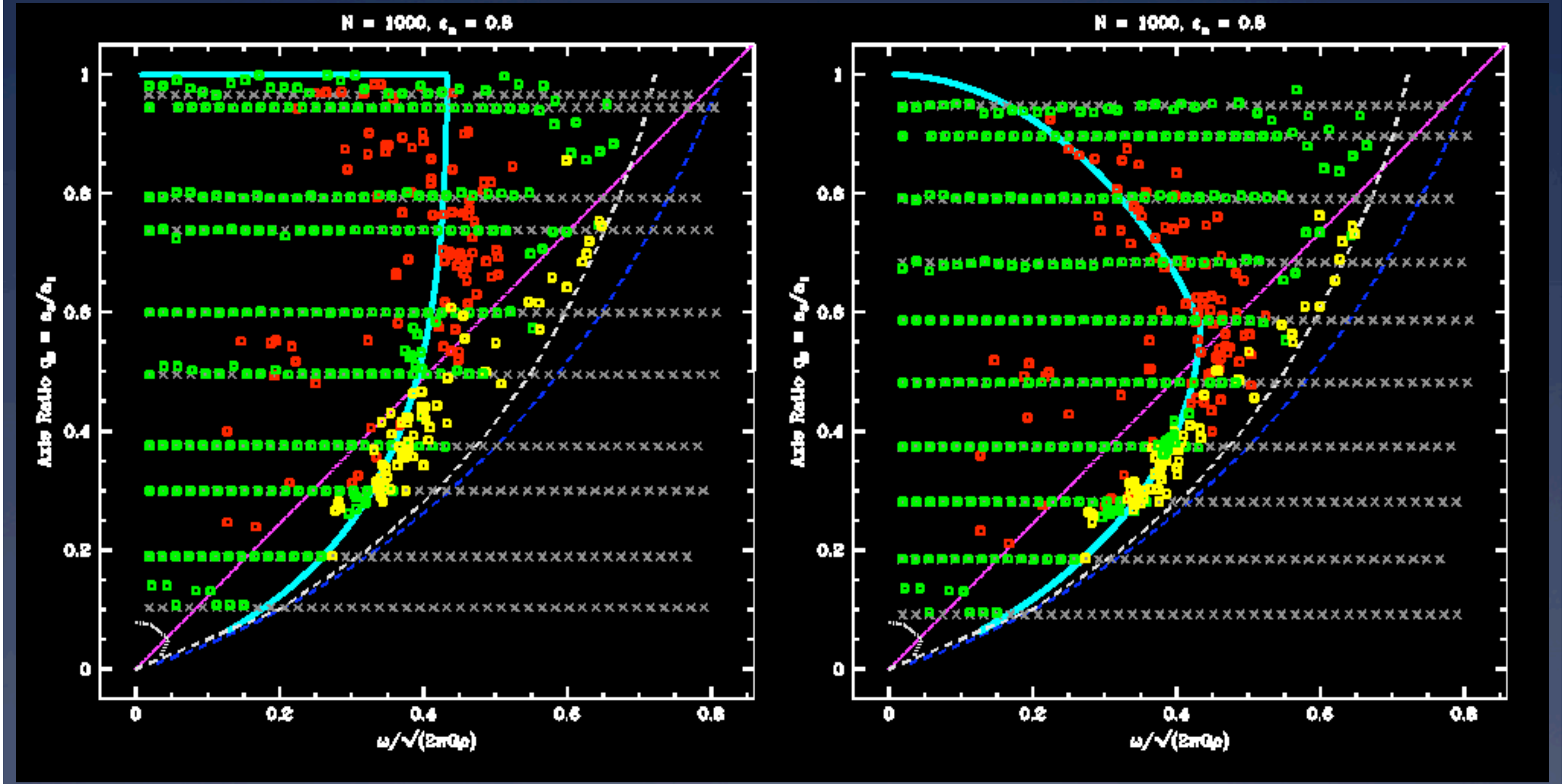
* Rubble piles do not require cohesion to retain non-equilibrium shapes.
* Finite particle effects provide shear strength.

Shear Strength

Rubble cubes colliding

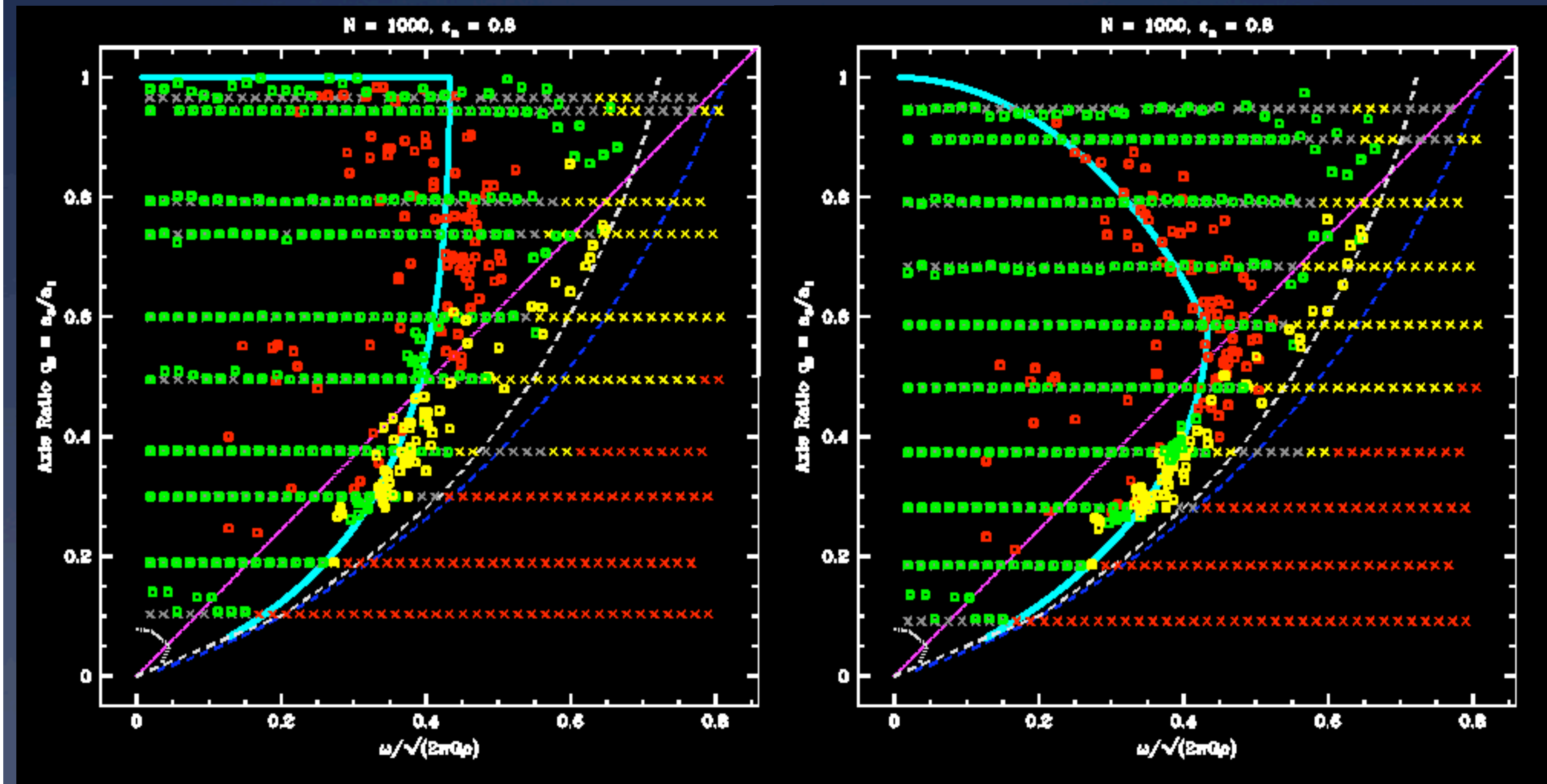


Rubble Pile Equilibrium Shapes



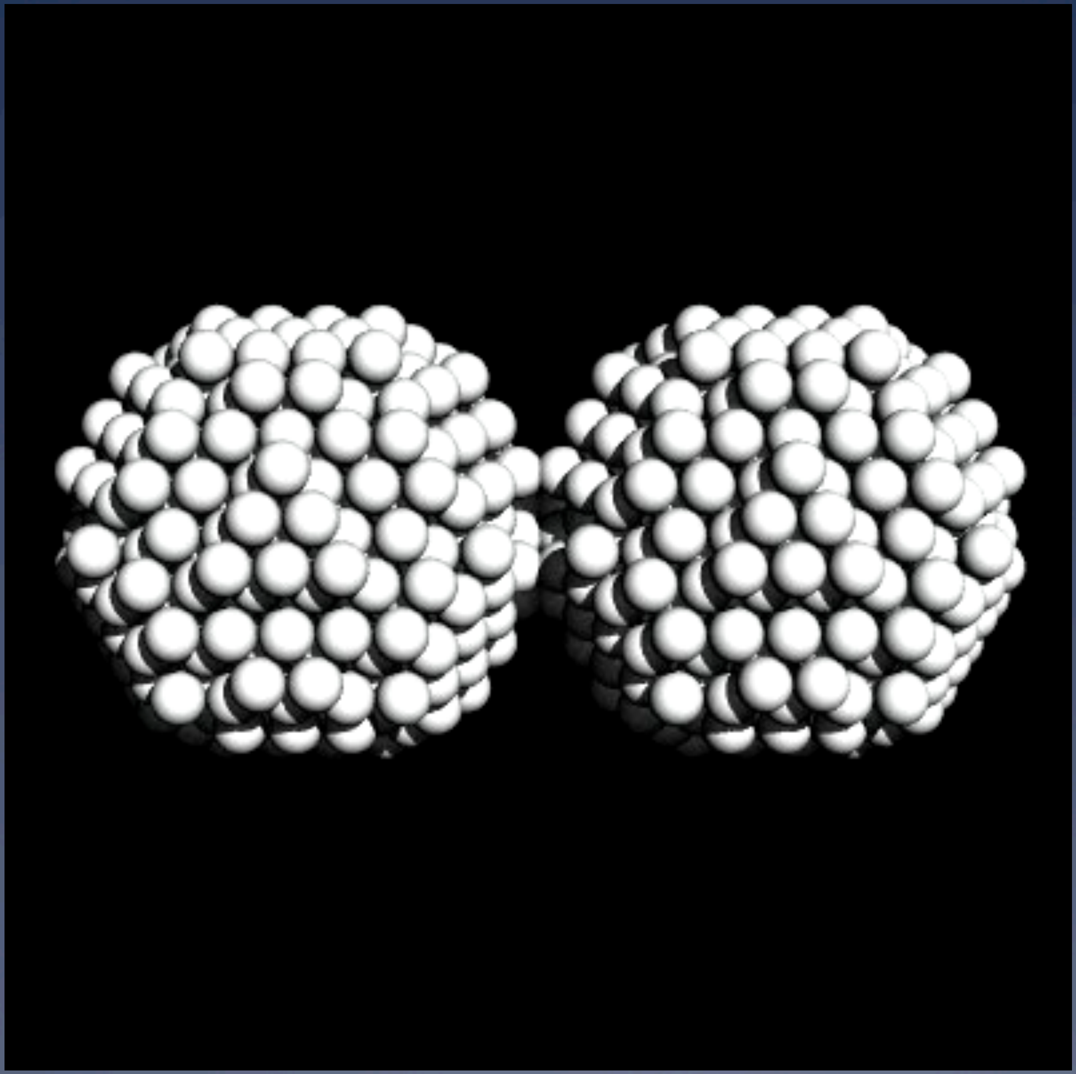
Mass loss: 0% < 10% > 10%   X = initial condition

Rubble Pile Equilibrium Shapes



Mass loss: 0% < 10% > 10%   X = initial condition

Rubble Fission



Rigid Models

- Can "fuse" spheres together to form more complex shapes.
- Either as an initial condition, or as a sticking rule ($v_{impact} < v_{stick}$).
- Need to solve Euler's equations of rigid-body motion with external torques.
  - Use 5th-order (time adaptive) Runge-Kutta.

Rigid Models

$$I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3) = N_1$$
$$I_2\dot{\omega}_2 - \omega_3\omega_1(I_3 - I_1) = N_2$$
$$I_3\dot{\omega}_3 - \omega_1\omega_2(I_1 - I_2) = N_3$$

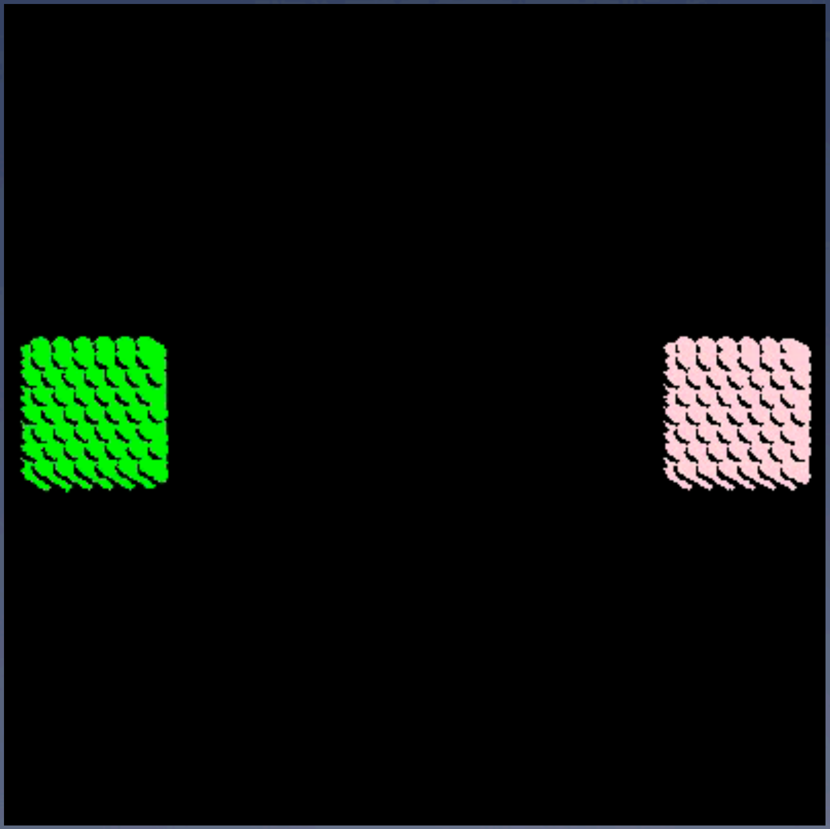
$$\dot{\hat{\mathbf{p}}}_1 = \omega_3\hat{\mathbf{p}}_2 - \omega_2\hat{\mathbf{p}}_3$$
$$\dot{\hat{\mathbf{p}}}_2 = \omega_1\hat{\mathbf{p}}_3 - \omega_3\hat{\mathbf{p}}_1$$
$$\dot{\hat{\mathbf{p}}}_3 = \omega_2\hat{\mathbf{p}}_1 - \omega_1\hat{\mathbf{p}}_2$$

Rigid Models

* Collision detection and resolution considerably more complicated.
* Predict time to collision between spheres on rotating aggregates. Only an approximation.
* Solve outcome using method of generalized coefficients for non-central impacts.
  * Surface friction not supported.

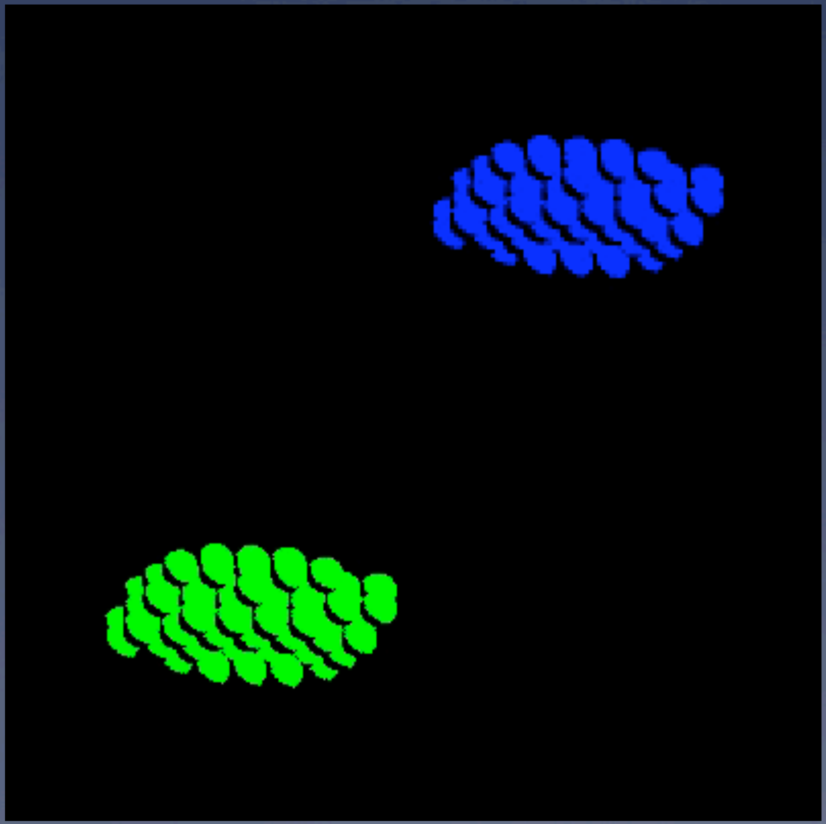
Rigid Models

Rigid cubes colliding



Rigid Models

Rigid bodies torquing



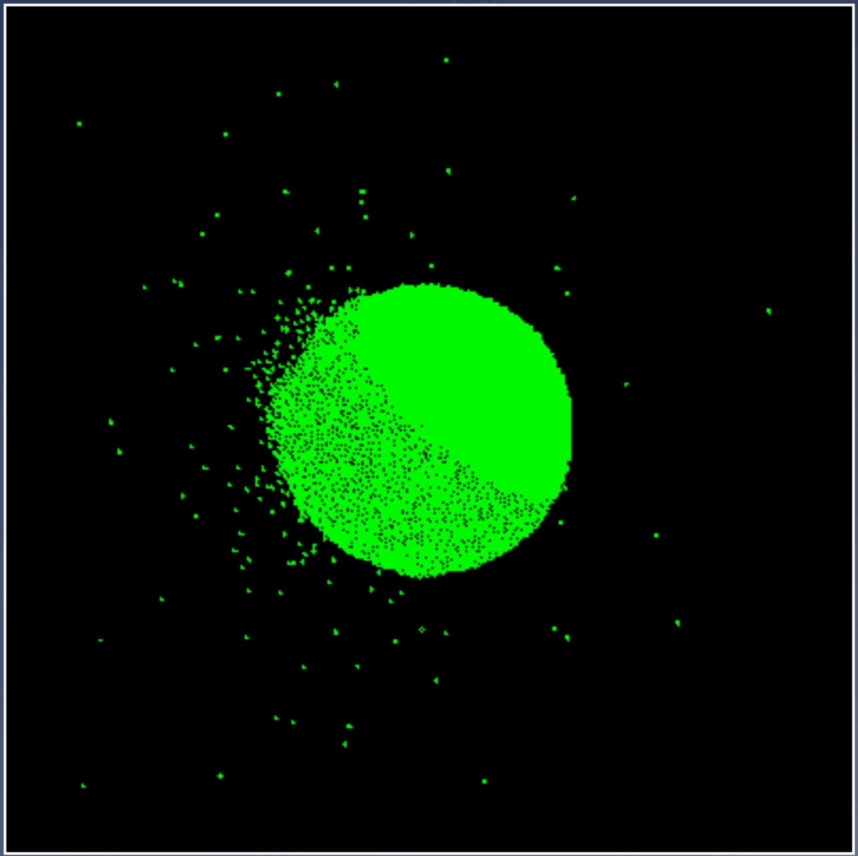
Models With Variable Cohesion

- Specify either…
  1. …size-dependent strength (and optional splitting threshold, $v_{impact} > v_{split}$) with only rigid failure;
  2. or, Young's modulus and maximum strain to simulate elastic failure.

Models With Variable Cohesion

* In case 1 (rigid failure), strength $S \sim R^{\alpha}$, and stress arises from rotation and tides.
* Implementation:
  * Compare acceleration acting on constituent particle relative to center of mass with strength multiplied by $\pi R^2 / m$.
  * Particles experiencing excessive stress are liberated (but may stick again later, if desired).

Models With Variable Cohesion



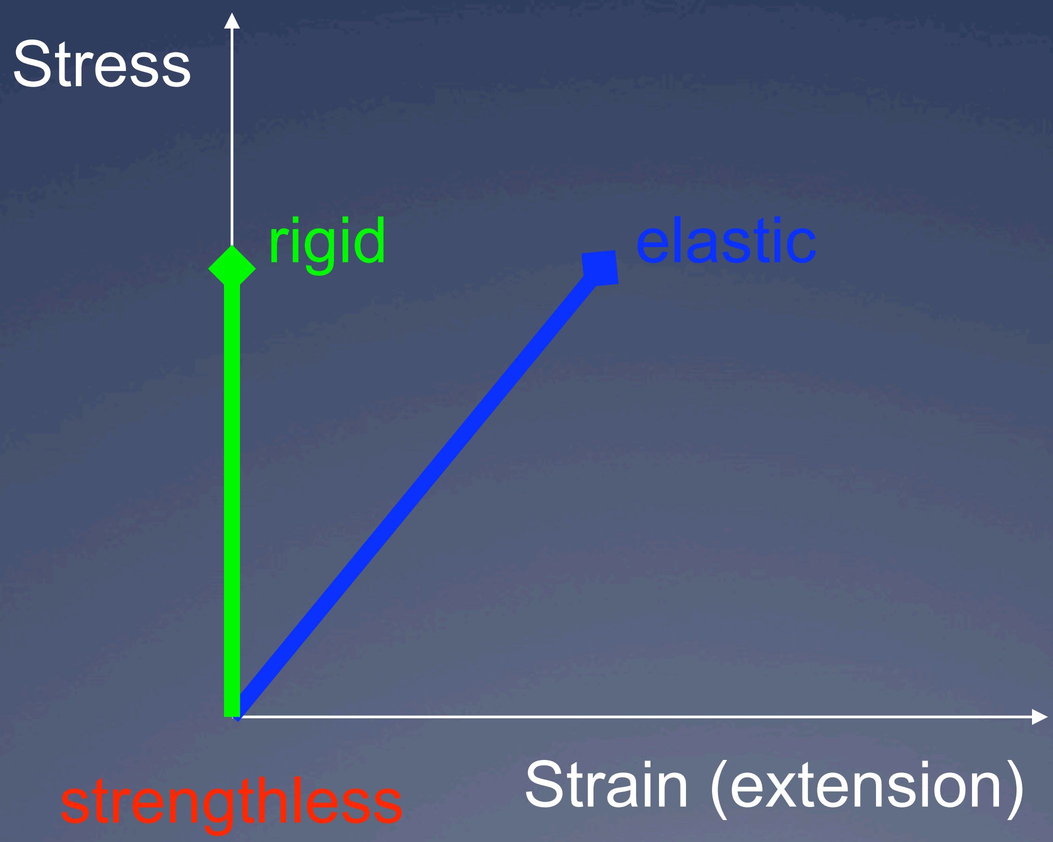
Post-catastrophic disruption gravitational reaccumulation with sticking, bouncing, splitting, and strength.

Models With Variable Cohesion

* In case 2 (elastic failure), particles can move with respect to one another, up to a maximum displacement (strain).

* Implementation:
  * Particles must be free to move, so Euler's equations *not* used.
  * Add restoring force between neighboring particles proportional to strain (= Young's modulus × stress).
  * If maximum strain exceeded, particle permanently liberated (all particles start at close to zero strain).

Stress-strain Curve

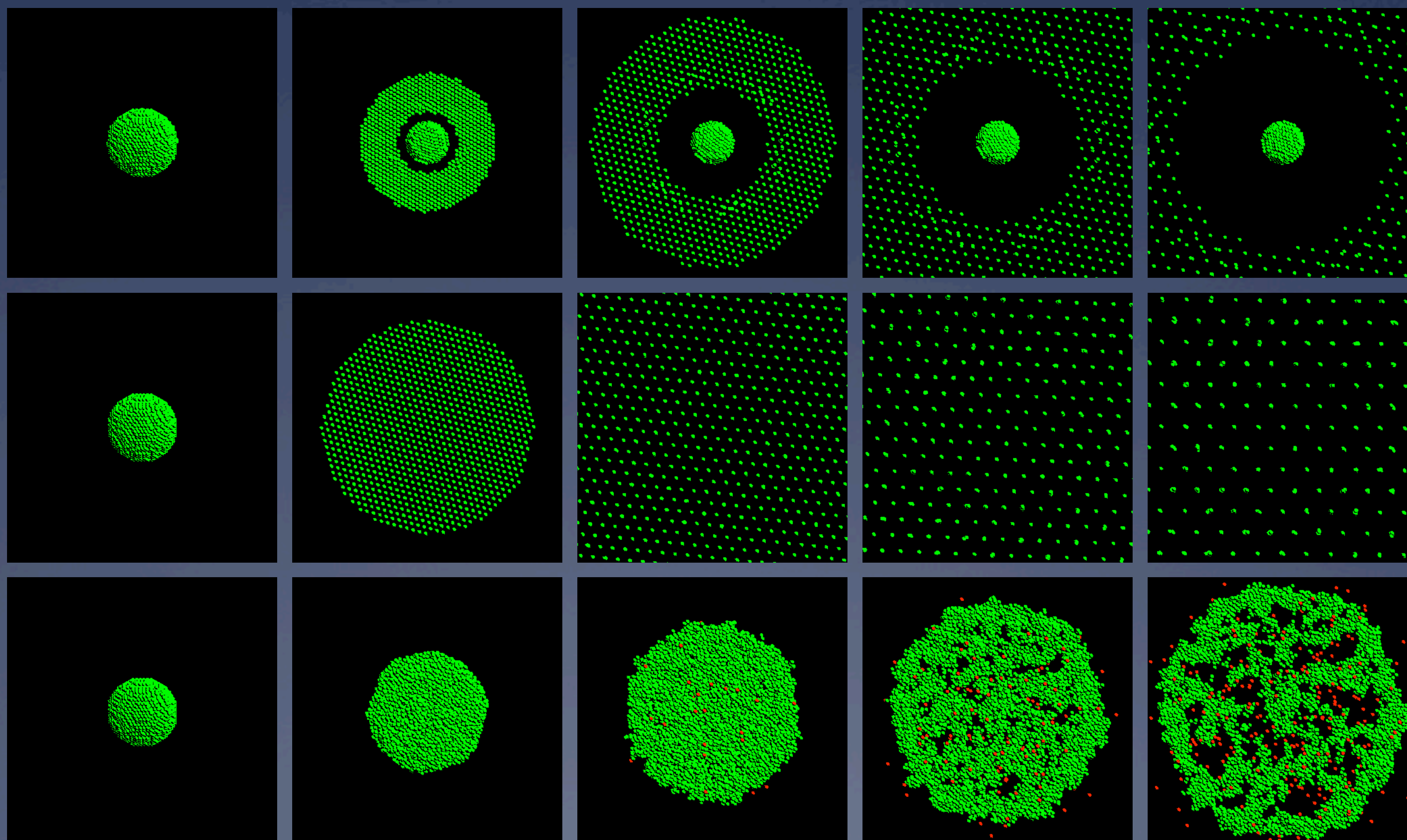


Response to Excessive Spin

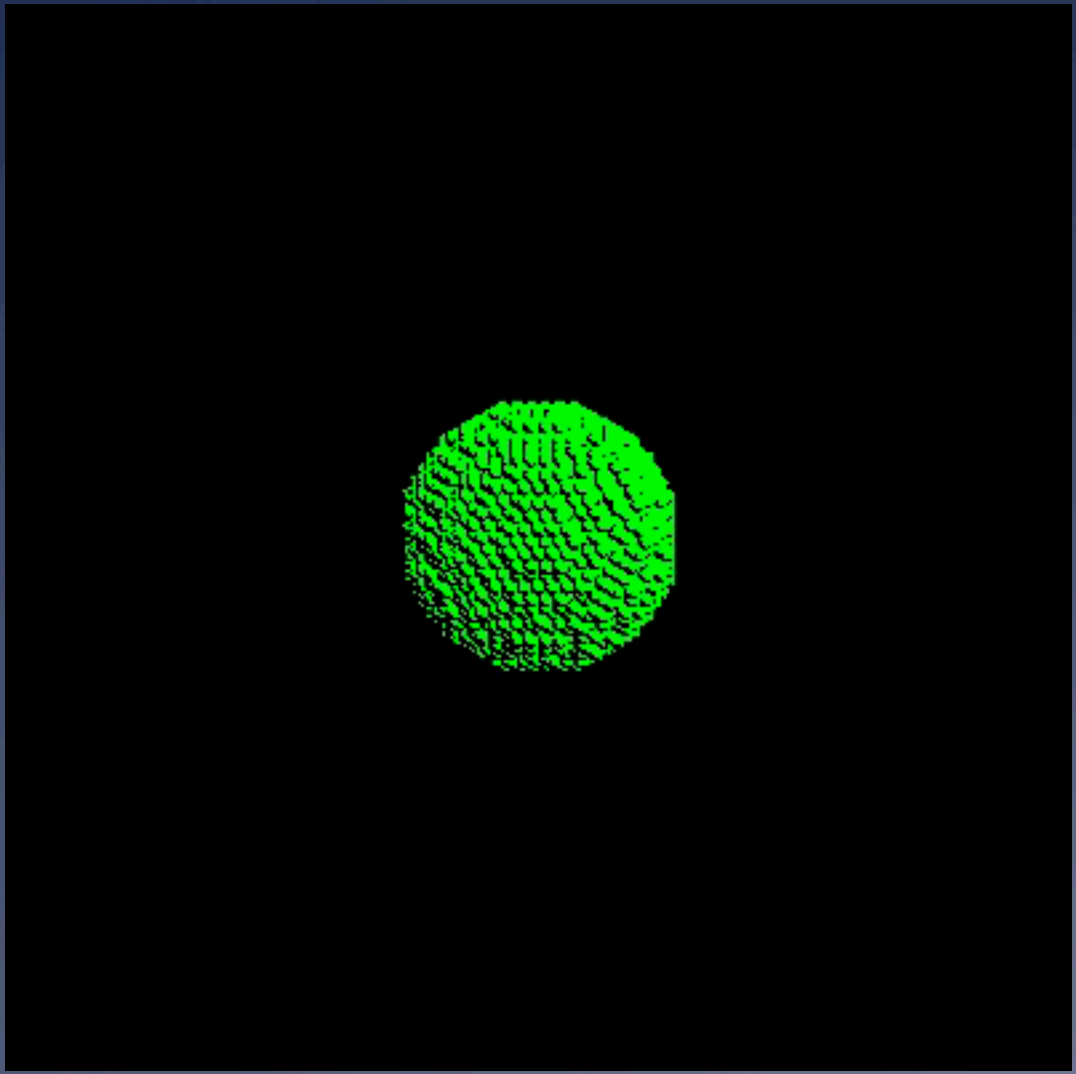
No cohesion

Rigid

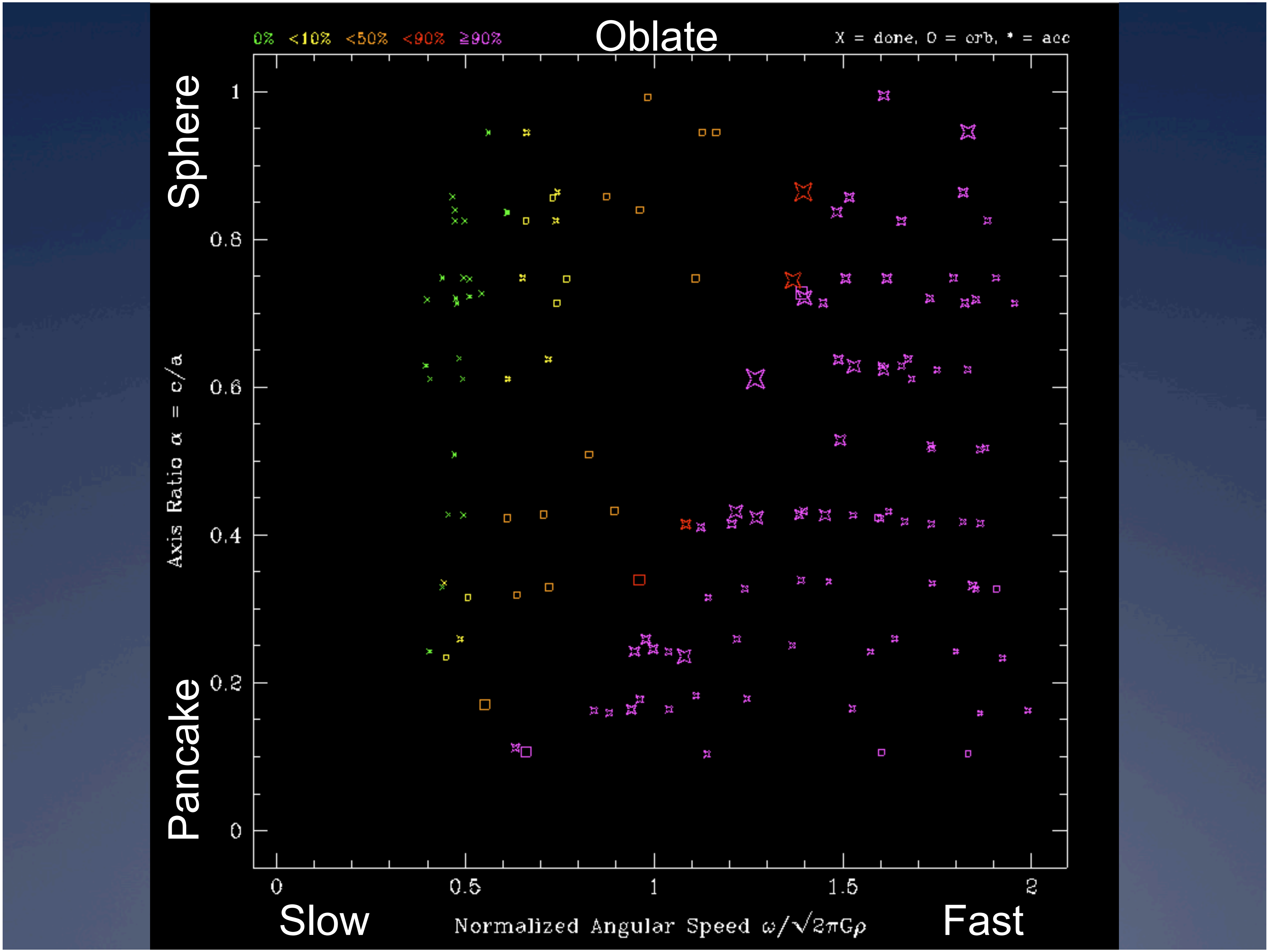
Elastic

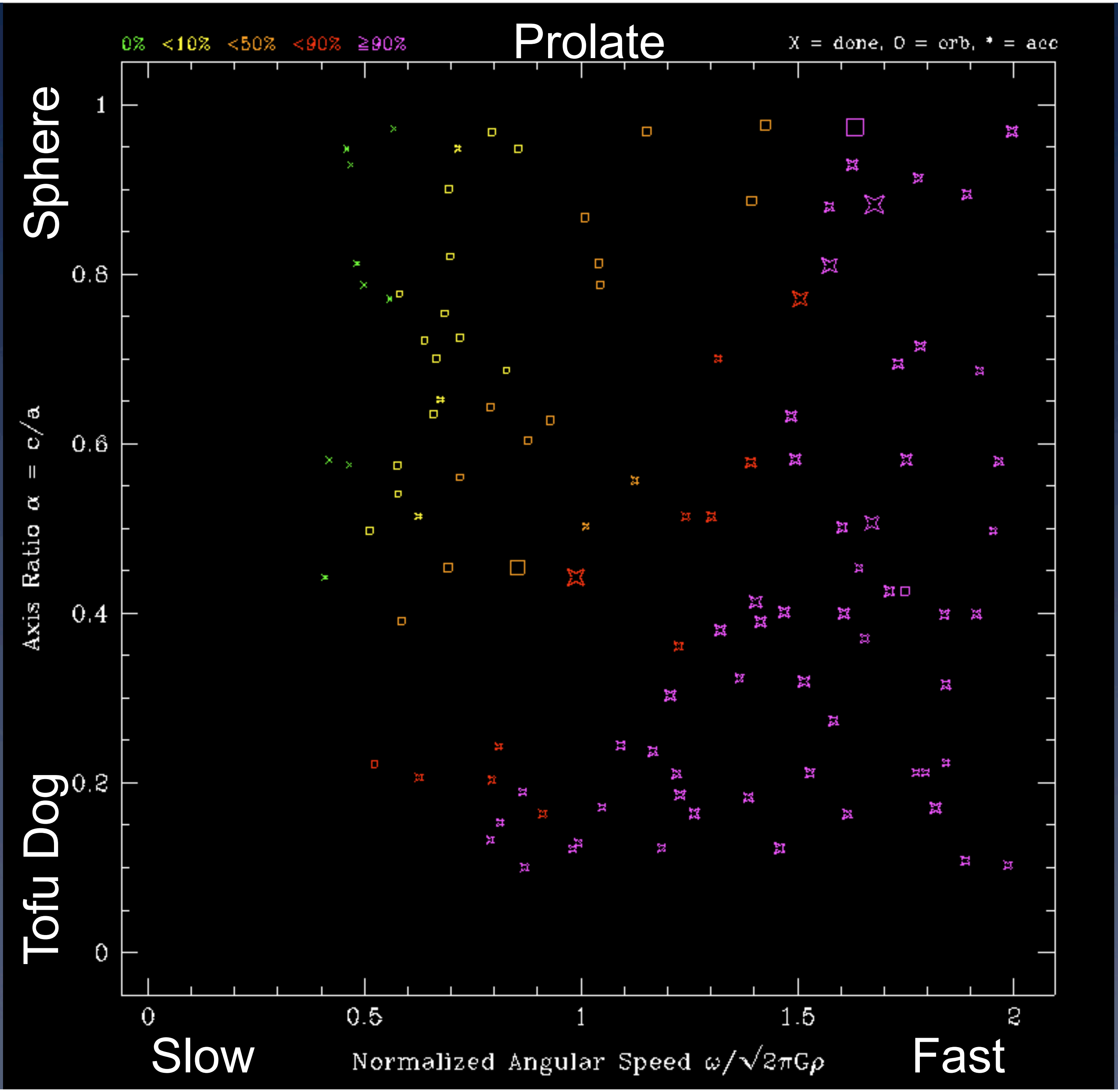


Models With Variable Cohesion



Elastic strain model at very high initial spin.


Oblate
0% <10% <50% <90% ≥90%
X = done, O = orb, * = acc
Sphere
Pancake
Axis Ratio α = c/a
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
Slow
Normalized Angular Speed ω/√2πGρ
Fast


0% <10% <50% <90% ≥90%
Prolate
X = done, O = orb, * = acc
Sphere
Tofu Dog
Axis Ratio α = c/a
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
Slow
Normalized Angular Speed ω/√2πGρ
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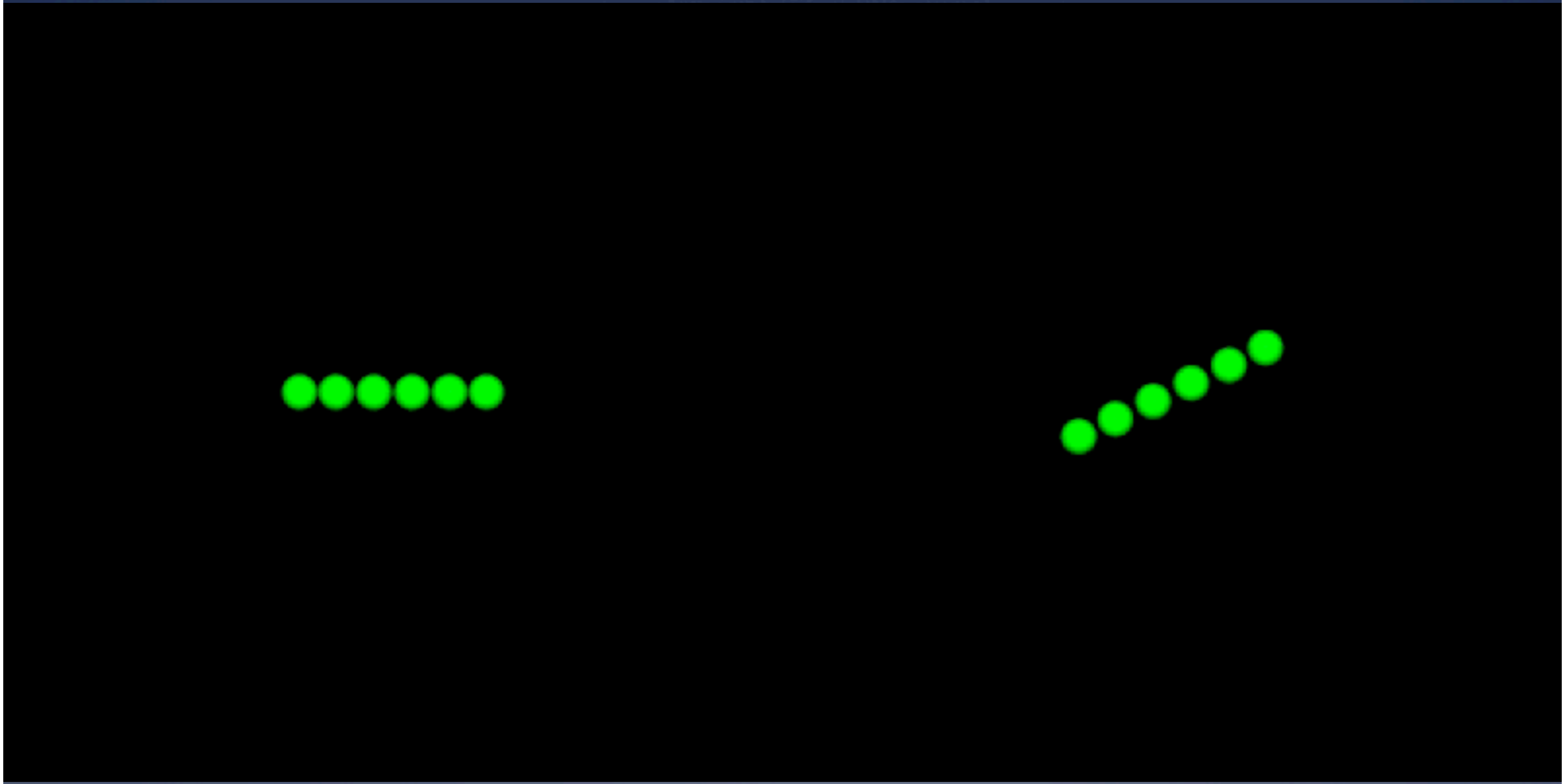
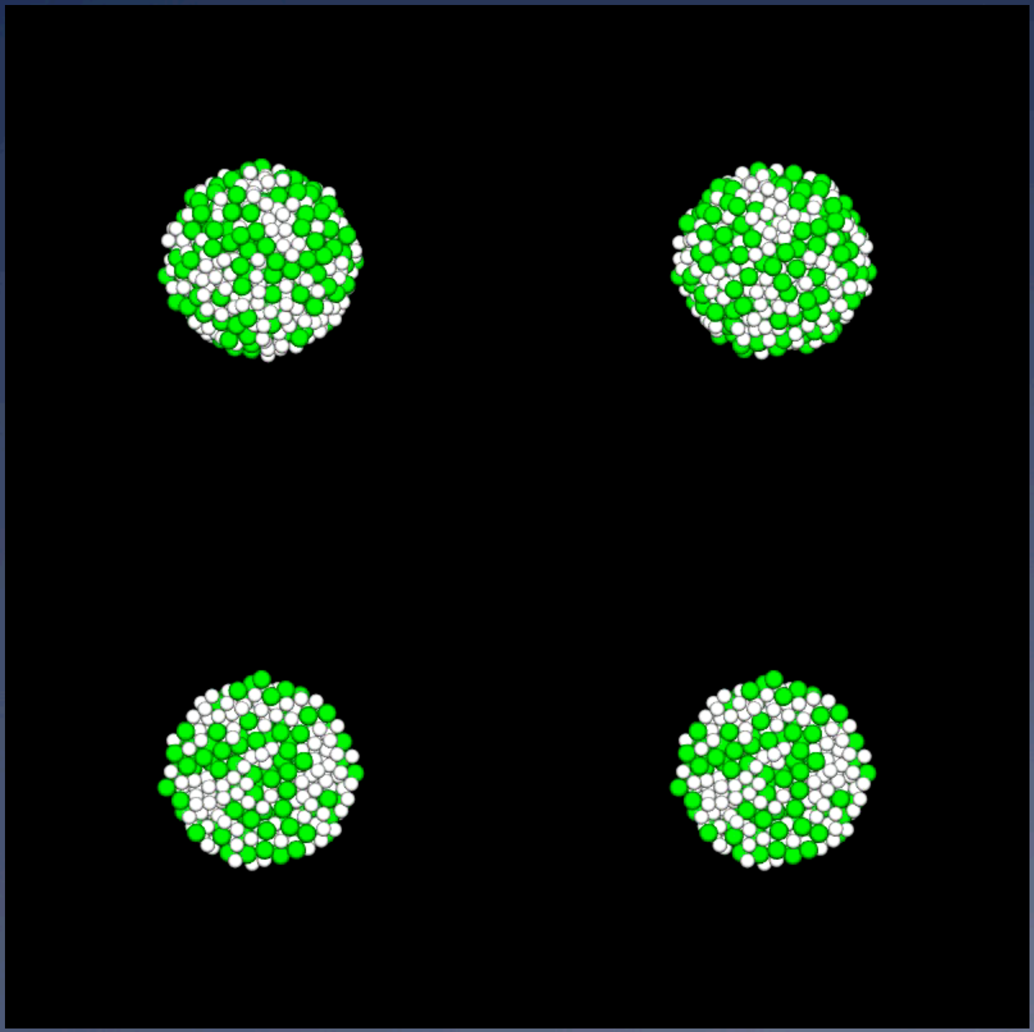
Models With Variable Cohesion



Models With Variable Cohesion



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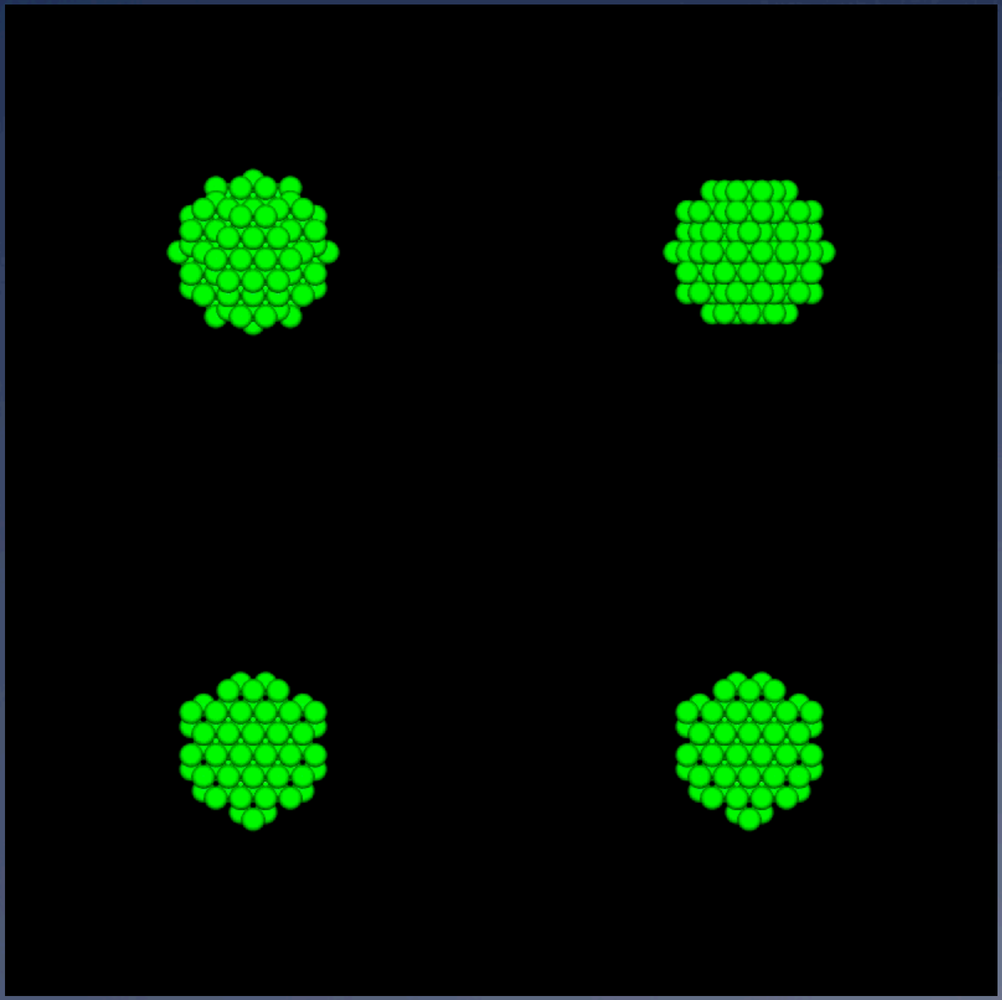

The Future

* Investigate effect of particle size/shape on gravitational aggregate dynamics (see next talk!).
* Compare strength models with Holsapple.
* Express strain relative to initial lattice for elastic models.
* Implement particle memory for modeling weak points, cracks, etc.

Extra Slides…

Van der Waals Force